

## GMAT 数学课程安排

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## 零. 课程安排和题型介绍

### 【课程安排】

第一周：知识点 1.算数和代数基础；2.集合、韦恩图和数列；3. 多项式、方程和坐标轴；4. 质数和因数；5. 余数。请按以下顺序准备：

- ① 学生先自己做完知识点 1/2/3 的题目
- ② 看知识点 1/2/3 的录屏
- ③ 课后,将知识点 1/2/3 的题目再做一次      以上为【录屏】
- ④ 学生自己做完知识点 4/5 的题目
- ⑤ 上直播课,学习知识点 4/5
- ⑥ 课后,将知识点 4/5 的题目再做一次      以上为【直播】

第二周：知识点 6.几何和应用题；7. 统计和排列组合基础；8. 排列组合和概率以及 9.答疑。请按以下顺序准备：

- ① 学生先自己做完知识点 6/7 的题目
- ② 看知识点 6/7 的录屏
- ③ 课后,将知识点 6/7 的题目再做一次      以上为【录屏】
- ④ 如果做过 OG,可以将 OG 中需要讲解的题号提前发给助教,答疑课讲解；如果没有做过 OG,由老师自行选择题目讲解
- ⑤ 学生自己做完知识点 8 的题目
- ⑥ 上直播课,学习知识点 8 以及 OG 中的难题
- ⑦ 课后,将知识点 8 的题目和 OG 难题再做一次      以上为【直播】

### 【题型介绍】

GMAT 分为两种题型,都是选择题(五选一)：

1. PS 题：类似高考数学题,需要考生算出答案(本练习册中部分题目选项省去,学生直接算出答案即可)
2. DS 题：在题干已知信息的基础上,加上条件 1 或/和条件 2,是否可以回答题干中提出的问题,如果可以则称为充分 sufficient；如果不能则称为不充分 insufficient. 所有 DS 题选项固定如下：  
A. Statement (1) **ALONE** is sufficient, but statement (2) alone is not sufficient.  
B. Statement (2) **ALONE** is sufficient, but statement (1) alone is not sufficient.  
C. **BOTH** statements **TOGETHER** are sufficient, but **NEITHER** statement **ALONE** is sufficient.  
D. **EACH** statement **ALONE** is sufficient.  
E. Statements (1) and (2) **TOGETHER** are **NOT** sufficient.  
(以下练习册中 DS 题选项全部省略)

### 例题

1.  $x$  and  $y$  are integers, whether  $x > y$ ?

(1)  $x = 2$

(2)  $y$  是正整数

先只看条件 1：只知道  $x$  的值,无法判断  $xy$  的大小关系,不充分

再只看条件 2：只知道  $y$  是正整数,无法判断  $xy$  的大小关系,不充分

再看条件 1&2： $x$  已知,但是  $y$  可以取不同的值(例如：1 或 3),无法判断  $xy$  的大小关系,不充分

选择 E

2.  $x$  and  $y$  are integers, whether  $x > y$ ?

(1)  $x = 2$

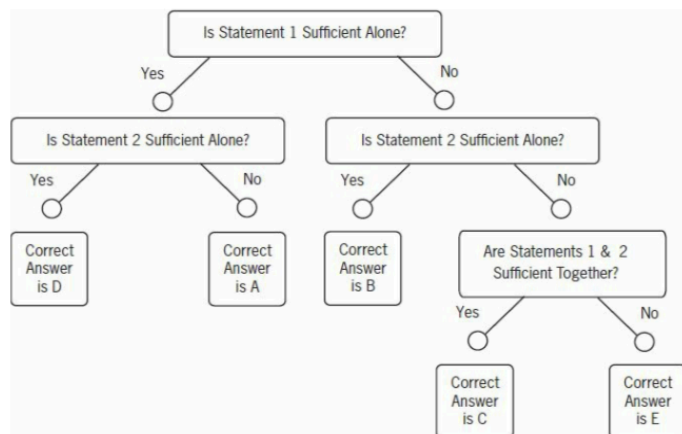
(2)  $y = 4$

条件 1 或者条件 2 单独都不行

条件 1&2 : 可以推出  $x < y$  , 所以  $x > y$  不成立 , 答案是 no , 充分

选择 C

思考过程



## 一. 【录屏】算数和代数基础

1.1 If  $n$  and  $k$  are positive integers, is  $n/k$  an even integer?

- (1)  $n$  is divisible by 8.
- (2)  $k$  is divisible by 4.

1.2 If  $x$  is an integer, which of the following must be an odd integer?

- A.  $2x + 2$
- B.  $4x + 3$
- C.  $12x - 6$
- D.  $13x$
- E.  $14x$

1.3 已知  $n$  是正整数, 以下条件能否判断  $n$  是偶数?

- (1) 前  $n$  个连续正整数的和是偶数
- (2) 前  $2n$  个连续正整数的和是偶数

2.1 If  $x$  and  $y$  are positive, is  $4x > 3y$ ?

- (1)  $x > y - x$
- (2)  $x/y < 1$

2.2 If  $w$  and  $c$  are integers, is  $w > 0$ ?

- (1)  $w + c > 50$
- (2)  $c > 48$

2.3 Is  $xy > x/y$ ?

- (1)  $xy > 0$
- (2)  $y < 0$

2.4 已知  $a$  的取值范围是  $(19, 31)$ ,  $b$  的取值范围是  $(29, 41)$ ,  $a/b$  的取值范围是多少?

2.5 已知正整数  $0 < x < y < z < 10$  且  $x+y+z$  是小于 12 的奇数, 以下条件能否判断  $x=1$ ?

- (1)  $x, y, z$  之中任意两个数的乘积是奇数
- (2)  $x, y$  和  $z$  中有一个数是另一个数的 3 倍

2.6 已知  $x > 0$  且  $0.6x < y < 0.8x$ , 以下条件能否判断  $x-y < 20$ ?

- (1)  $x > 50$
- (2)  $x < 100$

2.7 已知实数  $x, y$  和  $z$  满足  $xyz > 0$ , 以下条件能否判断  $xy^2z^3 > 0$ ?

- (1)  $x > 0$

(2)  $y > 0$

2.8 已知实数  $x, y$  和  $z$  满足  $xyz > 0$ , 以下条件能否判断  $xy^2z^3 > 0$

(1)  $xz > 0$

(2)  $yz > 0$

3.1 If  $|2x+3| < 5$ , what's the possible value of  $x$ ?

3.2 已知实数  $r$  和  $s$ , 以下条件能否判断  $r < s$ ?

(1)  $|r| \leq s$

(2)  $|s| \leq r$

3.3 已知  $x$  是实数, 以下条件能否判断  $|x+1| < |x+5|$

(1)  $x < 3$

(2)  $x > -3$

3.4 已知实数  $r$  和  $t$ , 以下条件能否判断  $|r-t| > ||r|-|t||$

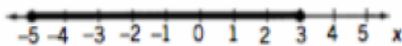
(1)  $rt < 0$

(2)  $r-t < 0$

4.1 The number line shown contains three points R, S and T, whose coordinates have absolute values  $r$ ,  $s$  and  $t$ , respectively. What is the average (arithmetic mean) of the coordinates of the points R, S and T?



4.2 what is the algebraic expression for the shaded part of the number line below (hint: inequality)?



4.3  $|x+1| + |x-2| > 3$ ?

(1)  $x > -2$

(2)  $x < 2$

4.4 On the number line, point R has coordinate  $r$  and point T has coordinate  $t$ . Is  $t < 0$ ?

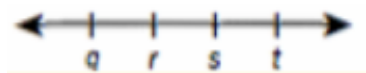
(1)  $-1 < r < 0$

(2) The distance between R and T is equal to  $r^2$

4.5 Of the four numbers represented on the number line above, is  $r$  closest to zero?

(1)  $q = -s$

(2)  $-t < q$



4.6 If  $a < x < b$  and  $c < y < d$ , is  $x < y$ ?

(1)  $a < c$

(2)  $b < c$

5.1 If  $x$  and  $k$  are integers and  $(12^x)(4^{2x+1}) = (2^k)(3^2)$ , what is the value of  $k$ ?

5.2 If  $-1 < h < 0$ , which of the following has the greatest value?

A.  $1 - h$

B.  $1 + h$

C.  $1 + h^2$

D.  $1 - 1/h$

E.  $1 - 1/h^2$

5.3 If  $n = 8^{11} - 8$ , what is the units digit of  $n$ ?

5.4 If  $10^{50} - 74$  is written as an integer in base 10 notation, what is the sum of the digits in that integer?

5.5 What is the value of  $[1/(\sqrt{3} - \sqrt{2})]^2$ ?

Answer

1.1	E	3.3	B
1.2	B	3.4	A
1.3	B	4.1	$(s+t-r)/3$
2.1	E	4.2	$ x+1  <= 4$
2.2	E	4.3	E
2.3	E	4.4	C
2.4	$(19/41, 31/29)$	4.5	A
2.5	A	4.6	B
2.6	E	5.1	14
2.7	B	5.2	D
2.8	A	5.3	4
3.1	$-4 < X < 1$	5.4	440
3.2	B	5.5	$5+2\sqrt{6}$

## 二. 【录屏】集合、韦恩图和数列

1.1 A set of numbers has the property that for any number  $t$  in the set,  $t + 2$  is in the set. If  $-1$  is in the set, which of the following must also be in the set(可以多选)?

- a.  $-3$     b.  $1$     c.  $5$

1.2 现在定义  $A \# B$  代表在  $A \cup B$  中, 但是不在  $A \cap B$  中的原色组成的集合。现在已知  $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ ; 问: 以下条件能否判断  $B$  集合是什么?

- (1)  $A = \{1, 2, 3, 5, 7\}$   
(2)  $A \# B = \{1, 2, 4, 5, 6\}$

2.1 On a certain transatlantic crossing, 20 percent of a ship's passengers held round-trip tickets and also took their cars abroad the ship. If 60 percent of the passengers with round-trip tickets did not take their cars abroad the ship, what percent of the ship's passengers held round-trip tickets?

2.2 Of the students in a certain school, 15 percent are enrolled in an art class and 10 percent are enrolled in a music class. What percent of the students in the school are enrolled in neither an art class nor a music class?

- (1)  $\frac{2}{3}$  of the students who are enrolled in an art class are also enrolled in a music class.  
(2) There are more than 100 students in the school.

3.1 Last year 26 members of certain club traveled to England, 26 members traveled to France, and 32 members traveled to Italy. Last year no members of the club traveled to both England and France, 6 members traveled to both England and Italy, and 11 members traveled to both France and Italy. How many members of the club traveled to at least one of these three countries last year?

3.2 有 53 个人参加一个聚会, 每个人至少选三个食物中的一个: pizza, cheese, hamburger。问三种食物都吃的人有多少?

- (1) 没有人只吃了 pizza, 并且只吃 cheese 的人有 9 个  
(2) 没有人吃了两种食物, 并且只吃了 hamburger 的人有 7 个

4. The table below shows the results of a survey of 100 voters each responded "favorable" or "unfavorable" or "not sure" when asked about their impressions of candidate M and of candidate N. What was the number of voters who responded "favorable" for both candidates?

- (1) The number of voters who did not respond "favorable" for either candidate was 40

(2) The number of voters who responded "unfavorable" for both candidates was 10

	Favorable	Unfavorable	Not Sure
Candidate <i>M</i>	40	20	40
Candidate <i>N</i>	30	35	35

5. Every object in a box is either a sphere or a cube, and every object in the box is either red or green. How many objects are in the box?

- (1) There are six cubes and five green objects in the box.
- (2) There are two red spheres in the box.

6.1 Of the 1,400 college teachers surveyed, 42 percent said that they considered engaging in research an essential goal. How many of the college teachers surveyed were women?

- (1) In the survey, 36 percent of the men and 50 percent of the women said that they considered engaging in research an essential goal.
- (2) In the survey, 288 men said that they considered engaging in research an essential goal.

6.2 Are at least 10 percent of the people in Country X who are 65 years old or older employed?

- (1) In Country X, 11.3 percent of the population is 65 years old or older.
- (2) In Country X, of the population 65 years old or older, 20 percent of the men and 10 percent of the women are employed.

7.1 If  $r$  is a constant and  $a_n = r \cdot n$  for all positive integers  $n$ , for how many values of  $n$  is  $a_n < 100$ ?

- (1)  $a_{50} = 500$
- (2)  $a_{100} + a_{105} = 2,050$

7.2 What is the sum of the integers from -190 to 195, inclusive?

8.1 In the sequence of nonzero numbers  $t_1, t_2, t_3, \dots, t_n, \dots, t_{n+1} = t_n / 2$  for all positive integers  $n$ . What is the value of  $t_5$ ?

- (1)  $t_3 = 1/4$
- (2)  $t_1 - t_5 = 15/16$

8.2 In the sequence 1, 2, 4, 8, 16, 32, ..., each term after the first is twice the previous term. What is the sum of the 16th, 17th, and 18th terms in the sequence?

9. If the sequence  $x_1, x_2, x_3, \dots, x_n, \dots$  is such that  $x_1 = 3$  and



$x_{n+1} = 3x_n - 2$  for  $n \geq 1$ , then  $x_{20} - x_{19} =$

10.1 对任意的  $n>2$ ,  $a_n=a_{n-1}-a_{n-2}$  ,  $a_1=-1$  ,  $a_2=1$  , 求前 1000 项的和

10.2 一列数列  $S_1, S_2, S_3.....$ , 其中  $S_1=\{1\}$ ,  $S_2=\{2\ 3\ 4\}$ ,  $S_3=\{5\ 6\ 7\ 8\ 9\}$ ,  $S_{n+1}$  中数的个数等于  $S_n$  的个数加 2 , 里面全部是连续整数 , 且后一个数列的第一个数是前一个数列最大的数+1。求  $S_{101}$  中最小的数是几？

10.3 一个数列  $4*10^n, 4*10^{n-1}, .....$ ,  $4*10^{n-m}$ ,  $m$  和  $n$  均为正整数 , 这个数列的平均值是否是整数？  
 (1)  $m<6$   
 (2)  $n=12$

Answer

1.1	b.c	6.2	B
1.2	C	7.1	D
2.1	50%	7.2	965
2.2	A	8.1	D
3.1	67	8.2	$7*2^{15}$
3.2	C	9	$4*3^{18}$
4	A	10.1	3
5	E	10.2	10001
6.1	A	10.3	C

### 三.【录屏】多项式、方程和坐标轴

1.1 已知以下关于  $f(x)$ ,  $h(x)$  和  $g(x)$  的函数情况：

(1)  $f(x)$  的定义域是  $|x| < 2$

(2)  $h(x)$  的定义域是  $0 < |x| < 6$

(3)  $g(x)$  的定义域是  $x < -7$

请问:  $f(x) \cdot h(x) \cdot g(x)$  的定义域是什么?

1.2 已知一个分段函数:

(1) 当  $0 \leq x \leq 4$  时,  $f(x) = 7 - x$

(2) 当  $4 < x \leq 8$  时,  $f(x) = 0.5x - 2$

请问: 这个函数的值域是多少?

1.3 已知  $f(x) = 2x$ ,  $g(y) = 2y + 1$ 。问:  $f(g(2))$  的值

2.1 已知以下关于  $x$ ,  $y$  和  $z$  的关系式:

(1)  $x + y = 1$

(2)  $xy + z = 1$

(3)  $xyz = 3/16$

问:  $(1/x + 1/y + 1/z - 1/3)^{0.5}$  等于多少?

2.2 已知  $x, y$  和  $z$  是三个不同的实数, 问一下条件能否判断  $x + y + z$  的值?

(1)  $x^2 + y^2 + z^2 = 27$

(2)  $xy + yz + xz = 11$

2.3 已知  $x, y, z$  和  $w$  满足一下条件:

(1)  $w/x = x/y = y/z$

(2)  $xyz$  均不等于 0

问: 如何用  $w$  和  $x$  表示  $w/z$ ?

3.1 In the  $xy$  plane, does the line with equation  $y = 3x + 2$  contain the point  $(r, s)$

(1)  $(3r + 2 - s)(4r + 9 - s) = 0$

(2)  $(4r - 6 - s)(3r + 2 - s) = 0$

3.2 If  $b, c$  and  $d$  are constants and  $x^2 + bx + c = (x + d)^2$  for all values of  $x$ , what is the value of  $c$ ?

(1)  $d = 3$

(2)  $b = 6$

3.3 已知  $x = -1$ ,  $n$  是前 404 个质数之和。问:  $x + x^n + x^{n+1} + x^{n+2}$  等于多少?

4.1 If  $s$  is the product of the integers from 100 to 200, inclusive, and  $t$  is the product of the integers from 100 to 201, inclusive, what is  $1/s + 1/t$  in terms of  $t$ ?

- A.  $(201)^2/t$       B.  $[(202)(201)]/t$       C.  $201/t$       D.  $202/t$       E.  $[(202)(201)]/t^2$

4.2 For any positive integer  $x$ , the 2-height of  $x$  is defined to be the greatest nonnegative integer  $n$  such that  $2^n$  is a factor of  $x$ . If  $k$  and  $m$  are positive integers, is the 2-height of  $k$  greater than the 2-height of  $m$ ?

- (1)  $k > m$   
(2)  $k/m$  is an even integer.

4.3 If  $A @ B = A^2 + B^2$  for all real numbers  $A$  and  $B$ , then  $(A @ B) @ C =$

4.4 The function  $f$  is defined for each positive three-digit integer  $n$  by  $f(n) = 2^x 3^y 5^z$ , where  $x$ ,  $y$ , and  $z$  are the hundreds, tens, and units digits of  $n$ , respectively. If  $m$  and  $v$  are three-digit positive integers such that  $f(m) = 9 f(v)$ , then  $m - v =$

4.5 The operation  $\otimes$  is defined for all nonzero numbers  $a$  and  $b$  by  $a \otimes b = a/b - b/a$ . If  $x$  and  $y$  are nonzero numbers, which of the following statements must be true?

- I.  $x \otimes xy = x(1 \otimes y)$   
II.  $x \otimes y = -(y \otimes x)$   
III.  $1/x \otimes 1/y = y \otimes x$

5.1 已知直线  $y=mx+b$ , 其中  $m$  和  $b$  是实数。问以下条件能否判断这条直线在  $x$  轴上的截距不是负的?

- (1)  $m+b>0$   
(2)  $mb>0$

5.2 In the  $xy$ -plane, region  $R$  consists of all the points  $(x, y)$  such that  $2x+3y \leq 6$ . Is the point  $(r,s)$  in the region  $R$ ?

- (1)  $3r+2s=6$   
(2)  $r \leq 3$  and  $s \leq 2$

5.3 已知关于  $x$  和  $y$  的方程  $|x|+|y|=6$ , 问这个方程在  $x,y$  坐标系围成的图形的面积

5.4 In the  $xy$ -coordinate plane, the slope of line  $I$  is  $3/4$ . Does line  $I$  pass through the point  $(-2/3, 1/2)$ ?

- (1) Line  $I$  passes through the point  $(4, 4)$   
(2) Line  $I$  passes through the point  $(-4, -2)$

5.5 已知一个二次函数  $f(x)=ax^2+bx+c$ ,问以下条件能否判断这个函数和  $x$  轴有多少个交点？

(1)  $a>0$

(2)  $c<0$

6.1 已知一个人出差有 11 天下雨,其中下雨的时候在上午或者下午,没有上午和下午都下雨的情况。这个人出差的时候有 16 天上午没下雨, 13 天下午没下雨。问: 这个人出差了多少天？

6.2 某考试一共有 50 题,评分规则如下：

(1)对一题+5 分

(2)错一题或者不做-2 分

以下哪个选项是可能的得分？

A.193      B.194      C.195      D.196      E. 197

6.3 At a certain company, the average number of years of experience is 9.8 years for male employees and 9.1 years for female employees. What is the ratio of the number of male employees to the number of female employees?

(1) There are 52 mal employees at the company

(2) The average number of years of experience for male and female employees combined is 9.3 years

7.1 已知  $xy>0$ , 问  $x/y+y/x>2$ ?

(1)  $x$  和  $y$  不相等

(2) $x=1.1y$

7.2  $AB+BA=AAC$  and  $A, B$ , and  $C$  are different digits, what is the units digit of the integer  $AAC$ ?

7.3 If  $x$  and  $y$  are positive, which of the following must be greater than  $1/(x+y)^{0.5}$ ?

I.  $(x+y)^{0.5}/2x$       II.  $(x^{0.5}+y^{0.5})/(x+y)$       III.  $(x^{0.5}-y^{0.5})/(x+y)$

Answer

1.1	空集	4.1	D	5.5	C
1.2	$0<x\leq 2 \cup 3\leq x\leq 7$	4.2	B	6.1	20
1.3	10	4.3	$A^4+B^4+2A^2B^2+C^2$	6.2	B
2.1	$5^{0.5}$	4.4	20	6.3	B
2.2	E	4.5	II III	7.1	D
2.3	$(w/x)^3$	5.1	B	7.2	0
3.1	C	5.2	E	7.3	II
3.2	D	5.3	72		
3.3	-2	5.4	D		

#### 四. 【直播】质数和因数

1. If  $p$ ,  $s$ , and  $t$  are positive prime numbers, what is the value of  $p^3s^3t^3$ ?

- (1)  $p^3st=728$
- (2)  $t=13$

2. If  $x$  and  $y$  are positive integers, what is the value of  $(x+y)^2$ ?

- (1)  $x = y - 3$
- (2)  $x$  and  $y$  are prime numbers.

3. If  $0 < x < y < z$  and  $x$ ,  $y$ ,  $z$  are consecutive odd integers, what is the value of  $xyz$ ?

- (1)  $y=5$
- (2)  $x$ ,  $y$ ,  $z$  are prime numbers

4. 已知  $k$  和  $n$  是正整数,  $g$  是  $k+7$  和  $n$  的公约数。

问: 以下条件能否判断整数  $g$  的大小

- (1)  $n=9843$
- (2)  $n=k+6$

5.1. PS :  $n=2^53^45^37^211$ ,  $n$  有多少个正因数?

5.2. The positive integer  $k$  has exactly two positive prime factors, 3 and 7. If  $k$  has a total of 6 positive factors, including 1 and  $k$ , what is the value of  $k$ ?

- (1)  $3^2$  is a factor of  $k$
- (2)  $7^2$  is not a factor of  $k$

5.3.  $n$  是整数, 以下条件能否判断  $12n$  有多少个因子?

- (1)  $n^2$  有 3 个正整数因子
- (2)  $n > 3$

6.1. How many different prime numbers are factors of the positive integer  $n$ ?

- (1) Four different prime numbers are factors of  $2n$
- (2) Four different prime numbers are factors of  $n^2$

6.2. 30 是不是  $P$  的因子?

- (1) 30 是  $P^2$  的因子
- (2) 30 是  $2P$  的因子

7. An integer greater than 1 that is not prime is called composite. If the two-digit integer  $n$  is greater than 20, is  $n$

composite?  
 (1)The tens digit of n is a factor of the units digit of n.  
 (2)The tens digit of n is 2.

8.1 For every positive even integer n, the function h(n) is defined to be the product of all the even integers from 2 to n, inclusive. If p is the smallest prime factor of h(100)+1, then p is

A.  $P<10$   
 B.  $30>P>10$   
 C.  $50>P>30$   
 D.  $P>50$

8.2 The product of the first N consecutive 正整数等于 X, X 的末尾有 6 个 0 , 求 N 的最小值是多少

9. n 是大于 1 的整数 , 问 n 是不是 prime number ?  
 条件 1 : every prime factor of n 大于根号 n  
 条件 2 : every prime factor of n 大于 n/2

Answer:

1	2	3	4	5.1	5.2	5.3	6.1	6.2	7	8.1	8.2	9
A	C	D	B	720	D	C	B	A	A	D	25	D

## 五. 【直播】余数

1.1 When positive integer  $x$  is divided by positive integer  $y$ , the remainder is 9. If  $x/y = 96.12$ , what is the value of  $y$ ?

1.2 已知  $n$  是正整数, 以下条件能否判断  $n$  除以 27 的余数  $r$  的值?

(1)  $n$  除以 20 的余数是 7

(2)  $n$  除以 27 的商是 2, 余数是  $r$

2.1 What is the remainder when the two digit, positive integer  $x$  is divided by 3?

(1) The sum of the digits of  $x$  is 5

(2) The remainder when  $x$  is divided by 9 is 5

2.2 If  $n$  is a positive integer and  $r$  is the remainder when  $4 + 7n$  is divided by 3, what is the value of  $r$ ?

(1)  $n+1$  is divisible by 3

(2)  $n > 20$

3. What is the remainder when the sum of the positive integers  $x$  and  $y$  is divided by 6?

(1) When  $x$  is divided by 6, the remainder is 3

(2) When  $y$  is divided by 6, the remainder is 1

4.1 If  $n$  is a positive integer and  $r$  is the remainder when  $(n-1)(n+1)$  is divided by 24, what is the value of  $r$ ?

(1) 2 is not a factor of  $n$

(2) 3 is not a factor of  $n$

4.2 If  $n$  is an integer greater than 6, which of the following must be divisible by 3?

A.  $n(n+1)(n-4)$

B.  $n(n+2)(n-1)$

C.  $n(n+3)(n-5)$

D.  $n(n+4)(n-2)$

E.  $n(n+5)(n-6)$

5 If  $x$  is an integer, is  $x$  between 27 and 54?

(1) The remainder when  $x$  is divided by 7 is 2

(2) The remainder when  $x$  is divided by 3 is 2

6.1 If  $x$  is a positive integer, is the remainder 0 when  $3^x + 1$  is divided by 10?

(1)  $x = 4n + 2$ , where  $n$  is a positive integer

(2)  $x > 4$

6.2 What is the remainder when the positive integer n is divided by the positive integer k, where  $k > 1$ ?

(1) $n = (k+1)^3$   
 (2) $k = 5$

6.3 (1) $3^{50}$  除以 8 的余数 ? (2) $3^{11}$  除以 8 的余数 ? (3) $3^{50}$  除以 7 的余数 ? (4) $7^{50}$  除以 15 的余数 ?

Answer:

1.1	1.2	2.1	2.2	3	4.1	4.2	5	6.1	6.2	6.3
75	C	D	A	C	C	A	E	A	A	1 3 2 4

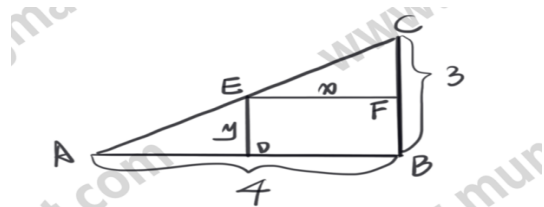


## 六.【录屏】几何和应用题

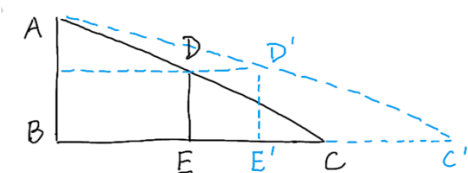
### 几何部分

1.1 已知三角形的顶点在直角坐标系中的坐标分别是  $A(-2, 4)$ ,  $B(3, 2)$ ,  $C(-6, -6)$ 。请问三角形  $ABC$  的面积是多少？

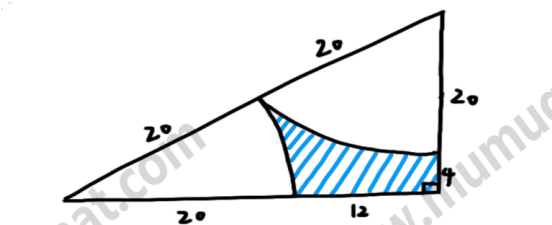
1.2 如图，已知  $AB=4, BC=3$ ，如何用  $y$  来表示  $x$ ？



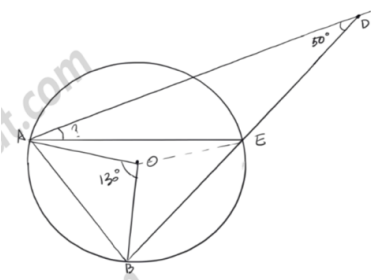
1.3 如图  $AB$  是高为 25feet 的一盏灯， $DE$  是高为 5feet 的一个人， $CE$  是人的投影长为 5feet。现在已知  $DE:AB=CE:CB$ 。请问如果这个人以 2feet/秒的速度朝灯塔反方向走，那么他的影子每秒增加多少 feet？



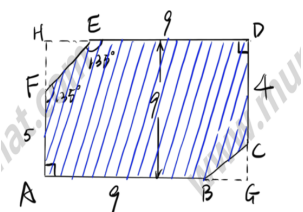
1.4 如图，一个直角三角形里有两个扇形，求阴影部分的面积。



1.5 已知  $\angle ADE=50^\circ$ ， $\angle AOB=130^\circ$ ，求  $\angle DAE$  的度数



1.6 如图所示， $\angle E=\angle F=135^\circ$ ， $DE=AB=9$ ， $AF=5$ ， $CD=4$ ，直线

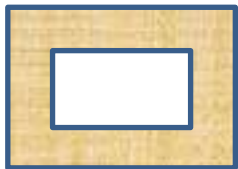


DE 到直线 AB 的距离为 9，求阴影部分的面积。

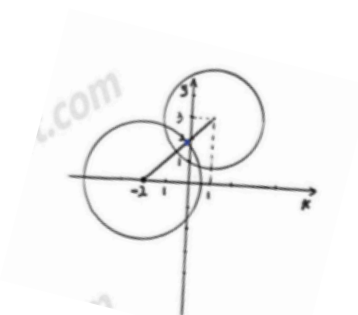
1.7 已知圆内有一个三角形，三角形一条边长为 6，请问以下条件能否判断这个三角形是直角三角形？

- (1) 圆的周长=6π
- (2) 三角形是等腰三角形

1.8 已知长方形的地区里有一个长方形的花园，两个长方形的周长分别是 102 和 30.求两个长方形的距离 w.



1.9 如图已知有两个圆，圆  $C_1$  的圆心在  $(-2, 0)$  半径为  $2\sqrt{0.5}$ ，圆  $C_2$  的圆心在  $(1, 3)$ ，问在圆  $C_1$  上距离圆  $C_2$  圆心最近的点的纵坐标是多少？

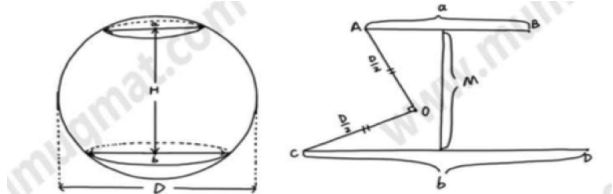


坐标是多少？

2.1 有一个高为 6inches 的罐头和体积 12 cube feet 的箱子，请问这个箱子可以装多少个罐头？

- (1) 箱子其中一个边是 2feet
- (2) 罐头的直径是 3 inches

2.2 已知一个球被切掉上半部分和下半部分，球的直径是  $D$ ，上半部分圆的直径是  $a$ ，下半部分是  $b$ ，两个圆心之间的距离是  $H$ ，请用

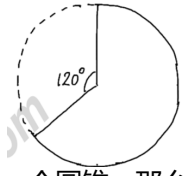


$D, a, b$  来表示  $H$

2.3 已知一个长方体的长宽高分别是  $L, W, H$ ，这个长方体拆开以后的图形的周长可能是多少？

- A.  $8H+4L+2W$
- B.  $6H+6L+2W$
- C.  $4H+4L+6W$

2.4 如图,已知有一个半径为 12 的扇形,如果将这个扇形卷起做成



一个圆锥,那么这个圆锥的高度是多少?

### 应用题部分

1.1. ①M 从家里去机场, bus 的平均速度是 60km/h, 火车比 bus 晚半个小时出发。火车出发以后 2 小时, 与 bus 同时到达机场, 求火车的速度?

1.1. ②已知两个人 A 和 B 开车过州界, 其中: (1)A 开车的速度是  $r$  (m/h), 在中午 12 点开过了州界; (2)B 开车的速度是  $1.5r$  (m/h), 在下午 1 点开过了州界。问: 第二个人超过第一个人时, 两人距离州界的距离?

1.2. 已知 A 和 B 两地距离 610km, 早上 8 点 a 从 A 地出发去 B 地, 时速是 90km/h; 早上 9:30 b 从 B 地出发去 A 地 时速是 100km/h. 请问 :a 和 b 在 b 出发后多久相遇?

1.3. 一艘船逆水中走 80miles 和顺水中走 120miles 所需的时间一样, 已知船在静水中的速度是 20mile/h, 求水流的速度?

1.4. 某段路 60miles, 某人先用 24miles/h 的速度跑 12miles, 再用 50miles/h 的速度跑完剩下的路程, 求这个人的平均速度?

2.1 已知有 A 和 B 两种机器: (1)生产 1 吨螺丝 A 单独用的时间是 B 单独用的时间的 75%; (2)机器 A 和 B 一起生产一吨螺丝需要 6 个小时。问: 机器 B 单独生产 1 吨螺丝用时多少?

2.2 已知有 x 和 y 两个机器要完成一个工程, 先用 x 机器完成工程的一半, 再用 y 机器完成剩下的一半, 共花费 16 小时。问以下条件能否求出 x 机器单独完成全部工程的时间?

(1) x 和 y 一起工作完成这个工程需要 6 小时

(2) x 单独比 y 单独快

2.3 A 工人和 B 工人上午一起用时 X 小时完成一项工作, 下午他们两人一起用 Y 小时完成了同样的工作, 是否能求出 Y/X 的值?

(1) A 工人下午比上午效率高 20%

(2) B 工人下午比上午效率高 10%

3.1 m 升的浓度为 x% 的溶液和 n 升的浓度为 y% 的溶液，混合后成为 m+n 升的浓度为 20% 的溶液，问 m/n 的值？

- (1)  $x=2y$
- (2)  $x=30$

4.1 已知一个人贷款数额为 N，simple annual interest rate 为 p%。贷款后他马上投资相同的数额 N，投资的收益率为 r%，compound semi-annually。问：他贷款第一年需要还的利息 < 投资第一年收到的利息吗？

- (1)  $p=10, r=9.8$
- (2)  $p\% < (1+0.5r\%)^2 - 1$

5.1 已知一个人做了工具之后出售，其中：①如果做 80 个以内，单价为 6.25 美元；②如果超过 80 个，则全部商品单价为 6.5 美元。问：以下条件能否判断他一共做了多少个？

- (1) 他多做了 4 个可以多赚 26 美元
- (2) 他一共赚了 533 美元

5.2 一个人工作的前 40 个小时工资为 10 块/小时，超过 40 小时以后的工资为 15 块/小时。上个礼拜没到 40 小时，这个礼拜超过了 40 小时，请问这个礼拜工作了多少小时？

- (1) 这个礼拜比上个礼拜多工作 20 个小时，多赚了 250 元
- (2) 工作时间是上个礼拜的  $\frac{5}{3}$ ，收入是上个礼拜的  $\frac{11}{6}$

5.3 已知商家进了一批椅子和桌子，其中椅子售价 18.2 美元，桌子售价 52 美元。椅子的利润是进价的 40%，桌子的利润是进价的 30%。商家进货总共用了 420 美元。求商家的总利润是多少？

6.1 志愿者要整理一些捐赠的书，其中：①一人一天最少整理 1 本，最多 10 本；②昨天一共整理 1600 本，请问志愿者是否超过 300 人？

- (1) 整理 1-2 本的有 80 人
- (2) 整理 3-4 本的有 150 人

6.2 有 100 个学生，其中生日在 10 月份的学生人数大于生日在其他任意月份的学生人数，问生日在 10 月份的学生人数最小可以是多少？

6.3 已知一个卖车商 30 天内每天卖车，请问以下条件能否判断这个卖车商这 30 天中至少有一天卖出少于 9 辆车？

- (1) 这 30 天卖出的车的平均值小于 10
- (2) 这 30 天中有 16 天至少卖出 11 辆车

7.1 On a certain sight-seeing tour, the ratio of the number of women to the number of children was 5 to 2. What was the number of men in the sight-seeing tour?

- (1) On the sight-seeing tour, the ratio of the number of children to the number of men was 5 to 11.

(2) The number of women on the sight-seeing tour was less than 30.

7.2 The rate of a certain chemical reaction is directly proportional to the square of the concentration of chemical A present and inversely proportional to the concentration of chemical B present. If the concentration of chemical B is increased by 100 percent, what is the percentage change in the concentration of chemical A required to keep the reaction rate unchanged?

7.3 Last year the price per share of Stock X increased by  $k$  percent and the earning per share of Stock X increased by  $m$  percent, where  $k$  is great than  $m$ . By what percent did the ration of price per share to earnings per share increase, in terms of  $k$  and  $m$ ?

Answer

几何部分

- |                   |   |
|-------------------|---|
| 1.1 29            | 2.1 E   |
| 1.2 $(3-y)^{4/3}$ | 2.2 $[(D/2)^2-(a/2)^2]^{0.5}+[(D/2)^2-(b/2)^2]^{0.5}$ |
| 1.3 0.5           | 2.3 A   |
| 1.4 $384-100\pi$  | 2.4 $80^{0.5}$  |
| 1.5 $15^\circ$    |   |
| 1.6 99            |   |
| 1.7 A             |   |
| 1.8 9             |   |
| 1.9 2             |   |

应用题部分

- |             |                     |
|-------------|---------------------|
| 1.1 ①75 ②3r | 5.1 B               |
| 1.2 2.5     | 5.2 D               |
| 1.3 4       | 5.3 152             |
| 1.4 41      | 6.1 C               |
| 2.1 14      | 6.2 10              |
| 2.2 C       | 6.3 C               |
| 2.3 E       | 7.1 C               |
| 3.1 C       | 7.2 41%             |
| 4.1 D       | 7.3 $(k-m)/(100+m)$ |

七.【录屏】统计和排列组合基础

1.1 以下条件是否能判断 s 是 r, s, t 三个数的中位数?

- (1)s 是这三个数的平均数
- (2)s=t

1.2 以下条件是否能判断一组数的中位数是多少?

- (1)这些数的极差是 40
- (2)这些数的平均数是 40

1.3 已知有 25 人参加某项考试,请问以下条件能否判断这 25 个人考试成绩的中位数和平均数中哪一个比较大?

- (1)所有人的分数都在 70-100 之间
- (2)有一半以上的人的出成绩超出 85 分

2.1 下表第一列表示信用卡拥有数,第二列表示对应的人数,问以下条件能否判断信用卡拥有数的中位数是多少?

信用卡拥有数	人数
1	3
2	B
3	4
4	10
A	2
8	1

- (1) A=6
- (2) B=7

2.2 卖车公司统计汽车价格情况,以下条件能否判断这些车价格的中位数是多少?

- (1)有至少一半的车价格低于 18500 美元
- (2)有至少一半的车价格高于 18500 美元

2.3 下表显示一年前 182 天和后 183 天中每天温度的最低值、中值和最大值,问这一年温度的中位数可能是多少?

	Lowest	Median	Highest
182	3x		41
183	37		9x

- A.<37      B.39      C.>41

3.1 已知 12 个月降水量的最大值是 12.2, 最小值是 0.2 , 平均数是 3.4 , 请问降水量的中位数最大可能是多少？

3.2 已知 8 个整数相加的和是 440 , 以下条件能否判断这 8 个数中最大的数是多少？  
(1)这 8 个数是连续的偶数  
(2)这 8 个数的 range 是 14

3.3 周一股市开盘的时候 , 某公司的股价为  $x$ , 下图显示了这个公司周一到周五股票的涨跌情况 , 请问这 5 天的平均股票休市价格是多少？

Day	price change
周一	$m$
周二	$-t$
周三	$-v$
周四	$n$
周五	$p$

4.1 已知有两个数列 A 和 B , 并且 A 数列的标准差是 1.75 , B 数列中所有的数都是数列 A 的-2 倍 , 问数列 B 的标准差是多少？

4.2 已知 33 , 34 , 35 , 36 , 37 这 5 个数的标准差是  $y$ , 请问 433 , 434 , 435 , 436 , 437 这 5 个数的标准差是多少？

Answer

1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	4.1	4.2
D	E	E	B	E	B	4.6	A	$x+m-4t/5-3v/5+2n/5+p/5$	3.5	$y$

## 八. 【直播】排列组合和概率

1.1 A three-digit code for certain locks uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 according to the following constraints. The first digit cannot be 0 or 1, the second digit must be 0 or 1, and the second and third digits cannot both be 0 in the same code. How many different codes are possible?

1.2 Six cards numbered from 1 to 6 are placed in an empty bowl. First one card is drawn and then put back into the bowl; then a second card is drawn. If the cards are drawn at random and if the sum of the numbers on the cards is 8, what is the probability that one of the two cards drawn is numbered 5?

1.3 A certain company assigns employees to offices in such a way that some of the offices can be empty and more than one employee can be assigned to an office. In how many ways can the company assign 3 employees to 2 different offices?

1.4 有 16 支队伍举行足球赛,每一个队伍都要跟其他所有的队伍踢一场比赛, 请问一共要踢多少场比赛?

2.1 On Saturday morning, Malachi will begin a camping vacation and he will return home at the end of the first day on which it rains. If on the first three days of the vacation the probability of rain on each day is 0.2, what is the probability that Malachi will return home at the end of the day on the following Monday?

2.2 A string of 10 lightbulbs is wired in such a way that if any individual lightbulb fails, the entire string fails. If for each individual lightbulb the probability of failing during time period  $T$  is 0.06, what is the probability that the string of lightbulbs will fail during time period  $T$ ?

2.3 If a certain coin is flipped, the probability that the coin will land heads is  $\frac{1}{2}$ . If the coin is flipped 5 times, what is the probability that it will land heads up on the first 3 flips and not on the last 2 flips?



2.4 A jar contains 16 marbles, of which 4 are red, 3 are blue, and the rest are yellow. If 2 marbles are to be selected at random from the jar, one at a time without being replaced, what is the probability that the first marble selected will be red and the second marble selected will be blue?

3.1 There are 11 women and 9 men in a certain club. If the club is to select a committee of 2 women and 2 men, how many different such committees are possible?

3.2 There are 8 books on a shelf, of which 2 are paperbacks and 6 are hardbacks. How many possible selections of 4 books from this shelf include at least one paperback?

3.3 The membership of a committee consists of 3 English teachers, 4 Mathematics teachers, and 2 Social Studies teachers. If 2 committee members are to be selected at random to write the committee's report, what is the probability that the two members selected will both be English teachers?

4.1 If a committee of 3 people is to be selected from among 5 married couples so that the committee does not include two people who are married to each other, how many such committees are possible?

4.2 A certain restaurant offers 6 kinds of cheese and 2 kinds of fruit for its dessert platter. If each dessert platter contains an equal number of kinds of cheese and kinds of fruit, how many different dessert platters could the restaurant offer?

4.3 12 个人分成 2 队 ,一队 6 人 ,一共有多少种 6 人 vs 6 人组合 ?

4.4 If 2 different representatives are to be selected at random from a group of 10 employees and if  $p$  is the probability that both representatives selected will be women, is  $p > 1/2$  ?

- (1) More than  $1/2$  of the 10 employees are women.
- (2) The probability that both representatives selected will be men is less than  $1/10$ .

5.1 Each signal that a certain ship can make is comprised of 3 different flags hanging vertically in a particular order. How many unique signals can be made by using 4 different flags?

5.2 A certain roller coaster has 3 cars, and a passenger is equally likely to ride in any 1 of the 3 cars each time that passenger rides the roller coaster. If a certain passenger is to ride the roller coaster 3 times, what is the probability that the passenger will ride in each of the 3 cars?

5.3 A gardener is going to plant 2 red rosebushes and 2 white rosebushes. If the gardener is to select each of the bushes at random, one at a time, and plant them in a row, what is the probability that the 2 rosebushes in the middle of the row will be the red rosebushes?

5.4 Of the three-digit positive integers that have no digits equal to zero, how many have two digits that are equal to each other and the remaining digit different from the other two?

6.1 There are seven different car keys, namely A/B/C/D/E/F/G, to put in a key ring. What is the probability that A and B will be adjacent?

6.2 In a meeting of 3 representatives from each of 6 different companies, each person shook hands with every person not from his or her own company. If the representatives did not shake hands with people from their own company, how many handshakes took place?

## Answer

1.1	152	4.2	27
1.2	$2/5$	4.3	$C_{12}^6 / 2$
1.3	8	4.4	E
1.4	120	5.1	24
2.1	0.128	5.2	$2/9$
2.2	$1-0.94^{10}$	5.3	$1/6$
2.3	$0.5^5$	5.4	216
2.4	$1/20$	6.1	$1/3$
3.1	1980	6.2	135
3.2	55	6.3	$21/64$
3.3	$1/12$	6.4	5
4.1	80		

## 九. 【直播】OG 2018 数学答疑