Problem Solving

Counting/Sets/Series

PS54110.01

1. The letters C, I, R, C, L, and E can be used to form 6-letter strings such as CIRCLE or CCIRLE. Using these letters, how many different 6-letter strings can be formed in which the two occurrences of the letter C are separated by at least one other letter?

   A. 96

   B. 120

   C. 144

   D. 180

   E. 240

Arithmetic **Elementary combinatorics**

This can be solved by using the Multiplication Principle. The answer is $m \times n$, where m is the number of ways to choose the 2 suitable positions in which to place the C's and n is the number of ways in which to place the 4 remaining letters in the 4 remaining positions.

The value of m can be found by a direct count of the number of suitable ways to choose the 2 positions in which to place the C's. In what follows, each * denotes one of the 4 remaining positions.

There are 4 possibilities when a C is in the first position:

C*C***	C**C**	C***C*	C****C

There are 3 more possibilities when a C is in the second position:

*C*C**	*C**C*	*C***C

There are 2 more possibilities when a C is in the third position:

**C*C*	**C**C

There is 1 more possibility when a C is in the fourth position:

***C*C

Therefore, $m = 4 + 3 + 2 + 1 = 10$.

Alternatively, the value of m can be found by subtracting the number of non-suitable ways to place the C's (i.e., the number of consecutive positions in the string) from the number of all possible ways to place the C's (suitable or not). This gives $m = 15 - 5 = 10$, where $15 = \binom{6}{2}$ is the number of all possible ways to place the C's ("6 choose 2") and 5 is the number of non-suitable ways to place the C's (shown below).

CC****	*CC***	**CC**	***CC*	****CC

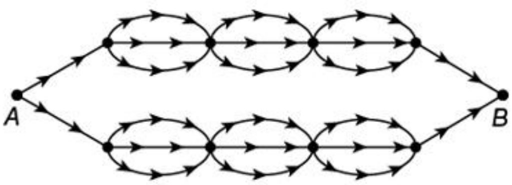
Tip

The alternative approach for finding m is useful when a direct count of the number of suitable ways is more difficult than a direct count of the number of non-suitable ways. An example is determining the number of 8-letter strings that can be formed from the letters in REPEATED in which there is at least one pair of E's having at least one other letter between them. For this example, $m = \binom{8}{3} - 6$ $= 56 - 6 = 50$.

The value of n is equal to the number of ways to place the 4 remaining letters into 4 positions, where order matters and the letters are selected without replacement. Thus, $n = 4! = 24$.

Therefore, the answer is $m \times n = 10 \times 24 = 240$.

The correct answer is E.



PS24831.01

PS24831.01

2. The map above shows the trails through a wilderness area. If travel is in the direction of the arrows, how many routes along the marked trails are possible from point *A* to point *B* ?

   A. 11

   B. 18

   C. 54

   D. 108

   E. 432

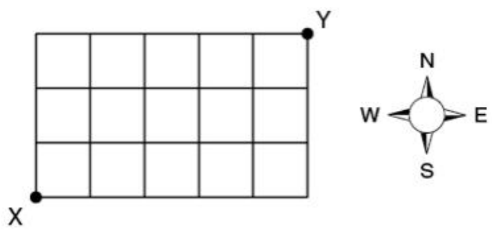
Arithmetic Elementary combinatorics

It is clear that the number of routes from point *A* to point *B* that begin by going up from point *A* ("up" relative to the orientation of the map) is the same as the number of routes from point *A* to point *B* that begin by going down from point *A*. Therefore, we only need to determine the number of routes from point *A* to point *B* that begin by going up and then double the result.

> **Tip**
>
> If these two numbers of routes were not the same, or at least if it was not clear whether they were the same, then we would simply determine each of the numbers separately and then add them.

To determine the number of routes that begin by going up from point *A*, we can apply the Multiplication Principle. There are 3 locations at which branches occur. Moreover, at each of these locations, there are 3 different trails that can be taken. Finally, the choices of which trail to take at each location can be made independently. Therefore, the Multiplication Principle applies and we get (3)(3)(3) = 27 for the number of routes that begin by going up from point *A*. Hence, the number of routes from point *A* to point *B* is 2(27) = 54.

The correct answer is C.



PS61551.01

3. In the figure above, X and Y represent locations in a district of a certain city where the streets form a rectangular grid. In traveling only north or east along the streets from X to Y, how many different paths are possible?

   A. 720

   B. 512

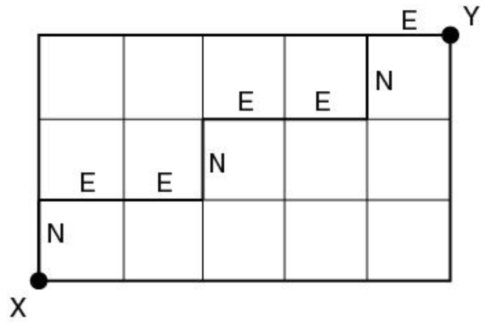
   C. 336

   D. 256

   E. 56

Arithmetic **Elementary combinatorics**

Each possible path will consist of traveling a total of 3 grid segments north and 5 grid segments east. Thus, letting 'N' represent traveling north by one grid segment and 'E' represent traveling east by one grid segment, each path can be uniquely represented by an appropriate 8-character string of N's and E's. For example, as shown in the figure below, NEENEENE represents grid segments traveled in the order north, east, east, north, east, east, north, and east.



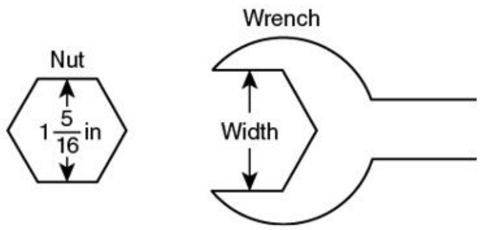
Therefore, the number of possible paths is equal to the number of appropriate 8-character strings of N's and E's, which is $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{(5!)(6)(7)(8)}{(2)(3)(5!)}$ = (7)(8) = 56, since each appropriate string is determined when a specification is made for the 3 positions in the string at which the N's are to be placed. Alternatively, the number of possible paths is equal to the number of permutations of 8 objects in which 3 are identical (the N's) and the remaining 5 are identical (the E's), and thus equal to $\frac{8!}{(3!)(5!)}$.

Tip

The alternative approach has a well-known generalization that can be used to calculate the number of permutations of n objects when various subsets of those objects consist of objects to be treated as identical. We give four examples in which such a calculation can be used.

1. The number of 8-letter words that can be formed using the letters of PEPPERER is equal to $\frac{8!}{(2!)(3!)(3!)}$ = 560.
2. Consider a 3-dimensional analog of the rectangular grid above, with dimensions 2 by 3 by 3. The number of paths from the front-left-down vertex to the back-right-up vertex such that each path consists of traveling only back, right, or up is equal to $\frac{8!}{(2!)(3!)(3!)}$ = 560.
3. The number of ways to distribute 8 different books to David, Liam, and Sophia so that David is given 2 of the books, Liam is given 3 of the books, and Sophia is given 3 of the books is equal to $\frac{8!}{(2!)(3!)(3!)}$ = 560.
4. The coefficient of $a^2b^3c^3$ in the expansion of $(a + b + c)^8$ after like terms are combined is equal to $\frac{8!}{(2!)(3!)(3!)}$ = 560.

The correct answer is E.



PS92751.01

4. The figures above show a hexagonal nut that has a width of $1\frac{5}{16}$ inches and a wrench that, in order to fit the nut, must have a width of at least $1\frac{5}{16}$ inches. Of all the wrenches that fit the nut and have widths that are whole numbers of millimeters, the wrench that fits the nut most closely has a width of how many millimeters?

   (*Note:* 1 inch ≈ 25.4 millimeters)

   A. 30

   B. 31

   C. 32

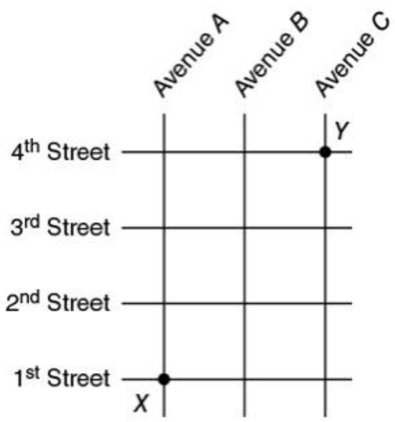
   D. 33

   E. 34

Arithmetic Measurement conversion

The width of the nut in millimeters is nearly equal to $\left(1+\frac{5}{16}\right)(25.4) = 25.4+\left(\frac{5}{16}\right)\left(\frac{254}{10}\right) = 25.4+\frac{127}{16}$. Since $25.4+\frac{127}{16} = 25.4+\frac{128}{16}-\frac{1}{16} = 33.4-\frac{1}{16}$, it follows that the width of the nut is between 33 mm and 34 mm.

The correct answer is E.



PS45461.01

PS45461.01

5. Pat will walk from intersection *X* to intersection *Y* along a route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from *X* to *Y* can Pat take that have the minimum possible length?

   A. Six

   B. Eight

   C. Ten

   D. Fourteen

   E. Sixteen

Arithmetic Elementary combinatorics

Each minimum-length route will consist of traveling a total of 3 grid segments up and 2 grid segments right. Thus, letting 'U' represent traveling up by one grid segment and 'R' represent traveling right by one grid segment, each minimum-length route can be uniquely represented by an appropriate 5-character string of U's and R's. For example, URUUR represents grid segments traveled in the order up, right, up, up, and right. Therefore, the number of possible minimum-length routes is equal to the number of appropriate 5-character strings of U's and R's, which is $\binom{5}{3} = \frac{5!}{3!(5-3)!} = 10$, since each appropriate string is determined when a specification is made for the 3 positions in the string at which the U's are to be placed.

Tip

The ideas involved in counting the number of 5-character strings of U's and R's can be extended to counting the number of n-character strings when various characters being used are the same. For example, MISSISSIPPI has one M, four I's, four S's, and two P's, and thus the number of 11-letter words that can be formed using the letters of MISSISSIPPI is equal to $\frac{11!}{(1!)(4!)(4!)(2!)} = \binom{11}{1} \cdot \binom{10}{4} \cdot \binom{6}{4} \cdot \binom{2}{2} = 34{,}650$. (Each of the two expressions we have given for 34,650 is intended to suggest a method for counting the number of 11-letter words.)

The correct answer is C.

PS95302.01

6. Rita and Sam play the following game with n sticks on a table. Each must remove 1, 2, 3, 4 or 5 sticks at a time on alternate turns, and no stick that is removed is put back on the table. The one who removes the last stick (or sticks) from the table wins. If Rita goes first, which of the following is a value of n such that Sam can always win no matter how Rita plays?

   A. 7
   B. 10
   C. 11
   D. 12
   E. 16

Arithmetic Elementary combinatorics

Let Player A be either Rita or Sam, and let Player B be the other player. If, after one of Player A's turns, there are exactly 6 sticks left, then Player A can win on his or her next turn. This is because if 6 sticks are left after Player A's turn, then regardless of whether Player B removes 1, 2, 3, 4, or 5 sticks, it follows that Player A can win on his or her next turn by removing, respectively, 5, 4, 3, 2, or 1 stick.

$n = 7$: If Rita begins by removing 1 stick, then there will be 6 sticks left after Rita's turn. Therefore, by the remarks above, Rita can win. Hence, Sam cannot always win.

$n = 10$: If Rita begins by removing 4 sticks, then there will be 6 sticks left after Rita's turn. Therefore, by the remarks above, Rita can win. Hence, Sam cannot always win.

$n = 11$: If Rita begins by removing 5 sticks, then there will be 6 sticks left after Rita's turn. Therefore, by the remarks above, Rita can win. Hence, Sam cannot always win.

$n = 12$: If Rita begins by removing 1 stick, then Sam can win by removing 5 sticks on his next turn, because 6 sticks will remain after Sam's turn. If Rita begins by removing 2 sticks, then Sam can win by removing 4 sticks on his next turn, because 6 sticks will remain after Sam's turn. By continuing in this manner, we see that if Rita begins by removing k sticks (where k is one of the numbers 1, 2, 3, 4, or 5), then Sam can win by removing $(6-k)$ sticks on his next turn because 6 sticks will remain after Sam's turn. Therefore, no matter how many sticks Rita removes on her first turn, Sam can win by removing appropriate numbers of sticks on his next two turns. Hence, Sam can always win.

$n = 16$: If Rita removes 4 sticks on her first turn, then Sam will be in the same situation as Rita for $n = 12$ above, and therefore Rita can win no matter what Sam does. Hence, Sam cannot always win.

The correct answer is D.

PS65402.01

7. When $\frac{2}{9}$ of the votes on a certain resolution have been counted, $\frac{3}{4}$ of those counted are in favor of the resolution. What fraction of the remaining votes must be against the resolution so that the total count will result in a vote of 2 to 1 against the resolution?

   A. $\frac{11}{14}$

   B. $\frac{13}{18}$

   C. $\frac{4}{7}$

   D. $\frac{3}{7}$

   E. $\frac{3}{14}$

Arithmetic Operations on rational numbers

For this problem, by assigning carefully chosen numbers to quantities given in the problem, it can be made more concrete and some of the computations with fractions can be avoided. Since $\frac{2}{9}$ of all the votes have been counted and $\frac{3}{4}$ of them are for the resolution, 36 (= 9 × 4) would be a good number to use as the total number of votes cast. Since the total count must result in a vote of 2 to 1 against the resolution, $\frac{2}{3}$ of all of the votes must be against the resolution. This information can be summarized in the following table.

	Total votes cast	Counted so far	Still to be counted
	36	$\frac{2}{9}(36) = 8$	$36 - 8 = 28$
For		$\frac{3}{4}(8) = 6$	
Against	$\frac{2}{3}(36) = 24$	$8 - 6 = 2$	$24 - 2 = 22$

From the table, it is clear that of the 28 votes still to be counted, 22 must be against the resolution. Therefore, the fraction of the votes still to be counted that must be against the resolution is $\frac{22}{28} = \frac{11}{14}$.

In general, letting T represent the total number of votes cast, since the total count must result in a vote of 2 to 1 against the resolution, $\frac{2}{3}T$ votes must be against the resolution. The information is summarized in the following table.

	Total votes cast	Counted so far	Still to be counted
	T	$\frac{2}{9}T$	$T - \frac{2}{9}T = \frac{7}{9}T$
For		$\frac{3}{4}\left(\frac{2}{9}T\right) = \frac{1}{6}T$	
Against	$\frac{2}{3}T$	$\frac{1}{4}\left(\frac{2}{9}T\right) = \frac{1}{18}T$	$\frac{2}{3}T - \frac{1}{18}T = \frac{11}{18}T$

From the table, it is clear that of the $\frac{7}{9}T$ votes still to be counted $\frac{11}{18}T$ must be against the resolution. Therefore, the fraction of the votes still to be counted that must be against the resolution is $\frac{\frac{11}{18}T}{\frac{7}{9}T} = \frac{11}{14}$.

The correct answer is A.

Alternative explanation:

Assign actual numbers to the problem to make the math more concrete. Since we are dealing with $\frac{2}{9}$ of something and also $\frac{1}{4}$ of something, we will want our numbers to be convenient. Look for multiples of 36 (9 times 4) for which $\frac{2}{9}$ and $\frac{1}{4}$ will result in whole numbers. A number that will work well is 180.

Of the 180 votes, $\frac{2}{9}$ have been counted.

$\frac{2}{9}$ (180) = 40 votes counted. This means 140 votes have not been counted.

Of those 40 counted votes, $\frac{3}{4}$ are in favor. $\frac{3}{4}$ (40) = 30 votes in favor (of the 40 counted).

This means 10 votes are not in favor (of the 40 counted).

Looking ahead to the desired end result, in order to achieve a 2:1 ratio against, $\frac{1}{3}$ of the votes will be for and $\frac{2}{3}$ will be against. Therefore we will need 120 votes against. So far we have 10 votes not in favor.

In order to reach a total of 120 uncounted votes, of the 140 uncounted votes, we will need 110 votes not in favor to combine with the 10 counted votes not in favor.

This is $\frac{110}{140}$ or $\frac{11}{14}$. The correct answer is A.

PS85402.01

8. The sum of the first 100 positive integers is 5,050. What is the sum of the first 200 positive integers?

A. 10,100

B. 10,200

C. 15,050

D. 20,050

E. 20,100

Arithmetic Sequences

The sum of the first n positive integers is given by $\frac{n(n+1)}{2}$, so the sum of the first 200 positive integers is $\frac{200(201)}{2}$ = 20,100.

Alternatively, letting $\sum_{i=1}^{100} i$ represent the sum of the first 100 positive integers, it is given that $\sum_{i=1}^{100} i = 5{,}050$. Using this notation, $\sum_{i=1}^{200} i = \sum_{i=1}^{100} i + \sum_{i=1}^{100}(100+i) = \sum_{i=1}^{100} i + \sum_{i=1}^{100} 100 + \sum_{i=1}^{100} i = 2\sum_{i=1}^{100} i + 100\sum_{i=1}^{100} 1 = 2(5{,}050) + 10{,}000$

= 10,100 + 10,000 = 20,100.

The correct answer is E.

Month	Average Price per Dozen
April	\$1.26
May	\$1.20
June	\$1.08

PS40502.01

9. The table above shows the average (arithmetic mean) price per dozen eggs sold in a certain store during three successive months. If $\frac{2}{3}$ as many dozen were sold in April as in May, and twice as many were sold in June as in April, what was the average price per dozen of the eggs sold over the three-month period?

   A. \$1.08

   B. \$1.10

   C. \$1.14

   D. \$1.16

   E. \$1.18

Arithmetic Statistics

Given that the numbers of eggs sold in each of the three months are in the ratio 2:3:4, it follows that $\frac{2}{9}$ of the eggs sold in the three-month period were sold at an average of \$1.26 per dozen, $\frac{3}{9} = \frac{1}{3}$ were sold at \$1.20 per dozen, and $\frac{4}{9}$ were sold at \$1.08 per dozen. Therefore, the average price per dozen of the eggs sold in the three-month period was $\frac{2}{9}(\$1.26) + \frac{1}{3}(\$1.20) + \frac{4}{9}(\$1.08) = \$1.16$.

The correct answer is D.

PS96602.01

10. Each of the integers from 0 to 9, inclusive, is written on a separate slip of blank paper and the ten slips are dropped into a hat. If the slips are then drawn one at a time without replacement, how many must be drawn to ensure that the numbers on two of the slips drawn will have a sum of 10 ?

   A. Three

   B. Four

   C. Five

   D. Six

   E. Seven

Arithmetic Elementary combinatorics

To simplify the discussion, we will refer to the drawing of the slip of paper with the integer n written on it as "drawing the integer n." The number of integers that must be drawn is at least seven, because if the six integers 0 through 5 were drawn, then no two of the integers drawn will have a sum of 10. In fact, it is easy to see that the sum of any two of these six integers is less than 10.

0, 1, 2, 3, 4, 5

Of the answer choices, only seven is not eliminated.

Although it is not necessary to show that seven is the least number of integers that must be drawn to ensure there exists a pair of the drawn integers that has a sum of 10, we provide a proof that seven is the least such number. Thus, we will show that if seven integers were drawn, then there exists a pair of the drawn integers that has a sum of 10. Since the integer 0 is such that none of the other integers can be paired with 0 to give a sum of 10, and similarly for the integer 5, it will suffice to show that if five integers were drawn from the eight integers 1, 2, 3, 4, 6, 7, 8, and 9, then there exists a pair of the drawn integers that has a sum of 10. Note that each of these eight integers differs from 5 by one of the numbers 1, 2, 3, or 4, as shown below.

1 = 5 − 4	6 = 5 + 1
2 = 5 − 3	7 = 5 + 2
3 = 5 − 2	8 = 5 + 3
4 = 5 − 1	9 = 5 + 4

With these preliminaries out of the way, assume that five integers have been drawn from these eight integers. Of the five integers that have been drawn, at least two must differ from 5 by the same number, say k, and since these two integers must be different, it follows that one of these two integers is $5 + k$ and the other is $5 - k$, and hence these two integers have a sum of 10.

The correct answer is E.

PS15402.01

11. King School has an enrollment of 900 students. The school day consists of 6 class periods during which each class is taught by one teacher. There are 30 students per class. Each teacher teaches a class during 5 of the 6 class periods and has one class period free. No students have a free class period. How many teachers does the school have?

 A. 25

 B. 30

 C. 36

 D. 60

 E. 150

Algebra Statistics

If each teacher has 5 class periods a day and each class has 30 students, then each teacher has (5)(30) students per day. Each of the 900 students has 6 classes per day, from which it follows that all the teachers combined have a total of (6)(900) students per day. Therefore, the school has $\frac{(6)(900)}{(5)(30)}$ or 36 teachers.

The correct answer is C.

PS07602.01

12. Ben and Ann are among 7 contestants from which 4 semifinalists are to be selected. Of the different possible selections, how many contain neither Ben nor Ann?

 A. 5

 B. 6

 C. 7

 D. 14

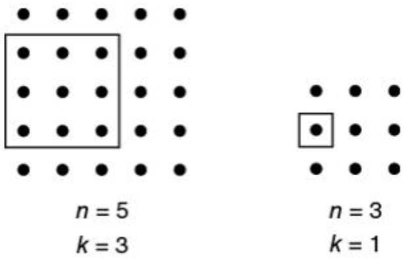
 E. 21

Arithmetic Elementary combinatorics

The number of possible selections of 4 semifinalists that do not contain Ben or Ann is equal to the number of possible selections of 4 semifinalists from the remaining 7 − 2 = 5 contestants, which is equal to $\binom{5}{4} = \frac{5!}{4!(5-4)!}$ = 5. Alternatively, the number of possible selections of 4 semifinalists from the remaining 5 contestants is equal to the number of possible selections of exactly 1 non-semifinalist, which is equal to 5.

The correct answer is A.

Equalities/Inequalities/Algebra



PS03551.01

13. Let n and k be positive integers with $k \leq n$. From an $n \times n$ array of dots, a $k \times k$ array of dots is selected. The figure above shows two examples where the selected $k \times k$ array is enclosed in a square. How many pairs (n, k) are possible so that exactly 48 of the dots in the $n \times n$ array are NOT in the selected $k \times k$ array?

    A. 1

    B. 2

    C. 3

    D. 4

    E. 5

Algebra **Factoring; Simultaneous equations**

The $n \times n$ array has n^2 dots and the $k \times k$ array has k^2 dots. The number of dots in the $n \times n$ array that are not in the $k \times k$ array is given by $n^2 - k^2 = (n - k)(n + k)$.

Therefore, $(n - k)(n + k) = 48$ is a necessary condition for there to be 48 dots not in the $k \times k$ array. This is also a sufficient condition, since it is clear that at least one $k \times k$ array of dots can be selected for removal from an $n \times n$ array of dots when $k \leq n$.

The equation $(n - k)(n + k) = 48$ represents two positive integers, namely $n - k$ and $n + k$, whose product is 48. Thus, the smaller integer $n - k$ must be 1, 2, 3, 4, or 6, and the larger integer $n + k$ must be 48, 24, 16, 12, or 8. Rather than solving five pairs of simultaneous equations (for example, $n - k = 2$ and $n + k = 24$ is one such pair), it is more efficient to observe that the solution to the system $n - k = a$ and $n + k = b$ is $n = \frac{1}{2}(a+b)$ (add the equations, then divide by 2) and $k = \frac{1}{2}(b-a)$ (substitute $n = \frac{1}{2}(a+b)$ for n in either equation and solve for k; or subtract the equations, then divide by 2). Therefore, the possible pairs (n, k) arise exactly when $48 = ab$ and both $a + b$ and $b - a$ are divisible by 2. This occurs exactly three times—$48 = (2)(24)$, $48 = (4)(12)$, and $48 = (6)(8)$.

The correct answer is C.

PS41471.01

14. If there is a least integer that satisfies the inequality $\frac{9}{x} \geq 2$, what is that least integer?

    A. 0

    B. 1

    C. 4

    D. 5

    E. There is not a least integer that satisfies the inequality.

Algebra Inequalities

It is clear that no negative integer satisfies the inequality (because $\frac{9}{\text{negative}} \geq 2$ is false) and zero does not satisfy the inequality (because $\frac{9}{0}$ is undefined). Thus, the integers, if any, that satisfy $\frac{9}{x} \geq 2$ must be among 1, 2, 3, 4, ... The least of these integers is 1, and it is easy to see that $x = 1$ satisfies the inequality $\frac{9}{x} \geq 2$. Therefore, the least integer that satisfies the inequality is 1.

Alternatively, the inequality can be solved algebraically. It will be convenient to consider three cases according to whether $x < 0$, $x = 0$, and $x > 0$.

Case 1: Assume $x < 0$. Then multiplying both sides of the inequality by x, which is negative, gives $9 \leq 2x$, or $x \geq 4.5$. Because we are assuming $x < 0$, there are no solutions to $x \geq 4.5$. Therefore, no solutions exist in Case 1.

Case 2: Assume $x = 0$. Then $\frac{9}{x}$ is not defined, and thus $x = 0$ cannot be a solution.

Case 3: Assume $x > 0$. Then multiplying both sides of the inequality by x, which is positive, gives $9 \geq 2x$, or $x \leq 4.5$. Because we are assuming $x > 0$, the solutions that exist in Case 2 are all real numbers x such that $0 < x \leq 4.5$.

The set of all solutions to the inequality $\frac{9}{x} \geq 2$ will be all solutions found in Cases 1, 2, and 3. Therefore, the solutions to the inequality consist of all real numbers x such that $0 < x \leq 4.5$. The least of these solutions that is an integer is 1.

The correct answer is B.

x	$C(x)$
0	25,000
10	24,919
20	24,846
30	24,781
40	24,724
50	24,675

PS18871.01

15. A certain manufacturer uses the function $C(x) = 0.04x^2 - 8.5x + 25{,}000$ to calculate the cost, in dollars, of producing x thousand units of its product. The table above gives values of this cost function for values of x between 0 and 50 in increments of 10. For which of the following intervals is the average rate of *decrease* in cost less than the average rate of *decrease* in cost for each of the other intervals?

    A. From $x = 0$ to $x = 10$

    B. From $x = 10$ to $x = 20$

    C. From $x = 20$ to $x = 30$

    D. From $x = 30$ to $x = 40$

    E. From $x = 40$ to $x = 50$

Arithmetic Applied problems

Since the average rate of decrease of $C(x)$ in the interval from $x = a$ to $x = a + 10$ is $\frac{C(a+10)-C(a)}{(a+10)-a} = \frac{C(a+10)-C(a)}{10}$, we are to determine for which value of a, chosen from the numbers 0, 10, 20, 30, and 40, the magnitude of $\frac{C(a+10)-C(a)}{10}$ is the least, or equivalently, for which of these values of a the magnitude of $C(a + 10) - C(a)$ is the least. Probably the most straightforward method is to simply calculate or estimate the difference $C(a + 10) - C(a)$ for each of these values of a, as shown in the table below.

a to $a + 10$	$C(a + 10) - C(a)$
0 to 10	−81
10 to 20	−73
20 to 30	−65
30 to 40	−57
40 to 50	−49

Alternatively, since the graph of $C(x) = 0.04x^2 - 8.5x + 25{,}000$ is a parabola with vertex at $x = -\frac{b}{2a} = -\frac{-8.5}{2(0.04)} \approx \frac{8}{0.08} = 100$, it follows that the graph levels out as the value of x approaches a number that is approximately equal to 100. Therefore, among the intervals given, the least magnitude in the average rate of change of $C(x)$ occurs for the interval closest to the vertex, which is the interval from $x = 40$ to $x = 50$.

The correct answer is E.

PS35302.01

16. On the day of the performance of a certain play, each ticket that regularly sells for less than \$10.00 is sold for half price plus \$0.50, and each ticket that regularly sells for \$10.00 or more is sold for half price plus \$1.00. On the day of the performance, a person purchases a total of y tickets, of which x regularly sell for \$9.00 each and the rest regularly sell for \$12.00 each. What is the amount paid, in dollars, for the y tickets?

A. $7y - 2x$

B. $12x - 7y$

C. $-\frac{1}{n} = \frac{n+3}{1-n}$

D. $7y + 4x$

E. $7y + 5x$

Algebra Applied problems

The amount paid for the y tickets is the sum of the amounts paid for two groups of tickets. The first group consists of x tickets, each of which regularly sells for \$9.00. The second group consists of the remaining $(y - x)$ tickets, each of which regularly sells for \$12.00. The amount paid for the first group was x(\$4.50 + \$0.50) = \$5x. The amount paid for the second group was $(y - x)$(\$6.00 + \$1.00) = \$7$(y - x)$, or \$7y − \$7x. Therefore, the amount paid for the y tickets was \$5x + (\$7y − \$7x) = \$$(7y - 2x)$.

The correct answer is A.

PS47302.01

17. If $N = \frac{K}{T + \frac{x}{y}}$, where $T = \frac{K}{5}$ and $x = 5 - T$, which of the following expresses y in terms of N and T?

A. $\frac{N(5-T)}{T(5-N)}$

B. $\frac{N(T-5)}{T(5-N)}$

C. $\frac{5-T}{T(5-N)}$

D. $\frac{5N(5-T)}{T(1-5N)}$

E. $\frac{N(5-T)}{5}$

Algebra **Simplifying algebraic expressions**

To eliminate K and x in the first equation (the only equation in which y appears), use the second and third equations to replace K and x with expressions involving only N and T. Then solve for y in terms of N and T.

N	=	$\dfrac{K}{T+\frac{x}{y}}$	given equation
N	=	$\dfrac{5T}{T+\frac{5-T}{y}}$	substitute using $K=5T$ and $x=5-T$
$N\left(T+\dfrac{5-T}{y}\right)$	=	$5T$	multiply both sides by $T+\dfrac{5-T}{y}$
$NT+\dfrac{N(5-T)}{y}$	=	$5T$	expand left side
$\dfrac{N(5-T)}{y}$	=	$5T-NT$	subtract NT from both sides
$\dfrac{N(5-T)}{y}$	=	$T(5-N)$	factor right side
$N(5-T)$	=	$yT(5-N)$	multiply both sides by y
$\dfrac{N(5-T)}{T(5-N)}$	=	y	divide both sides by $T(5-N)$

Alternatively, the algebraic manipulations involved in solving this type of problem as above can often be replaced with numerical computations by assigning values to the variables. The assigned values need to be consistent with all the constraints in the problem, and, for efficiency, the assigned values should be chosen to minimize the numerical computations. Letting $K = 10$, it follows from $T = \frac{K}{5}$ and $x = 5 - T$ that $T = 2$ and $x = 3$. Using these numerical values, the question can be rephrased as follows.

If $N = \dfrac{10}{2+\frac{3}{y}}$, then which of the following expresses y in terms of N?

A. $\dfrac{3N}{2(5-N)}$

B. $\dfrac{-3N}{2(5-N)}$

C. $\dfrac{3}{2(5-N)}$

D. $\dfrac{15N}{2(1-5N)}$

E. $\dfrac{3N}{5}$

Letting $y = 1$, it follows that $N = \dfrac{10}{2+3} = 2$. Plugging $N = 2$ into A, B, C, D, and E above gives, respectively, 1, −1, $\frac{1}{2}$, $-\frac{5}{3}$, and $\frac{6}{5}$.

The correct answer is A.

PS78302.01

18. If $2x + 5y = 8$ and $3x = 2y$, what is the value of $2x + y$?

A. 4

B. $\dfrac{70}{19}$

C. $DB=\dfrac{1}{2}(6)=3$

D. $\dfrac{56}{19}$

E. $\dfrac{40}{19}$

Algebra **Simultaneous equations**

From $3x = 2y$, it follows that $y = \frac{3}{2}x$, so $8 = 2x + 5\left(\frac{3}{2}x\right) = \frac{19}{2}x$. Then $x = \frac{16}{19}$, $2x = \frac{32}{19}$, and $y = \frac{24}{19}$. Thus $2x + y = \frac{56}{19}$.

The correct answer is D.

PS79302.01

19. If ${}_kS_n$ is defined to be the product of $(n + k)(n - k + 1)$ for all positive integers k and n, which of the following expressions represents ${}_{k+1}S_{n+1}$?

A. $(n + k)(n - k + 2)$

B. $(n + k)(n - k + 3)$

C. $(n + k + 1)(n - k + 2)$

D. $(n + k + 2)(n - k + 1)$

E. $(n + k + 2)(n - k + 3)$

Algebra **Substitution; Simplifying algebraic expressions**

Substituting $n + 1$ for n and $k + 1$ for k in the definition gives ${}_{k+1}S_{n+1}$

$= (n + 1 + k + 1)(n + 1 - (k + 1) + 1)$

$= (n + k + 2)(n - k + 1)$.

The correct answer is D.

PS20502.01

20. There were 36,000 hardback copies of a certain novel sold before the paperback version was issued. From the time the first paperback copy was sold until the last copy of the novel was sold, 9 times as many paperback copies as hardback copies were sold. If a total of 441,000 copies of the novel were sold in all, how many paperback copies were sold?

A. 45,000

B. 360,000

C. 364,500

D. 392,000

E. 396,900

Algebra **First-degree equations**

Let h be the number of hardback copies of the novel that were sold after the paperback version was issued. The following table summarizes the given information.

	Hardbacks sold	Paperbacks sold	Total
Before paperbacks	36,000	0	36,000
After paperbacks	h	$9h$	$10h$

Then, $441{,}000 = 36{,}000 + 10h$ or $h = 40{,}500$ and $9h = (9)(40{,}500) = 364{,}500$.

The correct answer is C.

Alternate Solution

Arithmetic Ratios

From the moment the paperback version was issued, 441,000 – 36,000 = 405,000 copies of the novel were sold. Test the answer choices to see whether the ratio of paperbacks sold to hardbacks sold is 9:1. Start with the middle value, because the answer choices are usually listed in numerical order. This way, if you pick an answer that does not give the 9:1 ratio, you can learn whether it was too high or too low, thus allowing you to eliminate other answers that are too high or too low.

Testing answer choice C, if 364,500 paperbacks were sold, then 405,000 – 364,500 = 40,500 hardbacks were sold after the initial 36,000 hardbacks were sold. Noting that 40,500 hardbacks is 10% of 405,000 and 364,500 paperbacks is 90% of 405,000, the ratio of paperbacks to hardbacks is 9:1.

The correct answer is C.

PS30502.01

21. In the formula $w = \frac{P}{\sqrt[t]{v}}$, integers p and t are positive constants. If $w = 2$ when $v = 1$ and if $w = \frac{1}{2}$ when $v = 64$, then $t =$

A. 1

B. 2

C. 3

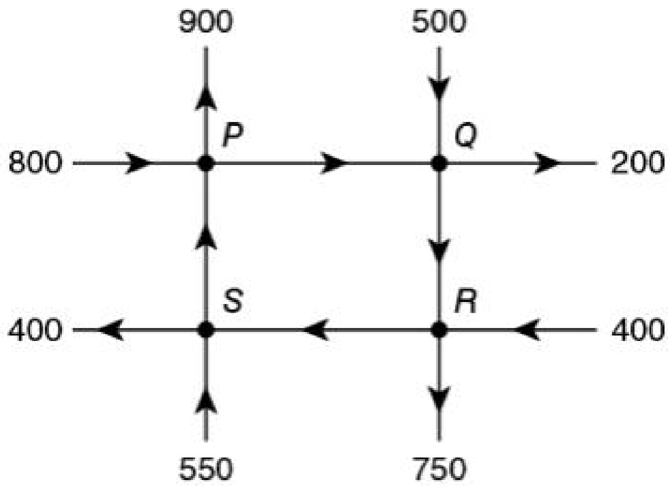
D. 4

E. 16

Algebra Exponents

It is given that $w = \frac{P}{\sqrt[t]{v}}$ and $w = 2$ when $v = 1$. Because 1 raised to any positive power is 1, it follows that $2 = P$. If $w = \frac{1}{2}$ when $v = 64$, then $\frac{1}{2} = \frac{2}{\sqrt[t]{64}}$ or $4 = \sqrt[t]{64}$. Then $\sqrt[t]{64} = 64^{\frac{1}{t}} = 4^{\frac{3}{t}}$ since $64 = 4^3$. So, $4 = 4^{\frac{3}{t}}$ and $t = 3$.

The correct answer is C.



22. The figure above represents a network of one-way streets. The arrows indicate the direction of traffic flow, and the numbers indicate the amount of traffic flow into or out of each of the four intersections during a certain hour. During that hour, what was the amount of traffic flow along the street from *R* to *S* if the total amount of traffic flow into *P* was 1,200 ? (Assume that none of the traffic originates or terminates in the network.)

A. 200

B. 250

C. 300

D. 350

E. 400

Arithmetic **Computation with integers**

In the following, the notation $A \to B$ will be used to represent the amount of traffic flow from A into B. Let x represent $R \to S$. From the figure, the amount of traffic flow into P was 800 plus $S \to P$. The total amount of traffic flow into P was 1,200, so $S \to P$ was 400. The amount of traffic flow into S was 550 plus $R \to S$, so $550 + x$. The amount of traffic flow out of S was 400 plus $S \to P$, or $400 + 400 = 800$. Since the amount of traffic flow into S must equal the amount of traffic flow out of S, $550 + x = 800$. Therefore, $x = 250$, so $R \to S$ was 250.

The correct answer is B.

PS23502.01

23. If C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit, then the relationship between temperatures on the two scales is expressed by the equation $9C = 5(F - 32)$. On a day when the temperature extremes recorded at a certain weather station differed by 45 degrees on the Fahrenheit scale, by how many degrees did the temperature extremes differ on the Celsius scale?

A. $\frac{65}{9}$

B. 13

C. 25

D. 45

E. 81

Algebra **Formulas**

Let F represent the larger extreme on the Fahrenheit scale. Then, $F - 45$ is the smaller extreme. It follows that the difference in the temperatures on the Celsius scale is $\frac{5}{9}(F-32) - \frac{5}{9}[(F-45)-32] = 25$.

The correct answer is C.

PS93502.01

24. If $d = \frac{a+b}{1+\frac{ab}{c^2}}$, $a = \frac{c}{2}$, and $b = \frac{3c}{4}$, what is the value of d in terms of c ?

A. $\frac{10c}{11}$

B. $\frac{5c}{2}$

C. $\frac{10c}{3}$

D. $\frac{10}{11c}$

E. $\frac{5}{2c}$

Algebra **Simplifying algebraic expressions**

First, $a + b = \frac{c}{2} + \frac{3c}{4} = \frac{5c}{4}$ and $1 + \frac{ab}{c^2} = 1 + \frac{\left(\frac{c}{2}\right)\left(\frac{3c}{4}\right)}{c^2} = 1 + \frac{3}{8} = \frac{11}{8}$. Then, $d = \frac{a+b}{1+\frac{ab}{c^2}} = \frac{\frac{5c}{4}}{\frac{11}{8}} = \frac{10c}{11}$.

The correct answer is A.

PS04502.01

25. A school supply store sells only one kind of desk and one kind of chair, at a uniform cost per desk or per chair. If the total cost of 3 desks and 1 chair is twice that of 1 desk and 3 chairs, then the total cost of 4 desks and 1 chair is how many times that of 1 desk and 4 chairs?

A. 5

B. 3

C. $x^4 + x^2 + 1 = \frac{1}{x^4 + x^2 + 1}$

D. $\frac{5}{2}$

E. $\frac{7}{3}$

Algebra Simultaneous equations

Let d represent the cost of 1 desk and let c represent the cost of 1 chair. It is given that $3d + c = 2(d + 3c)$. It follows that $d = 5c$. Then $4d + c = 21c$ and $d + 4c = 9c$. Since $\frac{21c}{9c} = \frac{7}{3}$, the total cost of 4 desks and 1 chair is $\frac{7}{3}$ times that of 1 desk and 4 chairs.

The correct answer is E.

PS35502.01

26. A certain truck traveling at 55 miles per hour gets 4.5 miles per gallon of diesel fuel consumed. Traveling at 60 miles per hour, the truck gets only 3.5 miles per gallon. On a 500-mile trip, if the truck used a total of 120 gallons of diesel fuel and traveled part of the trip at 55 miles per hour and the rest at 60 miles per hour, how many miles did it travel at 55 miles per hour?

A. 140

B. 200

C. 250

D. 300

E. 360

Algebra Applied problems

Let m be the number of miles the truck traveled at 55 miles per hour. It follows that $500 - m$ is the number of miles the truck traveled at 60 miles per hour. Then, $\frac{m}{4.5}$ is the number of gallons the truck used while traveling at 55 miles per hour and $\frac{500-m}{3.5}$ is the number of gallons the truck used while traveling at 60 miles per hour, so $\frac{m}{4.5} + \frac{500-m}{3.5} = 120$. Solving this equation gives $m = 360$.

The correct answer is E.

PS45502.01

27. A merchant paid $300 for a shipment of x identical calculators. The merchant used two of the calculators as demonstrators and sold each of the others for $5 more than the average (arithmetic mean) cost of the x calculators. If the total revenue from the sale of the calculators was $120 more than the cost of the shipment, how many calculators were in the shipment?

A. 24

B. 25

C. 26

D. 28

E. 30

Algebra Second-degree equations

The merchant paid $300 for a shipment of x calculators, so the average cost, in dollars, per calculator was $\frac{300}{x}$. The merchant sold $(x - 2)$ of them at the price of $5 + \frac{300}{x}$ dollars each, for a total revenue of $(x-2)\left(5 + \frac{300}{x}\right)$ dollars, which was $120 + 300 = 420$. Manipulating the equation $(x-2)\left(5 + \frac{300}{x}\right) = 420$ gives $x^2 - 26x - 120 = 0$ or $(x - 30)(x + 4) = 0$, which can be solved by factoring. It follows that there were 30 calculators in the shipment.

The correct answer is E.

PS06502.01

28. A car traveled 462 miles per tankful of gasoline on the highway and 336 miles per tankful of gasoline in the city. If the car traveled 6 fewer miles per gallon in the city than on the highway, how many miles per gallon did the car travel in the city?

A. 14

B. 16

C. 21

D. 22

E. 27

Algebra Applied problems

Let g be the number of gallons of gasoline in 1 tankful. Then the number of miles per gallon while traveling on the highway is $\frac{462}{g}$ and this number is 6 more than $\frac{336}{g}$, which is the number of miles per gallon while traveling in the city. Solving $\frac{462}{g} = 6 + \frac{336}{g}$ gives $g = 21$. Therefore, the number of miles per gallon while the car was traveling in the city was $\frac{336}{21} = 16$.

The correct answer is B.

PS56502.01

29. Machines X and Y run at different constant rates, and machine X can complete a certain job in 9 hours. Machine X worked on the job alone for the first 3 hours and the two machines, working together, then completed the job in 4 more hours. How many hours would it have taken machine Y, working alone, to complete the entire job?

A. 18

B. $13\frac{1}{2}$

C. $7\frac{1}{5}$

D. $4\frac{1}{2}$

E. $3\frac{2}{3}$

Algebra Applied problems

Machine X can complete the job in 9 hours, so it completed $\frac{1}{3}$ of the job before machine Y started working. Since machine X worked 4 more hours, it completed $\frac{7}{9}$ of the job, which left $\frac{2}{9}$ of the job for machine Y to do during the 4 hours it worked together with machine X to complete the job. At this rate, machine Y would take $\left(\frac{9}{2}\right)(4) = 18$ hours to do the entire job working alone.

Alternatively, machine X can complete the job in 9 hours, so it completed $\frac{1}{3}$ of the job before machine Y started working. This leaves $\frac{2}{3}$ of the job, which took machines X and Y, working together, 4 hours to complete. So, if working together the machines took 4 hours to complete $\frac{2}{3}$ of the job, it would take them 6 hours to do the whole job working together. Furthermore, rates are additive, so we should convert working times to rates. Machine X's rate is $\frac{1}{9}$ jobs per hour. Both machines together have a rate of $\frac{1}{6}$ jobs per hour. We must determine machine Y's individual rate $\frac{1}{Y}$ for the whole job. First, add machine X's rate to machine Y's rate to get the combined rate. Then,

$$\frac{1}{9} + \frac{1}{Y} = \frac{1}{6}$$
$$\frac{1}{Y} = \frac{1}{6} - \frac{1}{9}$$
$$\frac{1}{Y} = \frac{2}{36}$$
$$Y = 18$$

The correct answer is A.

PS77602.01

30. If $\pi\left(\frac{d_1}{2}\right)+\pi\left(\frac{d_2}{2}\right)+\pi\left(\frac{d_3}{2}\right)=\frac{\pi}{2}(d_1+d_2+d_3)$, then the value of which of the following can be determined?

I. $\frac{2t}{s}$

II. $\frac{s-t}{t}$

III. $\frac{t-1}{s-1}$

A. I only

B. III only

C. I and II only

D. II and III only

E. I, II, and III

Algebra **Simplifying algebraic expressions**

Substitute 2t for s in the expressions given in I, II, and III.

✓ I. Value can be determined: $\frac{2t}{s}=\frac{2t}{2t}=1$

✓ II. Value can be determined: $\frac{s-t}{t}=\frac{2t-t}{t}=\frac{t}{t}=1$

× III. Value cannot be determined: $\frac{t-1}{s-1}=\frac{t-1}{2t-1}$ equals 0 if $t=1$, and equals $\frac{1}{3}$ if $t=2$

The correct answer is C.

PS58602.01

31. If $k\neq 0$ and $k-\frac{3-2k^2}{k}=\frac{x}{k}$, then $x=$

A. $-3-k^2$

B. k^2-3

C. $3k^2-3$

D. $k-3-2k^2$

E. $k-3+2k^2$

Algebra **Simplifying algebraic expressions**

Multiplying both sides of the equation by k gives $k^2-(3-2k^2)=x$ or $x=3k^2-3$.

The correct answer is C.

PS68602.01

32. The sum of the ages of Doris and Fred is y years. If Doris is 12 years older than Fred, how many years old will Fred be y years from now, in terms of y ?

A. $y-6$

B. $2y-6$

C. $\frac{y}{2}-6$

D. $\frac{3y}{2}-6$

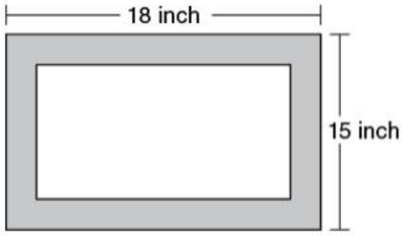
E. $\frac{5y}{2}-6$

Algebra **Applied problems**

Let D and F represent Doris's and Fred's current ages, respectively. It is given that $D+F=y$ and $D=F+12$. It follows that $(F+12)+F=y$ and $F=\frac{y-12}{2}=\frac{y}{2}-6$. Therefore, Fred's age y years from now will be $y+\left(\frac{y}{2}-6\right)$ $=\frac{3y}{2}-6$.

The correct answer is D.

Geometry



PS35461.01

33. The shaded region in the figure above represents a rectangular frame with length 18 inches and width 15 inches. The frame encloses a rectangular picture that has the same area as the frame itself. If the length and width of the picture have the same ratio as the length and width of the frame, what is the length of the picture, in inches?

A. $9\sqrt{2}$

B. $\frac{3}{2}$

C. $\frac{9}{\sqrt{2}}$

D. $15(1-\frac{1}{\sqrt{2}})$

E. $\frac{9}{2}$

Geometry Rectangles; Area

Let k be the proportionality constant for the fractional decrease from the dimensions of the frame to the dimensions of the picture. That is, let $18k$ be the length of the picture and let $15k$ be the width of the picture. We are given that $(18)(15) = 2(18k)(15k)$. Hence, $k^2 = \frac{1}{2}$ and $k = \frac{1}{\sqrt{2}}$. Therefore, the length of the picture is $18k = \frac{18}{\sqrt{2}} = 9\sqrt{2}$.

The correct answer is A.

PS56271.01

34. The *surface distance* between 2 points on the surface of a cube is the length of the shortest path on the surface of the cube that joins the 2 points. If a cube has edges of length 4 centimeters, what is the surface distance, in centimeters, between the lower left vertex on its front face and the upper right vertex on its back face?

A. 8

B. $4\sqrt{5}$

C. $8\sqrt{2}$

D. $12\sqrt{2}$

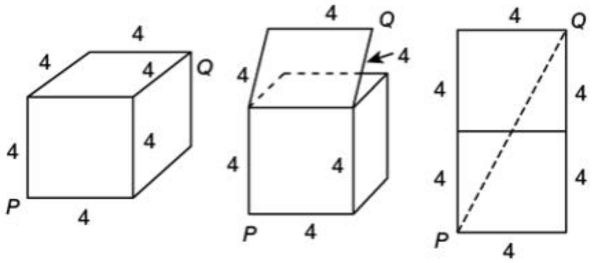
E. $4\sqrt{2}+4$

Geometry Rectangular solids

The left figure below shows a cube with edge length 4, where P is the lower left vertex on its front face and Q is the upper right vertex on its back face. It is clear that the shortest path on the surface between P and Q consists of a path on the front face joined to a path on the top face, or a path on the front face joined to a path on the right face. In fact, each of these two approaches can be used in essentially the same way to give a path from P to Q, whose length is the surface distance between P and Q. To simplify the exposition, we will consider the case where the surface distance is the length of a certain path P_F on the front face plus the length of a certain path P_T on the top face.

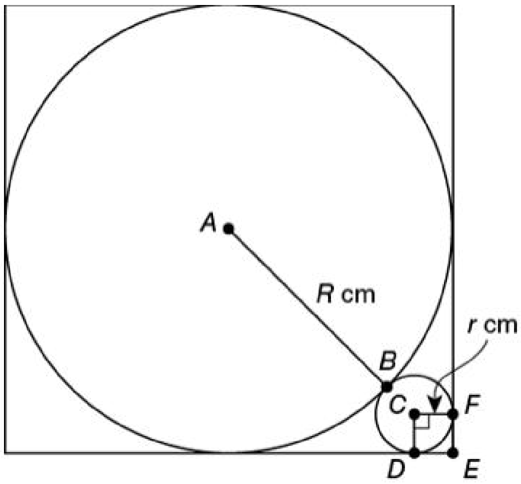
The middle figure below shows the top face of the cube lifted about 45 degrees, and the right figure below shows only the front and top faces of the cube after the top face has been lifted 90 degrees. Since rotations of the top face about this "hinge" do not change the length of any path on the front face or on the top face, this rotation by 90 degrees will not change the length of P_F or the length of P_T, and hence this rotation will not change the sum of the lengths of P_F and P_T.

In the right figure below, the shortest path from P to Q is the dashed segment shown in the figure. Moreover, the length of this dashed segment is the surface distance between P and Q because if P_F and P_T did not correspond to portions of this dashed segment on the front and top faces, respectively, then the sum of the lengths of P_F and P_T would be greater than the length of the dashed segment, since the shortest distance between P and Q in the right figure below is the length of the line segment with endpoints P and Q.



From the discussion above, it follows that the surface distance between P and Q is the length of the dashed segment in the right figure, which is easily found by using the Pythagorean theorem:
$\sqrt{4^2+8^2}=\sqrt{4^2(1+2^2)}=4\sqrt{5}$.

The correct answer is B.



PS75571.01

35. The figure above shows 2 circles. The larger circle has center A, radius R cm, and is inscribed in a square. The smaller circle has center C, radius r cm, and is tangent to the larger circle at point B and to the square at points D and F. If points A, B, C, and E are collinear, which of the following is equal to $\frac{R}{r}$?

A. $\frac{2}{\sqrt{2}+1}$

B. $\frac{2}{\sqrt{2}-1}$

C. $\frac{2}{2\sqrt{2}+1}$

D. $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

E. $\frac{2\sqrt{2}+1}{2\sqrt{2}-1}$

Geometry **Circles; Pythagorean theorem**

Because $\overline{CE}$ is a diagonal of square $CDEF$, which has side length r, it follows from the Pythagorean theorem that $r^2 + r^2 = (CE)^2$, and hence $CE = r\sqrt{2}$.

Tip

A sometimes useful shortcut is the fact that, for a square we have $d = s\sqrt{2}$, where d is the diagonal length and s is the side length. This can be obtained by applying the Pythagorean theorem as above or by using properties of a 45–45–90 triangle.

Therefore, $BE = r + r\sqrt{2} = r(1 + \sqrt{2})$ and $AE = R + r(1 + \sqrt{2})$. Since $2(AE)$ is the diagonal length of the large square, which has side length $2R$, it follows from the above tip that $2(AE) = (2R)\sqrt{2}$, or $AE = R\sqrt{2}$. Alternatively, an appropriate application of the Pythagorean theorem gives $R^2 + R^2 = (AE)^2$, or $AE = R\sqrt{2}$. Now substitute for AE and solve for $\frac{R}{r}$.

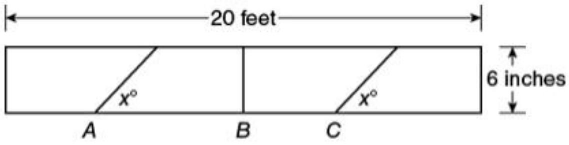
$$AE = R\sqrt{2}$$

$$R + r(1 + \sqrt{2}) = R\sqrt{2} \quad \text{substitute for } AE$$

$$r(\sqrt{2}+1) = R(\sqrt{2} - 1) \quad \text{rearrange terms}$$

From the last equation we get $\frac{R}{r} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$.

The correct answer is D.



Note: Figure not drawn to scale.

PS15302.01

36. The figure above shows the dimensions of a rectangular board that is to be cut into four identical pieces by making cuts at points A, B, and C, as indicated. If $x = 45$, what is the length AB ?

(1 foot = 12 inches)

A. 5 ft 6 in

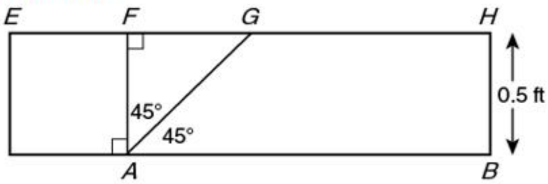
B. 5 ft $3\sqrt{2}$ in

C. 5 ft 3 in

D. 5 ft

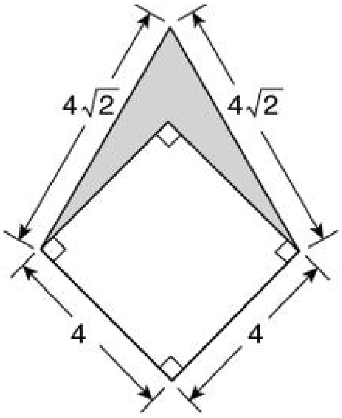
E. 4 ft 9 in

Geometry **Rectangles; Triangles**



The figure above shows the left side of the rectangular board with points E, F, G, and H added and segment $\overline{FA}$ added. We are to determine the value of AB, which equals the value of FH. Since ΔAFG is an isosceles triangle, it follows that $FG = FA = 0.5$ ft. Moreover, $EF = GH$ because the four pieces of the rectangular board have the same dimensions. Therefore, since EH is half the length of the 20 ft board, $EH = 10$ ft and we have $EF + FG + GH = 10$ ft, or $2(GH) + 0.5$ ft $= 10$ ft, or $GH = 4.75$ ft. Hence, $AB = FH = FG + GH = 0.5$ ft $+ 4.75$ ft $= 5.25$ ft, which equals 5 ft 3 in.

The correct answer is C.



PS57302.01

37. In the figure above, the area of the shaded region is

A. $8\sqrt{2}$

B. $4\sqrt{3}$

C. $4\sqrt{2}$

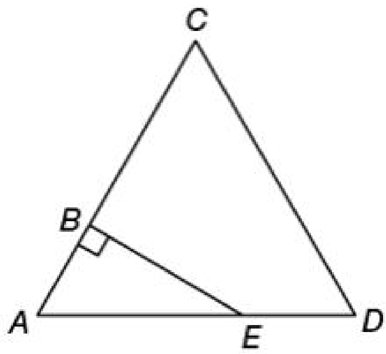
D. $8(\sqrt{3}-1)$

E. $8(\sqrt{2}-1)$

Geometry Triangles; Area

First, the diagonal of the square with sides of length 4 is $4\sqrt{2}$. From this it follows that the area of the shaded region consists of the area of an equilateral triangle with sides $4\sqrt{2}$ minus $\frac{1}{2}$ the area of a square with sides of length 4. Thus, the area of the shaded region is $\frac{\sqrt{3}}{4}(4\sqrt{2})^2 - \frac{1}{2}(4^2) = 8(\sqrt{3}-1)$.

The correct answer is D.



PS18302.01

38. If each side of ΔACD above has length 3 and if AB has length 1, what is the area of region $BCDE$?

A. $\frac{9}{4}$

B. $\frac{7}{4}\sqrt{3}$

C. $\frac{9}{4}\sqrt{3}$

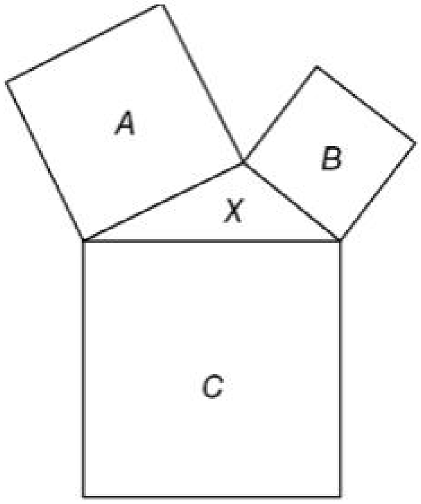
D. $\frac{7}{2}\sqrt{3}$

E. $6+\sqrt{3}$

Geometry **Triangles; Area**

The area of region *BCDE* is the area of ΔACD minus the area of ΔABE. Since ΔACD is equilateral, its area is $\frac{\sqrt{3}}{4}(3^2)=\frac{9\sqrt{3}}{4}$. ΔABE is a 30–60–90 triangle with side lengths 1, $\sqrt{3}$, and 2 and area $\frac{\sqrt{3}}{2}$. Thus, the area of region *BCDE* is $\frac{9\sqrt{3}}{4}-\frac{\sqrt{3}}{2}=\frac{7\sqrt{3}}{4}$.

The correct answer is B.



Note: Figure not drawn to scale.

PS76402.01

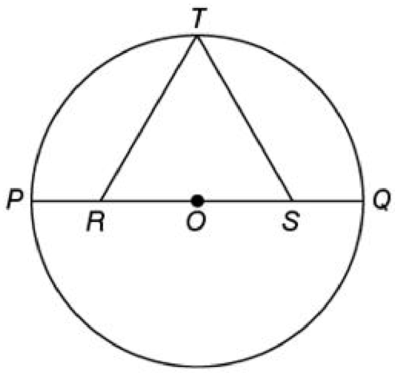
39. In the figure above, three squares and a triangle have areas of A, B, C, and X as shown. If $A = 144$, $B = 81$, and $C = 225$, then $X =$

    A. 150
    B. 144
    C. 80
    D. 54
    E. 36

Geometry **Triangles; Quadrilaterals; Area**

The side lengths of the squares with areas 144, 81, and 225 are 12, 9, and 15, respectively, so the triangle with area X has sides 12, 9, and 15. Because $12^2 + 9^2 = 15^2$, the triangle with area X is a right triangle with legs of length 12 and 9. Thus, $X = \frac{1}{2}(12)(9) = 54$.

The correct answer is D.



PS57402.01

40. In the figure above, PQ is a diameter of circle O, $PR = SQ$, and ΔRST is equilateral. If the length of PQ is 2, what is the length of RT ?

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

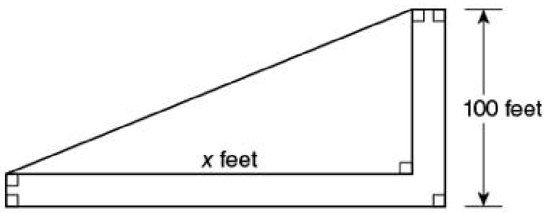
D. $\frac{2}{\sqrt{3}}$

E. $\sqrt{3}$

Geometry Triangles

Since $PR = SQ$, it follows that $RO = OS$, so O is the midpoint of $\overline{RS}$. Since ΔRST is equilateral and O is the midpoint of $\overline{RS}$, ΔROT is a 30°–60°–90° triangle, and since $\overline{OT}$ is a radius of the circle with diameter 2, $OT = 1$. Using the ratios of the sides of a 30°–60°–90° triangle, it follows that $RT = \frac{2}{\sqrt{3}}$.

The correct answer is D.



Note: Figure not drawn to scale.

PS22502.01

41. The figure above shows some of the dimensions of a triangular plaza with an L-shaped walk along two of its edges. If the width of the walk is 4 feet and the total area of the plaza and walk together is 10,800 square feet, what is the value of x ?

A. 200

B. 204

C. 212

D. 216

E. 225

Geometry Polygons; Area

The area of the triangular plaza is given by $\frac{1}{2}x(100-4) = 48x$, and the area of the walkway is given by $(x+4)(4) + (96)(4)$. Since the total area of the plaza and walkway is 10,800, it follows that $52x + 400 = 10{,}800$ and $x = 200$.

The correct answer is A.

PS88602.01

42. A circular rim 28 inches in diameter rotates the same number of inches per second as a circular rim 35 inches in diameter. If the smaller rim makes x revolutions per second, how many revolutions per minute does the larger rim make in terms of x ?

A. $\frac{48\pi}{x}$

B. $75x$

C. $48x$

D. $24x$

E. $\frac{x}{75}$

Geometry Circles; Circumference

Since the smaller rim has diameter 28 inches and rotates x revolutions per second, it rotates $28\pi x$ inches per second. If y represents the number of revolutions the larger rim rotates per second, then the larger rim rotates $35\pi y$ inches per second. Since the rims rotate the same number of inches per second, it follows that $28\pi x = 35\pi y$. Then $y = \frac{28x}{35} = \frac{4x}{5}$ inches per second or $\frac{4x}{5}(60) = 48x$ inches per minute.

The correct answer is C.

Rates/Ratios/Percent

PS17302.01

43. The annual stockholders' report for Corporation X stated that profits were up 10 percent over the previous year, although profits as a percent of sales were down 10 percent. Total sales for that year were approximately what percent of sales for the previous year?

A. 78%

B. 90%

C. 110%

D. 122%

E. 190%

Algebra Percents

Let P_1 and S_1 be the profit and sales for the previous year, and let P_2 and S_2 be the profit and sales for the following year. It is given that $P_2 = 1.1P_1$ and $\frac{P_2}{S_2} = 0.9\left(\frac{P_1}{S_1}\right)$. Substituting the first equation into the second equation gives $\frac{1.1\cancel{P_1}}{S_2} = 0.9\left(\frac{\cancel{P_1}}{S_1}\right)$, or $S_2 = \left(\frac{1.1}{0.9}\right)S_1 \approx (1.22)S_1$. Therefore, S_2 is approximately 122% of S_1.

The correct answer is D.

44. A certain brand of house paint must be purchased either in quarts at $12 each or in gallons at $18 each. A painter needs a 3-gallon mixture of the paint consisting of 3 parts blue and 2 parts white. What is the least amount of money needed to purchase sufficient quantities of the two colors to make the mixture?

(4 quarts = 1 gallon)

A. $54

B. $60

C. $66

D. $90

E. $144

Arithmetic Applied problems

To make 3 gallons of the mixture requires 12 quarts, and the least amount of money is achieved by purchasing the greatest number of gallons and least number of quarts. Letting B be the number of quarts of blue paint needed and W be the number of quarts of white paint needed, it follows that $B + W = 12$, where $B = \frac{3}{2}W$ since the mixture has a blue to white ratio of 3 to 2. This gives $B = 7\frac{1}{5}$ and $W = 4\frac{4}{5}$. Since the paint can be purchased in whole quarts only, the painter must purchase 8 quarts or 2 gallons of blue, and 5 quarts or 1 gallon plus 1 quart of white for a total of (2 + 1)($18) + $12 = $66.

The correct answer is C.

Month	Change in sales from previous month
February	+10%
March	−15%
April	+20%
May	−10%
June	+5%

PS43481.01

45. The table above shows the percent of change from the previous month in Company X's sales for February through June of last year. A positive percent indicates that Company X's sales for that month increased from the sales for the previous month, and a negative percent indicates that Company X's sales for that month decreased from the sales for the previous month. For which month were the sales closest to the sales in January?

A. February

B. March

C. April

D. May

E. June

Arithmetic Percents

Explicit calculation incorporating a few numerical shortcuts gives the following, where J is the January sales amount.

February sales: **1.1*J***

March sales: $0.85(1.1J) = 0.85J + 0.85(0.1J) = 0.85J + 0.085J =$ **0.935*J***

April sales: $1.2(0.935J) = 0.935J + 0.2(0.935J) = 0.935J + 0.187J =$ **1.122*J***

May sales: $0.9(1.122J) =$ **1.0098*J***

June sales: **1.05(1.0098*J*)** (May is clearly closer)

Alternatively, from

$$\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)=1+\frac{x+y}{100}+\frac{x}{100}\left(\frac{y}{100}\right)$$

it follows that a percent change of x% followed by a percent change of y% is equal to a percent change of $(x + y)$% plus x% of y percentage points (equivalently, plus y% of x percentage points).

Percent change from January through February: The percent change is given as **+10%**.

Percent change from January through March: Using the rule above for $x = +10$ and $y = -15$ gives $(+10 - 15)\% + (+0.10)(-15\%)$, or $-5\% - 1.5\% =$ **−6.5%**.

Percent change from January through April: This is equivalent to a −6.5% change (percent change from January through March) followed by a +20% change, and hence using the rule above for $x = -6.5$ and $y = +20$ gives $(-6.5 + 20)\% + (+0.20)(-6.5\%)$, or $+13.5\% - 1.3\% =$ **+12.2%**.

Percent change from January through May: This is equivalent to a +12.2% change (percent change from January through April) followed by a −10% change, and hence using the rule above for $x = +12.2$ and $y = -10$ gives $(+12.2 - 10)\% + (-0.10)(+12.2\%)$, or $+2.2\% - 1.22\% =$ **+0.98%**.

Percent change from January through June: This is clearly **greater than +0.98% + 5%**, and hence greater in magnitude than the result for May.

From the results above, the least change in the magnitude of the percent change from January occurred for May.

The correct answer is D.

PS56302.01

46. A store bought 5 dozen lamps at $30 per dozen and sold them all at $15 per lamp. The profit on each lamp was what percent of its selling price?

    A. 20%
    B. 50%
    C. $83\frac{1}{2}\%$
    D. 100%
    E. 500%

Arithmetic **Applied problems; Percents**

For this problem it is especially important to keep your focus on what is asked and to ignore extraneous details.

The cost per lamp is $\frac{\$30}{\text{dozen}} = \frac{\$30}{12}$ and the selling price per lamp is $15, so the profit per lamp is $\$\left(15 - \frac{30}{12}\right)$.

Therefore, for each lamp the profit as a percent of the selling price is $\frac{15 - \frac{30}{12}}{15} = 1 - \frac{{}^{2}\cancel{30}}{(12)(\cancel{15})} = 1 - \frac{1}{6} = 83\frac{1}{3}\%$.

The correct answer is C.

PS76302.01

47. Store N gives a 50 percent discount on the list price of all its items and Store W gives a 60 percent discount on the list price of all its items. If the list price of the same item is 20 percent higher in Store W, what percent (more or less) of the selling price in Store N is the selling price of the item in Store W ?

    A. 10% less
    B. 4% less
    C. 2% less
    D. 10% more
    E. 12% more

Arithmetic **Percents**

Let $\$P$ be the list price of the item in Store N. The table shows the list and selling prices of the item in the two stores. For example, the selling price of the item in Store W is 60 percent less than the item's list price of $\$1.2P$, or $(0.4)(\$1.2P) = \$0.48P$.

	List price ($)	Selling price ($)
Store N	P	$0.5P$
Store W	$1.2P$	$0.48P$

The amount, in dollars, by which the selling price of the item in Store W is less than the selling price of the item in Store N is $0.5P - 0.48P = 0.02P$, which is $\frac{0.02P}{0.5P} = 4\%$ less as a percent of the selling price in Store N.

Alternatively, we can assign a specific value to P and carry out the computations using this value. In percent problems, the computations are usually simpler when 100 is used, so let $P = 100$. (If, say, $33\frac{1}{3}\%$ of P had been involved, then $P = 300$ might be a better choice.) Therefore, the list price of the item in Store N is \$100, the discount price of the item in Store N is (50%)(\$100) = \$50, the list price of the item in Store W is (120%)(\$100) = \$120, and the discount price of the item in Store W is (40%)(\$120) = \$48. Thus, the question becomes the following. What percent (more or less) of 50 is 48 ? A simple computation shows that 48 is $\left(\frac{50-48}{50}\times 100\right)\% = 4\%$ less than 50.

The correct answer is B.

PS95402.01

48. A merchant purchased a jacket for \$60 and then determined a selling price that equaled the purchase price of the jacket plus a markup that was 25 percent of the selling price. During a sale, the merchant discounted the selling price by 20 percent and sold the jacket. What was the merchant's gross profit on this sale?

    A. \$0

    B. \$3

    C. \$4

    D. \$12

    E. \$15

Algebra **Percents**

The purchase price, in dollars, of the jacket was 60. If S represents the selling price, in dollars, then $S = 60 + 0.25S$, from which $S = 80$. During the sale, the discounted selling price, in dollars, was $0.8(80) = 64$, so the merchant's gross profit, in dollars, was $64 - 60 = 4$.

The correct answer is C.

Tip

Read carefully. Usually, markup is a percent of the purchase price, but in this problem, it is a percent of the selling price.

PS12502.01

49. When a certain stretch of highway was rebuilt and straightened, the distance along the stretch was decreased by 20 percent and the speed limit was increased by 25 percent. By what percent was the driving time along this stretch reduced for a person who always drives at the speed limit?

    A. 16%

    B. 36%

    C. $37\frac{1}{2}\%$

    D. 45%

    E. $56\frac{1}{4}\%$

Arithmetic **Applied problems; Percents**

Let D and r be the distance and speed limit, respectively, along the stretch of highway before it was rebuilt. Then, the distance and speed limit along the stretch of highway after it was rebuilt are given by $0.8D$ and $1.25r$. It follows that the percent reduction in time is

$$\left[\left(\frac{\frac{D}{r}-\frac{0.8D}{1.25r}}{\frac{D}{r}}\right)\times 100\right]\% = \left[\left(1-\frac{0.8}{1.25}\right)\times 100\right]\% = 36\%.$$

Alternatively, it helps to use actual numbers when calculating percent change, with 100 being the most mathematically convenient number to use. For this problem, set the speed limit at 100 miles per hour and the distance at 100 miles. A 20 percent decrease in distance makes the new distance 80 miles. A 25 percent increase in speed limit makes the new speed limit 125 miles per hour, which, of course, is unrealistic, but very convenient to work with. Then, the new time $\frac{\text{distance}}{\text{rate}}$ is $\frac{80}{125}$ hours. The percent change in time is $\left[\left(1-\frac{0.8}{1.25}\right)\times 100\right]\%$ $= 36\%$.

The correct answer is B.

Value/Order/Factors

	Number of components:		
Components	Monday	Tuesday	Wednesday
A	3	6	3
B	6	3	4
C	4	7	4

PS56441.01

50. A factory assembles Product X from three components, A, B, and C. One of each component is needed for each Product X and all three components must be available when assembly of each Product X starts. It takes two days to assemble one Product X. Assembly of each Product X starts at the beginning of one day and is finished at the end of the next day. The factory can work on at most five Product Xs at once. If components are available each day as shown in the table above, what is the largest number of Product Xs that can be assembled during the three days covered by the table?

A. 3

B. 5

C. 6

D. 9

E. 10

Arithmetic **Applied problems**

We will determine the largest number of Product Xs that can be assembled during all three days by considering separately the largest number that can be assembled if 0, 1, 2, or 3 Product Xs begin assembly on Monday.

0 Product Xs begin assembly on Monday: In this case, at most three Product Xs can begin assembly on Tuesday (because only three units of Component A are available on Wednesday), and hence at most 0 + 3 = **3 Product Xs** could be assembled during the three days.

1 Product X begins assembly on Monday: In this case, at most three Product Xs can begin assembly on Tuesday (because only three units of Component A are available on Wednesday), and hence at most 1 + 3 = **4 Product Xs** could be assembled during the three days.

2 Product Xs begin assembly on Monday: In this case, at most three Product Xs can begin assembly on Tuesday (because only three units of Component A are available on Wednesday), and hence at most 2 + 3 = **5 Product Xs** could be assembled during the three days.

3 Product Xs begin assembly on Monday: In this case, at most two Product Xs can begin assembly on Tuesday (because the factory can work on at most five Product Xs at once), and hence at most 3 + 2 = **5 Product Xs** could be assembled during the three days.

Therefore, the largest number of Product Xs that can be assembled during the three days is 5.

The correct answer is B.

PS04851.01

51. How many positive integers n have the property that both $3n$ and $\frac{n}{3}$ are 4-digit integers?

A. 111

B. 112

C. 333

D. 334

E. 1,134

Arithmetic Inequalities

If n is an integer, then $3n$ is always an integer. Also, $3n$ will be a 4-digit integer only when $1{,}000 \le 3n \le 9{,}999$. Therefore, n is an integer such that $333\frac{1}{3} \le n \le 3{,}333$. Equivalently, n is an integer such that $334 \le n \le 3{,}333$.

If n is an integer, then $\frac{n}{3}$ is an integer only when n is a multiple of 3. Also, $\frac{n}{3}$ will be a 4-digit integer only when $1{,}000 \le \frac{n}{3} \le 9{,}999$, or $3{,}000 \le n \le 29{,}997$. Therefore, n is a multiple of 3 such that $3{,}000 \le n \le 29{,}997$.

It follows that the values of n consist of all multiples of 3 between 3,000 = 3(1,000) and 3,333 = 3(1,111), inclusive. The number of such multiples of 3 is (1,111 − 1,000) + 1 = 112.

Tip

Be alert to possible easily overlooked constraints that may exist in a problem. For example, in applying the second requirement above, it is not sufficient to only consider integer values of n such that $1{,}000 \le \frac{n}{3} \le 9{,}999$. In addition, $\frac{n}{3}$ must also be an integer, and by applying this constraint it follows that the values of n must be multiples of 3.

The correct answer is B.

PS24851.01

52. If Whitney wrote the decimal representations for the first 300 positive integer multiples of 5 and did not write any other numbers, how many times would she have written the digit 5 ?

A. 150

B. 185

C. 186

D. 200

E. 201

Arithmetic Properties of integers

The number of times the digit 5 would be written is the number of times the digit 5 will appear in the units place plus the number of times the digit 5 will appear in the tens place plus the number of times the digit 5 will appear in the hundreds place.

Number of times the digit 5 will appear in the units place: This will be the number of terms in the sequence 5, 15, 25, 35, ..., 1485, 1495. Adding 5 to each member of this sequence does not change the number of terms, and doing this gives the sequence 10, 20, 30, 40, ..., 1490, 1500, which clearly has 150 terms (dividing the terms by 10 gives 1, 2, 3, 4, ..., 149, 150). Thus, the digit 5 appears **150 times** in the units place.

Number of times the digit 5 will appear in the tens place: This will be the number of terms in the sequence 50, 55, 150, 155, 250, 255, ..., 1450, 1455. The digit 5 appears in the tens place twice for each consecutive change in the hundreds digit. Thus, the digit 5 appears 2(15) = **30 times** in the tens place.

Number of times the digit 5 will appear in the hundreds place: This will be the number of terms in the sequence 500, 505, 510, 515, ..., 590, 595, 1500. Thus, the digit 5 appears 20 + 1 = **21 times** in the hundreds place.

Therefore, the number of times the digit 5 would be written is 150 + 30 + 21 = 201.

Tip

The method used above to count the number of terms in the sequence 5, 15, 25, 35, ..., 1,485, 1,495 can be applied to any arithmetic sequence, and it avoids the necessity of remembering certain formulas. For example, to determine the number of terms in the sequence 13, 19, 25, 31, 37, 43, ..., 301, we first observe that consecutive differences are equal to 6, so we subtract from each term a number chosen so that the first term becomes (1)(6) = 6. Thus, we subtract 7 from each term and obtain the sequence 6, 12, 18, 24, 30, 36, ..., 294, which has the same number of terms as the original sequence.The number of terms in this new sequence is now easy to find—divide each term of this new sequence by 6, and it will be clear that the number of terms is 49.

Alternatively, in the 2-digit multiples of 5, namely the multiples of 5 in the interval 5–95, there are twelve occurrences of the digit 5. The same number of occurrences of the digit 5 appear in the multiples of 5 in each of the intervals 100–195, 200–295, 300–395, and 400–495. For the multiples of 5 in the interval 500–595, there are the same corresponding twelve occurrences of the digit 5 plus twenty more for the digit in hundreds place for each of the twenty multiples of 5 in 500–595, for a total of thirty-two occurrences of the digit 5. For the multiples of 5 in each of the intervals 600–695, 700–795, 800–895, 900–995, 1,000–1,095, 1,100–1,195, 1,200–1,295, 1,300–1,395, and 1,400–1,495, there are twelve occurrences of the digit 5. Finally, there is one occurrence of the digit 5 in 1,500. Therefore, the total number of occurrences of the digit 5 in the first 300 multiples of 5 is 14(12) + 32 + 1 = 201.

The correct answer is E.

PS01661.01

53. The difference 942 − 249 is a positive multiple of 7. If *a*, *b*, and *c* are nonzero digits, how many 3-digit numbers *abc* are possible such that the difference *abc* − *cba* is a positive multiple of 7 ?

A. 142

B. 71

C. 99

D. 20

E. 18

Arithmetic Place value

Since *abc* is numerically equal to $100a + 10b + c$ and *cba* is numerically equal to $100c + 10b + a$, it follows that *abc* − *cba* is numerically equal to $(100 - 1)a + (10 - 10)b + (1 - 100)c = 99(a - c)$. Because 7 and 99 are relatively prime, $99(a - c)$ will be divisible by 7 if and only if $a - c$ is divisible by 7. This leads to two choices for the nonzero digits *a* and *c*, namely $a = 9$, $c = 2$ and $a = 8$, $c = 1$. For each of these two choices for *a* and *c*, *b* can be any one of the nine nonzero digits. Therefore, there is a total of 2(9) = 18 possible 3-digit numbers *abc*.

The correct answer is E.

PS41661.01

54. Let *S* be the set of all positive integers having at most 4 digits and such that each of the digits is 0 or 1. What is the greatest prime factor of the sum of all the numbers in *S* ?

A. 11

B. 19

C. 37

D. 59

E. 101

Arithmetic Properties of integers

By writing down all the positive integers in *S*, their sum can be found.

1	10	11	100	101
110	111	1,000	1,001	1,010
1,011	1,100	1,101	1,110	1,111

The sum of these integers is 8,888. Since this sum is $8 \times 1{,}111 = 2^3 \times 11 \times 101$ (note that $1{,}111 = (11 \times 100) + 11$), it follows that 101 is the largest prime factor of the sum.

Alternatively, we can simplify the description by letting the integers having fewer than four digits be represented by four-digit strings in which one or more of the initial digits is 0. For example, the two-digit number 10 can be written as 0010 = $(0 \times 10^3) + (0 \times 10^2) + (1 \times 10^1) + (0 \times 10^0)$. Also, we can include 0 = 0000, since the inclusion of 0 will not affect the sum. With these changes, it follows from the Multiplication Principle that there are $2^4 = 16$ integers to be added. Moreover, for each digit position (units place, tens place, etc.) exactly half of the integers will have a digit of 1 in that digit position. Therefore, the sum of the 16 integers will be $(8 \times 10^3) + (8 \times 10^2) + (8 \times 10^1) + (8 \times 10^0)$, or 8,888. Note that this alternative method of finding the sum is much quicker than the other method if "at most four digits" had been "at most seven digits." In the case of "at most seven digits," there will be $2^7 = 128$ integers altogether, and for each digit position, half of the integers will have a digit of 1 in that digit position and the other half will have a digit of 0 in that digit position. Thus, the sum will be $(64 \times 10^6) + (64 \times 10^5) + \ldots + (64 \times 10^0) =$
71,111,104. Incidentally, finding the greatest prime factor of 71,111,104 is not appropriate for a GMAT problem, but in this case a different question about the sum could have been asked.

The correct answer is E.

Age	Tax only	Tax and fees	Fees only
18–39	20	30	30
≥40	10	60	100

PS43661.01

55. The table above shows the number of residents in each of two age groups who support the use of each type of funding for a city initiative. What is the probability that a person randomly selected from among the 250 residents polled is younger than 40, or supports a type of funding that includes a tax, or both?

A. $\frac{1}{5}$

B. $\frac{8}{25}$

C. $\frac{12}{25}$

D. $\frac{3}{5}$

E. $\frac{4}{5}$

Arithmetic Probability

The requested probability is the number of residents described divided by the total number (250) of residents. The number of residents described is equal to the number of residents of age 18–39 (20 + 30 + 30 = 80) PLUS the number of residents of age ≥ 40 who support tax only (10) PLUS the number of residents of age ≥ 40 who support tax and fees (60). Therefore, the requested probability is $\frac{80+10+60}{250} = \frac{3}{5}$.

Alternatively, the requested probability is 1 minus the number of residents NOT described (100) divided by 250, or $1 - \frac{100}{250} = \frac{3}{5}$.

The correct answer is D.

PS55471.01

56. Which of the following describes the set of all possible values of the positive integer k such that, for each positive odd integer n, the value of $\frac{n}{k}$ is midway between consecutive integers?

A. All positive integers greater than 2

B. All prime numbers

C. All positive even integers

D. All even prime numbers

E. All positive even multiples of 5

Arithmetic **Properties of integers**

The logical complexities involved in this question are lessened by testing individual values of k.

Does k = *1 satisfy the condition?* NO. If $n = 1$, then $\frac{n}{k} = \frac{1}{1} = 1$ is not midway between consecutive integers.

Does k = *2 satisfy the condition?* YES. For each positive odd integer n, the value of $\frac{n}{k} = \frac{n}{2}$ is an odd integer divided by 2, and hence is midway between consecutive integers.

Does k = *3 satisfy the condition?* NO. If $n = 1$, then $\frac{n}{k} = \frac{1}{3}$ is not midway between consecutive integers.

It is easy to see that no other positive integer satisfies the condition, by considering the value of $\frac{n}{k}$ when $n = 1$. Therefore, 2 is the only positive integer value of k that satisfies the condition.

The correct answer is D.

PS92981.01

57. A certain online form requires a 2-digit code for the day of the month to be entered into one of its fields, such as 04 for the 4th day of the month. The code is *valid* if it is 01, 02, 03, ..., 31 and *not valid* otherwise. The *transpose* of a code xy is yx. For example, 40 is the transpose of 04. If N is the number of valid codes having a transpose that is not valid, what is the value of N?

A. 12

B. 13

C. 18

D. 19

E. 20

Arithmetic **Operations with integers**

It will be quicker to count the number of valid codes whose transposes are valid codes, and then subtract the result from 31 (the number of valid codes) to obtain the value of N.

Three such codes begin with 0: 01, 02, 03. The rest are such that their transposes are invalid codes.

Four such codes begin with 1: 10, 11, 12, 13. The rest are such that their transposes are invalid codes.

Three such codes begin with 2: 20, 21, 22. The rest are such that their transposes are invalid codes.

Two such codes begin with 3: 30, 31. The rest are invalid codes.

Therefore, the total number of valid codes whose transposes are valid codes is $3 + 4 + 3 + 2 = 12$, and hence the total number of valid codes whose transposes are not valid is $31 - 12 = 19$.

The correct answer is D.

PS25302.01

58. If $x < y < z$ and $y - x > 5$, where x is an even integer and y and z are odd integers, what is the least possible value of $z - x$?

A. 6

B. 7

C. 8

D. 9

E. 10

Algebra **Inequalities**

Since $y - x > 5$, it follows that y must be one of the integers

$x + 6, x + 7, x + 8, x + 9, \ldots$

Also, because x is even and y is odd, y cannot be an even integer added to x, and thus y must be one of the integers

$x + 7, x + 9, x + 11, x + 13, \ldots$

Since $z > y$ and both y and z are odd integers, it follows that z must be one of the integers

$y + 2, y + 4, y + 6, y + 8, \ldots$

Therefore, the least possible value of $z - x$ occurs when y is 7 greater than x and z is 2 greater than y, which implies that z is $7 + 2 = 9$ greater than x.

The correct answer is D.

PS36302.01

59. An “Armstrong number” is an n-digit number that is equal to the sum of the nth powers of its individual digits. For example, 153 is an Armstrong number because it has 3 digits and $1^3 + 5^3 + 3^3 = 153$. What is the digit k in the Armstrong number 1,6k 4 ?

A. 2

B. 3

C. 4

D. 5

E. 6

Arithmetic Operations with integers

If $k = 1$, then $1{,}6k4 = 1{,}614$ and $1^4 + 6^4 + 1^4 + 4^4$ is equal to $1 + 1{,}296 + 1 + 256 = 1{,}554 \neq 1{,}614$. Therefore, $k = 1$ does not produce an Armstrong number.

If $k = 2$, then $1{,}6k4 = 1{,}624$ and $1^4 + 6^4 + 2^4 + 4^4$ is equal to $1 + 1{,}296 + 16 + 256 = 1{,}569 \neq 1{,}624$. Therefore, $k = 2$ does not produce an Armstrong number.

If $k = 3$, then $1{,}6k4 = 1{,}634$ and $1^4 + 6^4 + 3^4 + 4^4$ is equal to $1 + 1{,}296 + 81 + 256 = 1{,}634$. Therefore, $k = 3$ produces an Armstrong number.

Alternatively, the condition that 1,6k4 is an Armstrong number can be expressed by the equation $1^4 + 6^4 + k^4 + 4^4 = 1{,}000 + 600 + 10k + 4$, or $1{,}553 + k^4 = 1{,}604 + 10k$. This can be rewritten as $k(k^3 - 10) = 51$. Therefore, k must be a factor of 51, and 3 is the only answer choice that is a factor of 51.

The correct answer is B.

PS30402.01

60. Five integers between 10 and 99, inclusive, are to be formed by using each of the ten digits exactly once in such a way that the sum of the five integers is as small as possible. What is the greatest possible integer that could be among these five numbers?

A. 98

B. 91

C. 59

D. 50

E. 37

Arithmetic Place value

Note that 0 cannot be the tens digit of an integer between 10 and 99, inclusive. In order for the sum of the five integers to be as small as possible, their tens digits should be the five smallest remaining digits (that is, 1, 2, 3, 4, and 5), leaving the digits 0, 6, 7, 8, and 9 to be used as the ones digits. Let a, b, c, d, and e represent distinct digits chosen from the digits 0, 6, 7, 8, and 9. The sum of the five integers formed in this way is as small as possible and equals $(10 + a) + (20 + b) + (30 + c) + (40 + d) + (50 + e)$ $= 150 + (a + b + c + d + e) = 150 + 30 = 180$ regardless of how the digits 0, 6, 7, 8, and 9 are assigned to a, b, c, d, and e. By assigning 9 to e, it follows that one of the integers could be 59. Therefore, 59 is the greatest possible integer among the five integers whose sum is 180.

The correct answer is C.

PS66402.01

61. When the integer n is divided by 17, the quotient is x and the remainder is 5. When n is divided by 23, the quotient is y and the remainder is 14. Which of the following is true?

A. $23x + 17y = 19$

B. $17x - 23y = 9$

C. $17x + 23y = 19$

D. $14x + 5y = 6$

E. $5x - 14y = -6$

Algebra Remainders; Simplifying algebraic expressions

It is given that $n = 17x + 5$ and $n = 23y + 14$. It follows that $17x + 5 = 23y + 14$, so $17x - 23y = 9$.

The correct answer is B.

PS17402.01

62. Of the following, which is greatest?

A. $3\sqrt{2}$

B. $2\sqrt{3}$

C. $\frac{4\sqrt{3}}{5}$

D. $\frac{5\sqrt{2}}{4}$

E. $\frac{7}{\sqrt{3}}$

Arithmetic Operations with radical expressions

Probably the easiest way to determine which of the given radical expressions is greatest is to compare their squares, which are 18, 12, $\frac{48}{25}$, $\frac{50}{16}$, and $\frac{49}{3}$. Clearly, 18 is the greatest of the squares, so $3\sqrt{2}$ is the greatest of the given radical expressions.

The correct answer is A.

PS37402.01

63. If $n = p^2$ and p is a prime number greater than 5, what is the units digit of n^2 ?

A. 1

B. 3

C. 4

D. 7

E. 9

Arithmetic Properties of integers

First, all prime numbers greater than 5 are odd numbers with units digit 1, 3, 7, or 9. Note that no prime number greater than 5 has units digit 5. The following table summarizes the results for the possible cases.

Units digit of p	Units digit of $p^2 = n$	Units digit of $p^4 = n^2$
1	1	1
3	9	1
7	9	1
9	1	1

The correct answer is A.

64. A computer can perform 1,000,000 calculations per second. At this rate, how many *hours* will it take this computer to perform the 3.6×10^{11} calculations required to solve a certain problem?

A. 60

B. 100

C. 600

D. 1,000

E. 6,000

Arithmetic Measurement conversion

It will take this computer $\frac{3.6 \times 10^{11}}{10^6} = 3.6 \times 10^5$ seconds to perform the calculations. Since there are 3,600 seconds in 1 hour, this is equivalent to $\frac{3.6 \times 10^5}{3.6 \times 10^3} = 100$ hours.

The correct answer is B.

PS66602.01

65. In an auditorium, 360 chairs are to be set up in a rectangular arrangement with x rows of exactly y chairs each. If the only other restriction is that $10 < x < 25$, how many different rectangular arrangements are possible?

A. Four

B. Five

C. Six

D. Eight

E. Nine

Arithmetic Properties of integers

Because a total of 360 chairs are distributed in x rows of exactly y chairs each, it follows that $xy = 360$. Also, $10 < x < 25$, and so x can only be an integer factor of $360 = 2^3 \times 3^2 \times 5$ that is between 10 and 25. Below is a list of all integers from 11 through 24. Since 2, 3, and 5 are the only prime factors of 360, any integer having a prime factor other than 2, 3, or 5 cannot be a factor of 360 and has been crossed out. For example, $21 = 3 \times 7$ has a prime factor of 7, and thus 21 has been crossed out.

~~11~~, 12, ~~13~~, ~~14~~, 15, 16, ~~17~~, 18, ~~19~~, 20, ~~21~~, ~~22~~, ~~23~~, 24

Of the six integers remaining, it is clear that each is a factor of $360 = 2^3 \times 3^2 \times 5$ except for $16 = 2^4$. Therefore, the number of possible rectangular arrangements is five.

The correct answer is B.

PS28602.01

66. If the product of the integers w, x, y, and z is 770, and if $1 < w < x < y < z$, what is the value of $w + z$?

A. 10

B. 13

C. 16

D. 18

E. 21

Arithmetic Properties of integers

The prime factorization of 770 is $2 \times 5 \times 7 \times 11$, so $w = 2$, $z = 11$, and $w + z = 13$.

The correct answer is B.

$$\begin{array}{r} 1{,}234 \\ 1{,}243 \\ 1{,}324 \\ \cdots\cdot \\ \cdots\cdot \\ +\,4{,}321 \\ \hline \end{array}$$

PS78602.01

67. The addition problem above shows four of the 24 different integers that can be formed by using each of the digits 1, 2, 3, and 4 exactly once in each integer. What is the sum of these 24 integers?

A. 24,000

B. 26,664

C. 40,440

D. 60,000

E. 66,660

Arithmetic Place value

Each digit 1, 2, 3, and 4 will appear six times in each of 1,000s place, 100s place, 10s place, and units place. Since $1 + 2 + 3 + 4 = 10$, it follows that the sum of the 24 integers is $(6)(10)(1{,}000) + (6)(10)(100) + (6)(10)(10) + (6)(10)(1) = 66{,}660$.

The correct answer is E.

Data Sufficiency
Counting/Sets/Series

DS19350.01

68. A country's per capita national debt is its national debt divided by its population. Is the per capita national debt of Country G within \$5 of \$500 ?

1. Country G's national debt to the nearest \$1,000,000,000 is \$43,000,000,000.
2. Country G's population to the nearest 1,000,000 is 86,000,000.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Estimation

Briefly, the question is whether

$495 \le \frac{\text{debt}}{\text{population}} \le 505$ is true.

1. We are given that 42.5 billion ≤ debt < 43.5 billion. However, no information is given about the population; NOT sufficient.
2. We are given that 85.5 million ≤ population < 86.5 million. However, no information is given about the debt; NOT sufficient.

Given (1) and (2), then to determine whether $\frac{\text{debt}}{\text{population}}$ lies between 495 and 505, we will investigate the minimum possible value of $\frac{\text{debt}}{\text{population}}$ and the maximum possible value of $\frac{\text{debt}}{\text{population}}$. To simplify the discussion that follows, we will assume that the upper bounds given in (1) and (2) can be achieved. This will not affect our conclusions because with this assumption, the estimates we obtain will still justify our conclusions. The minimum possible value of $\frac{\text{debt}}{\text{population}}$ is $\frac{\text{minimum debt}}{\text{maximum population}} = \frac{42.5 \text{ billion}}{86.5 \text{ million}} = \frac{42.5}{86.5} \times 1{,}000$. Since

$\frac{42.5}{86.5} = \frac{43.25}{86.5} - \frac{0.75}{86.5} = \frac{1}{2} - \frac{0.75}{86.5}$, which is less than $\frac{1}{2} - \frac{0.4325}{86.5} = \frac{1}{2} - \frac{1}{200}$, it follows that the minimum possible value of $\frac{\text{debt}}{\text{population}}$ is **less than** $\left(\frac{1}{2} - \frac{1}{200}\right) \times 1{,}000 = 500 - 5 = 495$. The maximum possible value of $\frac{\text{debt}}{\text{population}}$ is $\frac{\text{maximum debt}}{\text{minimum population}} = \frac{43.5 \text{ billion}}{85.5 \text{ million}} = \frac{43.5}{85.5} \times 1{,}000$. Since $\frac{43.5}{85.5} = \frac{42.75}{85.5} + \frac{0.75}{85.5} = \frac{1}{2} + \frac{0.75}{86.5}$, which is greater than $\frac{1}{2} + \frac{0.4275}{85.5} = \frac{1}{2} + \frac{1}{200}$, it follows that the maximum possible value of $\frac{\text{debt}}{\text{population}}$ is **greater than** $\left(\frac{1}{2} + \frac{1}{200}\right) \times 1{,}000 = 500 + 5 = 505$.

Since the minimum possible value of $\frac{\text{debt}}{\text{population}}$ is less than 495 and the maximum possible value of $\frac{\text{debt}}{\text{population}}$ is greater than 505, it is not possible to determine whether or not $\frac{\text{debt}}{\text{population}}$ lies between 495 and 505.

The correct answer is E;
both statements together are still not sufficient.

DS06351.01

69. The *cardinality* of a finite set is the number of elements in the set. What is the cardinality of set *A* ?

1. 2 is the cardinality of exactly 6 subsets of set *A*.
2. Set *A* has a total of 16 subsets, including the empty set and set *A* itself.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Concepts of sets**

Let n be the cardinality of the finite set A. What is the value of n ?

1. The number of 2-element subsets of A is equal to the number of unordered selections without replacement of 2 objects from a collection of n distinct objects, or "n choose 2." Therefore, we have $\binom{n}{2} = \frac{n(n-1)}{2} = 6$, or equivalently, $n^2 - n - 12 = 0$. Because this is a quadratic equation that may have two solutions, we need to investigate further to determine whether there is a unique value of n. Factoring leads to $(n - 4)(n + 3) = 0$, and thus $n = 4$ or $n = -3$. Since n must be a nonnegative integer, we discard the solution $n = -3$. Therefore, $n = 4$; SUFFICIENT.
2. The number of subsets of set A is 2^n, because each subset of A corresponds to a unique sequence of answers to yes-no questions about membership in the subset (one question for each of the n elements). For example, let $A = \{1, 2, 3, 4, 5\}$, let Y represent "yes," and let N represent "no." Then the sequence NYNNN corresponds to the subset $\{2\}$, since the answers to "is 1 in the subset," "is 2 in the subset," "is 3 in the subset," etc. are "no," "yes," "no," etc. Also, the subset $\{1, 3, 4\}$ of A corresponds to the 5-letter sequence YNYYN. Since the number of 5-letter sequences such that each letter is either N or Y is 2^5, it follows that there are $2^5 = 32$ subsets of $\{1, 2, 3, 4, 5\}$. For Statement (2), we are given that $2^n = 16$, and hence $n = 4$; SUFFICIENT.

Alternatively, observe that $\{1\}$ has two subsets, $\{1, 2\}$ has four subsets, and each addition of a new element doubles the number of subsets, because the subsets after adding the new element will consist of all the previous subsets along with the new element included in each of the previous subsets. Thus, $\{1, 2, 3\}$ has $2(4) = 8$ subsets, $\{1, 2, 3, 4\}$ has $2(8) = 16$ subsets, $\{1, 2, 3, 4, 5\}$ has $2(16) = 32$ subsets, etc.

The correct answer is D;
each statement alone is sufficient.

DS59851.01

70. For each positive integer k, let $a_k = \left(1 + \frac{1}{k+1}\right)$. Is the product $a_1 a_2 \ldots a_n$ an integer?

1. $n + 1$ is a multiple of 3.
2. n is a multiple of 2.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Series and sequences**

Since $1 + \frac{1}{k+1} = \frac{k+2}{k+1}$, it follows that the product can be written as

$$\left(\frac{\cancel{3}}{2}\right)\left(\frac{\cancel{4}}{\cancel{3}}\right)\left(\frac{\cancel{5}}{\cancel{4}}\right)\left(\frac{\cancel{6}}{\cancel{5}}\right)\cdots\left(\frac{\cancel{n}}{\cancel{n-1}}\right)\left(\frac{\cancel{n+1}}{\cancel{n}}\right)\left(\frac{n+2}{\cancel{n+1}}\right) =$$

$\frac{n+2}{2}$. Therefore, the product $a_1 a_2 \ldots a_n$ is an integer if and only if $\frac{n+2}{2}$ is an integer, or if and only if n is an even integer.

1. If $n = 2$, then $n + 1 = 3$ is a multiple of 3 and the product is $\frac{n+2}{2} = \frac{2+2}{2}$, which is an integer. However, if $n = 5$, then $n + 1 = 6$ is a multiple of 3 and the product is $\frac{n+2}{2} = \frac{5+2}{2}$, which is not an integer; NOT sufficient.
2. If n is a multiple of 2, then by the remarks above it follows that the product is an integer; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS95491.01

71. Let S be a set of outcomes and let A and B be events with outcomes in S. Let $\sim B$ denote the set of all outcomes in S that are not in B and let $P(A)$ denote the probability that event A occurs. What is the value of $P(A)$?

1. $P(A \cup B) = 0.7$
2. $P(A \cup \sim B) = 0.9$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

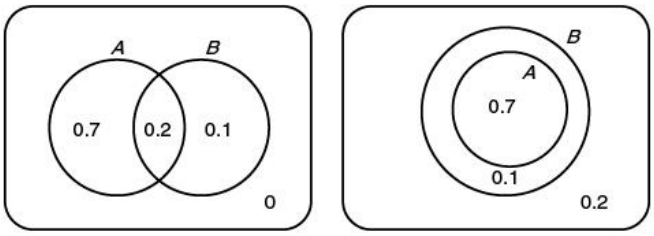
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Probability; Sets

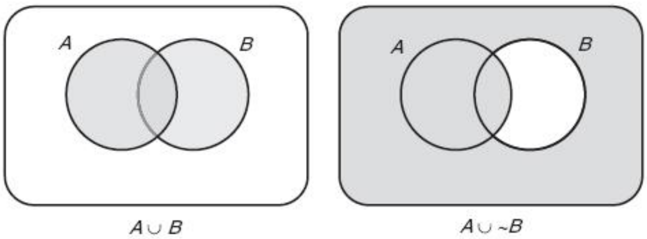
The general addition rule for sets applied to probability gives the basic probability equation

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

1. Given that $P(A \cup B) = 0.7$, it is not possible to determine the value of $P(A)$ because nothing is known about the relation of event A to event B. For example, if every outcome in event B is an outcome in event A, then $A \cup B = A$ and we have $P(A \cup B) = P(A) = 0.7$. However, if events A and B are mutually exclusive (i.e., $P(A \cap B) = 0$) and $P(B) = 0.2$, then the basic probability equation above becomes $0.7 = P(A) + 0.2 - 0$, and we have $P(A) = 0.5$; NOT sufficient.
2. Given that $P(A \cup \sim B) = 0.9$, it is not possible to determine the value of $P(A)$ because nothing is known about the relation of event A to event $\sim B$. For example, as indicated in the first figure below, if every outcome in event $\sim B$ is an outcome in event A, then $A \cup \sim B = A$ and we have $P(A \cup \sim B) = P(A) = 0.9$. However, as indicated in the second figure below, if events A and $\sim B$ are mutually exclusive (i.e., $P(A \cap \sim B) = 0$) and $P(\sim B) = 0.2$, then the basic probability equation above, with $\sim B$ in place of B, becomes $0.9 = P(A) + 0.2 - 0$, and we have $P(A) = 0.7$; NOT sufficient.



Given (1) and (2), if we can express event A as a union or intersection of events $A \cup B$ and $A \cup \sim B$, then the basic probability equation above can be used to determine the value of $P(A)$. The figure below shows Venn diagram representations of events $A \cup B$ and $A \cup \sim B$ by the shading of appropriate regions.



Inspection of the figure shows that the only portion shaded in both Venn diagrams is the region representing event A. Thus, A is equal to the intersection of $A \cup B$ and $A \cup \sim B$, and hence we can apply the basic probability equation with event $A \cup B$ in place of event A and event $A \cup \sim B$ in place of event B. That is, we can apply the equation

$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

with $C = A \cup B$ and $D = A \cup \sim B$. We first note that $P(C) = 0.7$ from (1), $P(D) = 0.9$ from (2), and $P(C \cap D) = P(A)$. As for $P(C \cup D)$, inspection of the figure above shows that $C \cup D$ encompasses all possible outcomes, and thus $P(C \cup D) = 1$. Therefore, the equation above involving events C and D becomes $1 = 0.7 + 0.9 - P(A)$, and hence $P(A) = 0.6$.

The correct answer is C;
both statements together are sufficient.

DS41402.01

72. What is the number of integers that are common to both set S and set T?

1. The number of integers in S is 7, and the number of integers in T is 6.
2. U is the set of integers that are in S only or in T only or in both, and the number of integers in U is 10.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Sets

In standard notation, $S \cap T$ and $S \cup T$ represent the intersection and union, respectively, of sets S and T, and $|S|$ represents the number of elements in a set S. Determine $|S \cap T|$.

1. It is given that $|S| = 7$ and $|T| = 6$. If, for example, $S = \{1, 2, 3, 4, 5, 6, 7\}$ and $T = \{1, 2, 3, 4, 5, 6\}$, then $|S \cap T| = 6$. However, if $S = \{1, 2, 3, 4, 5, 6, 7,\}$ and $T = \{11, 12, 13, 14, 15, 16\}$, then $|S \cap T| = 0$; NOT sufficient.
2. It is given that $|S \cup T| = 10$. If, for example, $S = \{1, 2, 3, 4, 5, 6, 7\}$ and $T = \{1, 2, 3, 8, 9, 10\}$, then $S \cup T = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $|S \cup T| = 10$, and $|S \cap T| = 3$. However, if $S = \{1, 2, 3, 4, 5, 6, 7\}$ and $T = \{11, 12, 13\}$, then $S \cup T = \{1, 2, 3, 4, 5, 6, 7, 11, 12, 13\}$, $|S \cup T| = 10$, and $|S \cap T| = 0$; NOT sufficient.

Taking (1) and (2) together along with the general addition rule for two sets A and B

($|A \cup B| = |A| + |B| - |A \cap B|$) applied to sets S and T gives $10 = 7 + 6 - |S \cap T|$, from which $|S \cap T|$ can be determined.

The correct answer is C;
both statements together are sufficient.

DS51402.01

73. What is the sum of 3 consecutive integers?

1. The sum of the 3 integers is less than the greatest of the 3 integers.
2. Of the 3 integers, the ratio of the least to the greatest is 3.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Sequences

Let k be the smallest of the three consecutive integers. It follows that $k + 1$ and $k + 2$ are the other two integers and $S = k + (k + 1) + (k + 2) = 3k + 3$, where S is the sum of the three consecutive integers. Determine S.

1. It is given that $3k + 3 < k + 2$. It follows that $k < -\frac{1}{2}$. If $k = -1$, then $S = 0$. However, if $k = -2$, then $S = -3$; NOT sufficient.
2. It is given that $\frac{k}{k+2} = 3$. It follows that $k = -3$ and S can be determined; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS54402.01

74. How many people in Town X read neither the *World* newspaper nor the *Globe* newspaper?

 1. Of the 2,500 people in Town X, 1,000 read no newspaper.
 2. Of the people in Town X, 700 read the *Globe* only and 600 read the *World* only.

 A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
 B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
 C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
 D. EACH statement ALONE is sufficient.
 E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Sets

Determine the number of people in Town X who read neither the *World* newspaper nor the *Globe* newspaper.

1. It is given that 1,000 of the 2,500 people in Town X read no newspaper, from which it follows that 1,500 people read at least one newspaper. It is possible that all of these 1,500 people read only the *Earth*, another newspaper that is read in Town X, so the number of people who read neither the *World* nor the *Globe* could be 2,500. It is also possible that all of the 1,500 people who read at least one newspaper read the *World*, so the number of people who read neither the *World* nor the *Globe* could be 1,000; NOT sufficient.
2. It is given that 600 people read only the *World* and 700 people read only the *Globe*, but there is no information about the number of people in Town X and no information about other newspapers that might be read by the people in Town X; NOT sufficient.

Neither (1) nor (2) gives information as to whether other newspapers are read by people in Town X, and without this information, the number of people who read neither the *World* nor the *Globe* cannot be determined.

The correct answer is E;
both statements together are still not sufficient.

> **Tip**
>
> Just because the *World* and the *Globe* are the only newspapers mentioned, do not assume that they are the only newspapers read in Town X.

DS16402.01

75. Bowls X and Y each contained exactly 2 jelly beans, each of which was either red or black. One of the jelly beans in bowl X was exchanged with one of the jelly beans in bowl Y. After the exchange, were both of the jelly beans in bowl X black?

 1. Before the exchange, bowl X contained 2 black jelly beans.
 2. After the exchange, bowl Y contained 1 jelly bean of each color.

 A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
 B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
 C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
 D. EACH statement ALONE is sufficient.
 E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Sets

Determine after an exchange of one jelly bean if both jelly beans in bowl *X* are black.

1. Bowl *X* has two black jelly beans, but bowl *Y* could have two red or two black or one of each color. If bowl *Y* has two red jelly beans, then bowl *X* will have one red and one black jelly bean after the exchange, not two black. However, if bowl *Y* has two black jelly beans, then bowl *X* will have two black after the exchange; NOT sufficient.
2. After the exchange, bowl *Y* has one red and one black. If, before the exchange, bowl *X* had two black and bowl *Y* had two red, then after the exchange, each of bowl *Y* and bowl *X* will have one of each color. However, if bowl *Y* had two black jelly beans, bowl *X* had one of each color, and bowl *X* obtained a black in exchange for the red, then bowl *Y* will have one of each color and bowl *X* will have two black; NOT sufficient.

Taking (1) and (2) together, bowl *X* had two black jelly beans to start with and bowl *Y* ended up with one of each color after the exchange. bowl *Y* had to have had at least one red because bowl *X* had no red jelly beans to give to bowl *Y*. If bowl *Y* has two red and exchanges a red for one of bowl *X*'s blacks, then, after the exchange, bowl *Y* will have one of each color, but bowl *X* will not have two black. On the other hand, if bowl *Y* has one of each color and exchanges its black for one of bowl *X*'s blacks, then after the exchange, bowl *Y* will have one of each color and bowl *X* will have two blacks.

The correct answer is E;
both statements together are still not sufficient.

DS27602.01

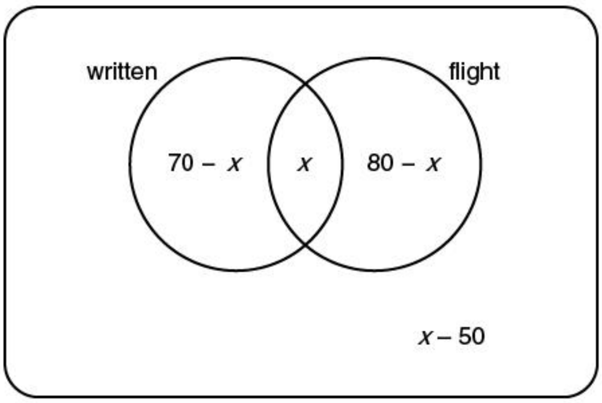
76. All trainees in a certain aviator training program must take both a written test and a flight test. If 70 percent of the trainees passed the written test, and 80 percent of the trainees passed the flight test, what percent of the trainees passed both tests?

    1. 10 percent of the trainees did not pass either test.
    2. 20 percent of the trainees passed only the flight test.

    A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
    B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
    C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
    D. EACH statement ALONE is sufficient.
    E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Sets

Let x be the percent of the trainees who passed both tests. The following Venn diagram represents the information that is given as well as information that can be derived from what is given. Note that $x – 50$ in the diagram can be found by using the requirement that the sum of the four values in the diagram is 100. What is the value of x ?



1. Given that $x – 50 = 10$, it follows that $x = 60$; SUFFICIENT.
2. Given that $80 – x = 20$, it follows that $x = 60$; SUFFICIENT.

Tip

A useful way to summarize the quantitative relations for a two-circle Venn diagram is

Total = A + B − Both + Neither,

where A is the number of elements in Circle A, B is the number of elements in Circle B, “Both” is the number of elements in the intersection of the circles, and “Neither” is the number of elements that do not belong to either of the circles. If we think of A and B as the numbers of elements in the two 1-way intersections (i.e., Circle A alone and Circle B alone) and “Both” as the number of elements in the single 2-way intersection (i.e., Circle A intersects Circle B), then this equation can be written as

Total = (sum of 1-way) − (sum of 2-way) + None.

This second way of expressing the quantitative relations for a two-circle Venn diagram can be modified to give a similar way of expressing the quantitative relations for a three-circle Venn diagram:

Total = (sum of 1-way) − (sum of 2-way)
+ (sum of 3-way) + None.

Although Venn diagrams involving more than three circles will not likely be needed for the GMAT, we recommend researching the *inclusion-exclusion principle* if the reader is interested in further extensions of these ideas.

The correct answer is D;
each statement alone is sufficient.

Equalities/Inequalities/Algebra

DS06110.01

77. Each of the five divisions of a certain company sent representatives to a conference. If the numbers of representatives sent by four of the divisions were 3, 4, 5, and 5, was the range of the numbers of representatives sent by the five divisions greater than 2 ?

1. The median of the numbers of representatives sent by the five divisions was greater than the average (arithmetic mean) of these numbers.
2. The median of the numbers of representatives sent by the five divisions was 4.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Statistics

Let x be the unspecified number of representatives. By considering individual positive integer values of x, the median of the numbers is found to be 4 when $x = 1, 2, 3$, or 4, and the median of the numbers is found to be 5 when $x \geq 5$. For example, the case in which $x = 2$ is shown below.

2, 3, **4**, 5, 5

1. In terms of x, the average of the numbers is $\frac{x+3+4+5+5}{5} = \frac{x+17}{5}$. If $x = 1$, then by the remarks above the median is 4, which is greater than $\frac{1+17}{5}$ (i.e., the median is greater than the average), and the range is $5 - 1 = 4$. If $x = 5$, then by the remarks above the median is 5, which is greater than $\frac{5+17}{5}$ (i.e., the median is greater than the average), and the range is $5 - 3 = 2$; NOT sufficient.
2. Given the assumption that the median of the numbers is 4, it follows from the previous remarks that x can be any one of the numbers 1, 2, 3, and 4. If $x = 1$, then the range is $5 - 1 = 4$, which is greater than 2. If $x = 4$, then the range is $5 - 3 = 2$, which is not greater than 2; NOT sufficient.

Given (1) and (2), then from the previous remarks and (2) it follows that x must be among the numbers 1, 2, 3, and 4. From (2) it follows that $4 > \frac{x+17}{5}$, or $x < 3$, and thus x is further restricted to be among the numbers 1 and 2. However, for each of these possibilities the range is greater than 2: If $x = 1$, then the range is $5 - 1 = 4 > 2$; and if $x = 2$, then the range is $5 - 2 = 3 > 2$.

The correct answer is C;
both statements together are sufficient.

DS24931.01

78. An investment has been growing at a fixed annual rate of 20% since it was first made; no portion of the investment has been withdrawn, and all interest has been reinvested. How much is the investment now worth?

1. The value of the investment has increased by 44% since it was first made.
2. If one year ago $600 had been withdrawn, today the investment would be worth 12% less than it is actually now worth.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Applied problems

If the investment was initially worth $\$P$, then after one year the investment was worth $\$1.2P$, after two years the investment was worth $1.2(\$1.2P) = \$1.44P$, after three years the investment was worth $1.2(\$1.44P) = \$1.728P$, etc.

1. Given that the investment was worth $\$(P + 0.44P) = \$1.44P$, it follows from the remarks above that the investment was first made two years ago. However, nothing is known about the value of P; NOT sufficient.
2. Let $\$X$ be how much the investment was worth one year ago. Then the investment is now worth $\$1.2X$. However, if $600 had been withdrawn from the investment one year ago, then the investment would have been worth $\$(X - 600)$ one year ago and the investment would have been worth $\$1.2(X - 600) = \$(1.2X - 720)$ today. It is given that this amount, namely $\$(1.2X - 720)$, is 12 percent less than $\$1.2X$. Therefore, $1.2X - 720 = (0.88)(1.2X)$. This equation can be solved for X, and using this value of X, the value of $1.2X$ can be determined; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS53841.01

79. X, 81, 73, 71, 98, 73, 64

What is the value of X in the above list of 7 numbers?

1. The average (arithmetic mean) of these 7 numbers is 80.
2. The range of these 7 numbers is 36.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Statistics

1. Since the average of the seven numbers is 80, it follows that the sum of the seven numbers is 7(80) = 560. Therefore, $X + 81 + 73 + 71 + 98 + 73 + 64 = 560$, which can be solved for a unique value of X; SUFFICIENT.

2. The range of the numbers when X is not included is 98 − 64 = 34. Therefore, if $X = 98 + 2 = 100$, then the range of the seven numbers is 100 − 64 = 36, and if $X = 64 - 2 = 62$, then the range of the seven numbers is 98 − 62 = 36. Therefore, more than one value of X satisfies the given information and statement 2; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS01451.01

80. In the first 2 hours after Meadow's self-service laundry opens, m large washing machines and n small washing machines are in continual use. Including the time for filling and emptying the washing machines, each load of laundry takes 30 minutes in a large washing machine and 20 minutes in a small washing machine. What is the total number of loads of laundry done at Meadow's self-service laundry during this 2-hour period?

1. $n = 3m$
2. $2m + 3n = 55$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simultaneous equations

During the two-hour period each large washer does four loads (the number of 30-minute periods in two hours is four) and each small washer does six loads (the number of 20-minute periods in two hours is six). Therefore, the total number of loads done during the two-hour period by m large washers and n small washers is $4m + 6n$.

1. If $m = 1$ and $n = 3$, then $n = 3m$ and the total number of loads of laundry done in the two-hour period is 4(1) + 6(3) = 22. However, if $m = 2$ and $n = 6$, then $n = 3m$ and the total number of loads of laundry done in the two-hour period is 4(2) + 6(6) = 44; NOT sufficient.

2. Since $2m + 3n = 55$, it follows that $4m + 6n = 110$. Therefore, by the remarks above, the total number of loads done during the two-hour period is 110; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS76851.01

81. A box of light bulbs contains exactly 3 light bulbs that are defective. What is the probability that a sample of light bulbs picked at random from this box will contain at least 1 defective light bulb?

1. The light bulbs in the sample will be picked 1 at a time without replacement.
2. The sample will contain exactly 20 light bulbs.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Statistics

It is clear that neither (1) alone nor (2) alone is sufficient.

Given (1) and (2), if the box contains 22 light bulbs, then a sample of 20 light bulbs must contain at least one defective light bulb, and hence the desired probability is equal to 1. However, if the box contains 22,000 light bulbs, then it is clear that the probability that a sample of 20 light bulbs contains at least one defective light bulb is less than 1.

The correct answer is E;
both statements together are still not sufficient.

DS01951.01

82. Khalil drove 120 kilometers in a certain amount of time. What was his average speed, in kilometers per hour, during this time?

1. If Khalil had driven at an average speed that was 5 kilometers per hour faster, his driving time would have been reduced by 20 minutes.
2. If Khalil had driven at an average speed that was 25% faster, his driving time would have been reduced by 20%.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Applied problems

Let r be Khalil's average speed in kilometers per hour and let t be Khalil's time in hours. Then $rt = 120$, or $t = \frac{120}{r}$. What is the value of r ?

1. If Khalil's speed had been $(r + 5)$ kilometers per hour, then his driving time would have been $\left(t-\frac{1}{3}\right)$ hours.

been $\left(t-\frac{1}{3}\right)$ hours.

$$
\begin{aligned}
(r+5)\left(t-\frac{1}{3}\right) &= 120 && \text{Statement (1)} \\
(r+5)\left(\frac{120}{r}-\frac{1}{3}\right) &= 120 && \text{substitute } t=\frac{120}{r} \\
(r+5)(360-r) &= 360r && \text{multiply both sides by } 3r \\
360r - r^2 + 1{,}800 - 5r &= 360r && \text{expand left side} \\
-r^2 + 1{,}800 - 5r &= 0 && \text{subtract } 360r \text{ from both sides} \\
-(r+45)(r-40) &= 0 && \text{factor the left side}
\end{aligned}
$$

Although there are two possible values for r, namely $r = -45$ and $r = 40$, only the positive value of r is consistent with the context. Therefore, $r = 40$; SUFFICIENT.

2. If Khalil's speed had been $1.25r$ kilometers per hour, then his driving time would have been $0.8t$ hours. It follows that $(1.25r)(0.8t) = 120$, or $rt = 120$, which provides no additional information to the given information; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS70061.01

83. What is the median of the data set S that consists of the integers 17, 29, 10, 26, 15, and x ?

1. The average (arithmetic mean) of S is 17.
2. The range of S is 24.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Statistics

1. Since the average of the six numbers is 17, it follows that the sum of the six numbers is 6(17). Therefore, $17 + 29 + 10 + 26 + 15 + x = 6(17)$, which can be solved for a unique value of x, after which the median can be determined; SUFFICIENT.
2. The range of the numbers when x is not included is $29 - 10 = 19$. Therefore, if $x = 29 + 5 = 34$, then the range of the seven numbers (10, 15, 17, 26, 29, 34) is $34 - 10 = 24$ and the median of the seven numbers is $\frac{17+26}{2} = 21.5$.

However, if $x = 10 - 5 = 5$, then the range of the seven numbers (5, 10, 15, 17, 26, 29) is $29 - 5 = 24$ and the median of the seven numbers is $\frac{15+17}{2} = 16$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS47661.01

84. If $n > 4$, what is the value of the integer n ?

1. $\frac{n!}{(n-3)!} = \frac{3!n!}{4!(n-4)!}$
2. $\frac{n!}{3!(n-3)!} + \frac{n!}{4!(n-4)!} = \frac{(n+1)!}{4!(n-3)!}$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simplifying algebraic expressions

1. Because the numerators of the two fractions have several common factors, and similarly for the denominators, a reasonable strategy is to begin by appropriately canceling these common factors.

$\frac{n!}{(n-3)!} = \frac{3!n!}{4!(n-4)!}$	given
$\frac{1}{(n-3)!} = \frac{3!}{4!(n-4)!}$	divide both sides by $n!$
$\frac{1}{(n-3)!} = \frac{1}{4(n-4)!}$	$4! = 3! \times 4$
$4(n-4)! = (n-3)!$	cross-multiply
$4(n-4)! = (n-4)! \times (n-3)$	
$4 = n-3$	divide both sides by $(n-4)!$
$n = 7$	

The manipulations above show that $n = 7$.

Alternatively, we could begin by reducing each of the fractions to lowest terms by using identities such as $n! = (n-3)! \times (n-2)(n-1)(n)$, and then performing operations on the resulting equation; SUFFICIENT.

2. For the same reason given in (1) above, we begin by canceling factors that are common on the left and right sides of the equality.

$\frac{n!}{3!(n-3)!} + \frac{n!}{4!(n-4)!} = \frac{(n+1)!}{4!(n-3)!}$	given
$\frac{1}{3!(n-3)!} + \frac{1}{4!(n-4)!} = \frac{n+1}{4!(n-3)!}$	divide both sides by $n!$
$\frac{4}{(n-3)!} + \frac{1}{(n-4)!} = \frac{n+1}{(n-3)!}$	multiply both sides by $4! = 3! \times 4$
$4 + (n-3) = n+1$	multiply both sides by $(n-3)! =$

sides by $(n-3)! =$
$(n-4)! \times (n-3)$

The manipulations above show that the original equation is identically true for all integers greater than 4, and thus n can be any integer greater than 4.

Alternatively, we could begin by reducing each of the fractions to lowest terms by using identities such as $n! = (n-3)! \times (n-2)(n-1)(n)$, and then performing operations on the resulting equation; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS50571.01

85. Tami purchased several identically priced metal frames and several identically priced wooden frames for a total pretax price of $144. What was the total pretax price of the metal frames that Tami purchased?

1. The price of each metal frame was 60% greater than the price of each wooden frame.
2. Tami purchased twice as many wooden frames as metal frames.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simultaneous equations

Let m and w, respectively, be the number of metal frames and wooden frames that Tami purchased, and let M and W, respectively, be their individual prices in dollars. We are given that $mM + wW = 144$. What is the value of mM?

1. Given that $M = 1.6W = \frac{8}{5}W$, or $W = \frac{5}{8}M$, we have $mM + w\left(\frac{5}{8}M\right) = 144$. Although it might seem there is not enough information to determine the value of mM, keep in mind that we are only seeking the value of the product mM, and not the individual values of m and M. Also, there are several implicit constraints involved, such as each of M and W must be a positive number less than 144 and each of m and w must be a positive integer. To see that more than one value of mM is possible, it will be convenient to specify a particular value for w, for example, $w = 16$. Then $mM + w\left(\frac{5}{8}M\right) = 144$ becomes $mM = 144 - 10M$, and non-sufficiency is now straightforward. If $w = 16$ and $M = 4$, then $mM = 104$. However, if $w = 16$ and $M = 8$, then $mM = 64$; NOT sufficient.
2. Given that $w = 2m$, we have $mM + (2m)W = 144$, or $mM = 144 - 2mW$. If $m = 10$ and $W = 4$, then $mM = 64$. However, if $m = 10$ and $W = 5$, then $mM = 44$; NOT sufficient.

Given (1) and (2), we have $W = \frac{5}{8}M$ and $w = 2m$. Therefore, $mM + wW = 144$ becomes $mM + (2m)\left(\frac{5}{8}M\right) = 144$, or $\left(1+\frac{5}{4}\right)mM = 144$, and hence the value of mM can be determined.

The correct answer is C;
both statements together are sufficient.

DS02871.01

86. A $10 bill (1,000 cents) was replaced with 50 coins having the same total value. The only coins used were 5-cent coins, 10-cent coins, 25-cent coins, and 50-cent coins. How many 5-cent coins were used?

1. Exactly 10 of the coins were 25-cent coins and exactly 10 of the coins were 50-cent coins.
2. The number of 10-cent coins was twice the number of 5-cent coins.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simultaneous equations

Let a, b, c, and d be the number, respectively, of 5-cent, 10-cent, 25-cent, and 50-cent coins. We are given that $a + b + c + d = 50$ and $5a + 10b + 25c + 50d = 1{,}000$, or $a + 2b + 5c + 10d = 200$. Determine the value of a.

$$a + b + c + d = 50$$

$$a + 2b + 5c + 10d = 200$$

> **Tip**
>
> Note that each of a, b, c, and d must be a nonnegative integer, and so care must be taken in deducing non-sufficiency. For example, there are many real number pairs (x,y) that satisfy the equation $2x + y = 1$, but if each of x and y must be a nonnegative integer, then $x = 0$ and $y = 1$ is the only solution.

1. We are given that $c = 10$ and $d = 10$. Substituting $c = 10$ and $d = 10$ into the two equations displayed above and combining terms gives $a + b = 30$ and $a + 2b = 50$. Subtracting these last two equations gives $b = 20$, and hence it follows that $a = 10$; SUFFICIENT.
2. We are given that $b = 2a$. Substituting $b = 2a$ into the two equations displayed above and combining terms gives $a + 2a + c + d = 50$ and $a + 4a + 5c + 10d = 200$, which are equivalent to the following two equations.

$$3a + c + d = 50$$
$$a + c + 2d = 40$$

Subtracting these two equations gives $2a - d = 10$, or $2a = d + 10$. Since $2a$ is an even integer, d must be an even integer. At this point it is probably simplest to choose various nonnegative even integers for d to determine whether solutions for a, b, c, and d exist that have different values for a. Note that it is not enough to find different nonnegative integer solutions to $2a = d + 10$, since we must also ensure that c and d are nonnegative integers. If $d = 8$, then $2a = 8 + 10 = 18$, and we have $a = 9$, $b = 18$, $c = 15$, and $d = 8$. However, if $d = 10$, then $2a = 10 + 10 = 20$, and we have $a = 10$, $b = 20$, $c = 10$, and $d = 10$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS56971.01

87. Merle's spare change jar has exactly 16 U.S. coins, each of which is a 1-cent coin, a 5-cent coin, a 10-cent coin, a 25-cent coin, or a 50-cent coin. If the total value of the coins in the jar is 288 U.S. cents, how many 1-cent coins are in the jar?

    1. The exact numbers of 10-cent, 25-cent, and 50-cent coins among the 16 coins in the jar are, respectively, 6, 5, and 2.
    2. Among the 16 coins in the jar there are twice as many 10-cent coins as 1-cent coins.

    A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
    B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
    C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
    D. EACH statement ALONE is sufficient.
    E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simultaneous equations

Let a, b, c, d, and e be the number, respectively, of 1-cent, 5-cent, 10-cent, 25-cent, and 50-cent coins. We are given the two equations shown below. Determine the value of a.

$$a + b + c + d + e = 16$$

$$a + 5b + 10c + 25d + 50e = 288$$

1. We are given that $c = 6$, $d = 5$, and $e = 2$. Substituting these values into the two equations displayed above and combining terms gives $a + b = 3$ and $a + 5b = 3$. Subtracting these last two equations gives $4b = 0$, and therefore $b = 0$ and $a = 3$; SUFFICIENT.
2. We are given that $c = 2a$. Substituting $c = 2a$ into the two equations displayed above and combining terms gives the following two equations.

$$3a + b + d + e = 16$$

$$21a + 5b + 25d + 50e = 288$$

From the first equation above we have $3a = 16 - b - d - e$. Therefore, $3a \le 16$, and it follows that the value of a must be among 0, 1, 2, 3, 4, and 5. From the second equation above we have $5(b + 5d + 10e) = 288 - 21a$, and thus the value of $288 - 21a$ must be divisible by 5.

a	$288 - 21a$
0	288
1	267
2	246
3	**225**
4	204
5	183

The table above shows that $a = 3$ is the only nonnegative integer less than or equal to 5 such that $288 - 21a$ is divisible by 5; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.

DS48391.01

88. At a certain university recreation center, a member can receive a 30-minute massage, a 60-minute massage, or a 90-minute massage, and is charged \$0.50 per minute for each massage. A member receiving a massage is charged the same fixed amount for each additional service, such as nutrition advice or a fitness evaluation. At this center, what is the total charge to a member for a 60-minute massage and 3 additional services?

1. At this recreation center, Jordan, a member, had a massage and 3 additional services for a total charge of \$37.50.
2. At this recreation center, Ryan, a member, had a massage and 2 additional services for a total charge of \$60.00.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **First-degree equations**

Let x be the charge, in dollars, of each additional service. Determine the value of $(60)(0.5) + 3x$, or equivalently determine the value of x.

1. The table below shows the value of x for each of the three possible massages that Jordan had.

Massage	Total charge	x
30-minute	$15 + 3x = 37.50$	7.50
60-minute	$30 + 3x = 37.50$	2.50
90-minute	$45 + 3x = 37.50$	−2.50

From the table it follows that there are two possible values of x, namely $x = 7.5$ and $x = 2.5$; NOT sufficient.

2. The table below shows the value of x for each of the three possible massages that Ryan had.

Massage	Total charge	x
30-minute	$15 + 2x = 60$	22.50
60-minute	$30 + 2x = 60$	15.00
90-minute	$45 + 2x = 60$	7.50

From the table it follows that there are three possible values of x, namely $x = 22.5$, $x = 15$, and $x = 7.5$; NOT sufficient.

Given (1) and (2), it follows that $x = 7.5$.

The correct answer is C;
both statements together are sufficient.

DS84302.01

89. If S is the sum of the first n positive integers, what is the value of n ?

1. $S < 20$
2. $S^2 > 220$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Sequences and series

1. Given that $S < 20$, n could be 5 since $\sum_{i=1}^{5} i = \frac{5(6)}{2} = 15 < 20$. However, n could also be 4 since $\sum_{i=1}^{4} i = \frac{4(5)}{2} = 10 < 20$; NOT sufficient.
2. Given that $S^2 > 220$, n could be 5 since $\sum_{i=1}^{5} i = \frac{5(6)}{2} = 15$ and $15^2 = 225 > 220$. However, n could also be 6 since $\sum_{i=1}^{6} i = \frac{6(7)}{2} = 21$ and $21^2 = 441 > 220$; NOT sufficient.

Given (1) and (2) together, it is clear from the following table that $n = 5$.

n	S	S^2	allowed/not allowed
≤ 4	≤ 10	≤ 100	not allowed by (2)
5	15	225	allowed by (1) and (2)
≥ 6	≥ 21	≥ 441	not allowed by (1)

The correct answer is C;
both statements together are sufficient.

DS48302.01

90. Is $x^2 - y^2$ a positive number?

1. $x - y$ is a positive number.
2. $x + y$ is a positive number.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Factoring**

Since $x^2 - y^2 = (x + y)(x - y)$, it follows that $x^2 - y^2$ will be a positive number if both $x + y$ and $x - y$ are positive numbers or if both are negative numbers.

1. It is given that $x - y$ is a positive number. If, for example, $x = 2$ and $y = -3$, then $x - y$ is a positive number, but $x + y$ is not a positive number, so $x^2 - y^2$ is not a positive number. On the other hand, if $x = 3$ and $y = 1$, then $x - y$ is a positive number and $x + y$ is a positive number, so $x^2 - y^2$ is a positive number; NOT sufficient.
2. It is given that $x + y$ is a positive number. If, for example, $x = -2$ and $y = 4$, then $x + y$ is a positive number, but $x - y$ is not a positive number, so $x^2 - y^2$ is not a positive number. On the other hand, if $x = 3$ and $y = 1$, then $x + y$ is a positive number and $x - y$ is a positive number, so $x^2 - y^2$ is a positive number; NOT sufficient.

Taking (1) and (2) together, both $x + y$ and $x - y$ are positive numbers, so $x^2 - y^2$ is a positive number.

The correct answer is C;
both statements together are sufficient.

DS89302.01

91. Alan and Sue have each been saving one dollar a day and will continue to do so for the next month. If Sue began saving several days before Alan, in how many days from today will Alan have saved one-half as much as Sue?

    1. As of today, Alan has saved 7 dollars and Sue has saved 27 dollars.
    2. Three days from today, Alan will have saved one-third as much as Sue.

    A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
    B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
    C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
    D. EACH statement ALONE is sufficient.
    E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **First-degree equations**

Let A be the amount Alan has saved as of today. Let S be the amount Sue had already saved when Alan started saving. Then, as of today, Sue has saved $S + A$ dollars. Determine d, the number of days from today that Alan will have saved half as much as Sue. That is, determine d, where $A + d = \frac{1}{2}(S + A + d)$ or, after algebraic manipulation, determine d such that $d = S - A$.

1. It is given that $A = 7$ and $S = 27$, so $d = 20$; SUFFICIENT.
2. It is given that three days from today, Alan will have saved one-third as much as Sue, from which it follows that $A + 3 = \frac{1}{3}(S + A + 3)$ or, after algebraic manipulation, $S = 2A + 6$. Then, $d = S - A = (2A + 6) - A = A + 6$. Since the value of A can vary, the value of d cannot be determined; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS64402.01

92. What is the value of x ?

    1. $x^4 + x^2 + 1 = \frac{1}{x^4 + x^2 + 1}$
    2. $x^3 + x^2 = 0$

    A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
    B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
    C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
    D. EACH statement ALONE is sufficient.
    E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Simplifying algebraic expressions**

1. If $x^4 + x^2 + 1 = \frac{1}{x^4 + x^2 + 1}$, then $x^4 + x^2 + 1$ is its own reciprocal. The only numbers that are their own reciprocals are −1 and 1. Since even powers of x are nonnegative, $x^4 + x^2 + 1 \neq -1$, so $x^4 + x^2 + 1 = 1$. It follows that $x^4 + x^2 = 0$ and $x = 0$; SUFFICIENT.

2. If $x^3 + x^2 = 0$, then $x^2(x + 1) = 0$. It follows that $x = 0$ or $x = -1$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS26402.01

93. Is x less than y ?

1. $x - y + 1 < 0$
2. $x - y - 1 < 0$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Inequalities**

Determine if $x < y$ is true.

1. If $x - y + 1 < 0$, then $x < y - 1$. Since $y - 1 < y$, it follows that $x < y$; SUFFICIENT.

2. If $x = 1$ and $y = 2$, then $x - y - 1 = -2 < 1$ and $x < y$ is true. However, if $x = 1.5$ and $y = 1$, then $x - y - 1 = -0.5 < 1$ and $x < y$ is not true; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

Tip

In (1), manipulating the given inequality leads to $x < y - 1$, which leads directly to $x < y$ since $y - 1 < y$. This is not the case in (2), where manipulating the given inequality leads to $x < y + 1$ but not to $x < y$ since $y + 1 > y$. Examples can then be used to verify that $x < y$ can be, but doesn't have to be, true.

DS08402.01

94. State X has a sales tax rate of k percent on all purchases and State Y has a sales tax rate of n percent on all purchases. What is the value of $k - n$?

1. The sales tax on a $15 purchase is 30 cents more in State X than in State Y.
2. The sales tax rate in State X is 1.4 times the sales tax rate in State Y.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Applied problems; Percents**

1. It is given that $15\left(\frac{k}{100}\right) = 15\left(\frac{n}{100}\right) + 0.30$. It follows that $15(k - n) = 30$, from which the value of $k - n$ can be determined; SUFFICIENT.

2. It is given that $k = 1.4n$, from which $k - n$ cannot be determined. For example, if $n = 5$, then $k = 7$ and $k - n = 2$. However, if $n = 10$, then $k = 14$ and $k - n = 4$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS28402.01

95. Is $-3 \le x \le 3$?

1. $x^2 + y^2 = 9$
2. $x^2 + y \le 9$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Properties of numbers; Inequalities**

1. Given that $x^2 + y^2 = 9$, if $x < -3$ or if $x > 3$, then $x^2 > 9$ and $y^2 < 0$, which is not possible. Therefore, $-3 \le x \le 3$; SUFFICIENT.

2. Given that $x^2 + y \le 9$, if $x = 0$ and $y = 4$, then $x^2 + y \le 9$, and $-3 \le x \le 3$ is true. However, if $x = 4$ and $y = -7$, then $x^2 + y \le 9$, and $-3 \le x \le 3$ is not true; NOT sufficient.

The correct answer is A;

statement 1 alone is sufficient.

DS69402.01

96. What is the value of x ?

1. $4^{x+1} + 4^x = 320$
2. $x^2 = 9$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Exponents**

Determine the value of x.

1. If $4^{x+1} + 4^x = 320$, then $4^x(4 + 1) = 320$, from which $4^x = 64$ and $x = 3$; SUFFICIENT.

2. If $x^2 = 9$, then $x = -3$ or $x = 3$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS50502.01

97. Three dice, each with faces numbered 1 through 6, were tossed onto a game board. If one of the dice turned up 4, what was the sum of the numbers that turned up on all three dice?

1. The sum of two of the numbers that turned up was 10.
2. The sum of two of the numbers that turned up was 11.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Operations with integers

Determine the sum of the numbers that turned up when three dice were tossed, given that one of the dice turned up 4.

1. Given that the sum of two of the numbers was 10, the numbers that turned up on the three dice could be 5, 4, and 5 ($5 + 5 = 10$) for a total sum of 14. On the other hand, they could be 2, 4, and 6 ($4 + 6 = 10$) for a total sum of 12; NOT sufficient.
2. The number 4 could not have been one of the two numbers whose sum was 11 because $11 - 4 = 7$ and 7 is not a number that can turn up on the dice. Therefore, since 11 can be obtained only when the other two numbers are 5 and 6, the numbers that turned up on the three dice must be 4, 5, and 6 for a total sum of 15; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS81502.01

98. Of the numbers q, r, s, and t, which is greatest?

1. The average (arithmetic mean) of q and r is s.
2. The sum of q and r is t.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Statistics

Determine which of the numbers q, r, s, and t is the greatest.

1. Given that $\frac{q+r}{2} = s$, then s is halfway between q and r and $q \le s \le r$ or $r \le s \le q$. So it is possible that q is the greatest, it is possible that r is the greatest, and it is possible that t is the greatest since no information is given about t; NOT sufficient.
2. Given that $t = q + r$, it is not possible to determine which of q, r, s, and t is greatest. For example, if $q = 1$, $r = 5$, $s = 3$, and $t = 6$, then t is the greatest, but if $q = 1$, $r = -5$, $s = -2$, and $t = -4$, then q is the greatest; NOT sufficient.

Taking (1) and (2) together, it is still not possible to determine which of q, r, s, and t is greatest because the examples used to show that (2) is not sufficient satisfy (1) also.

The correct answer is E;
both statements together are still not sufficient.

CAR RENTAL CHARGES AT THRIFTY AGENCY		
Car Type	Charge per day	Charge per Week (7 days)
Economy	\$28	\$100
Compact	\$30	\$120
Midsize	\$32	\$140
Standard	\$34	\$160
Luxury	\$39	\$200

DS94502.01

99. The table above shows the car rental charges at Thrifty Agency. The daily rate applies for each day or fraction of a day in excess of any multiple of a 7-day week, up to the charge per week. If Olga rented a car of one of the types indicated, which type was it?

1. Olga's total rental charge, based only on the rates specified, was \$184.
2. Olga rented the car for 10 days.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Interpretation of tables

Determine which type of car Olga rented.

1. Since Olga's total rental charge was \$184, determine for which type of car (economy, compact, midsize or standard) $\frac{184 - \text{charge per week}}{\text{charge per day}}$ is an integer. Note that Olga did not rent a luxury car because 184 < 200 and 184 is not a multiple of 39.

Economy:	184 − 100 = 84 = 3(28)
Compact:	184 − 120 = 64, not a multiple of 30
Midsize:	184 − 140 = 44, not a multiple of 32
Standard:	184 − 160 = 24, not a multiple of 34

Olga rented an economy car; SUFFICIENT.

2. Just knowing that Olga rented the car for 10 days is not enough information to determine which type of car she rented; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS76602.01

100. Is $xy < 6$?

1. $x < 3$ and $y < 2$.
2. $\frac{1}{2} < x < \frac{2}{3}$ and $y^2 < 64$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Inequalities

1. Given that $x < 3$ and $y < 2$, it is not possible to determine whether or not $xy < 6$. For example, if $x = 1$ and $y = 1$, then $x < 3$, $y < 2$, and $xy = 1$. However, if $x = -3$ and $y = -3$, then $x < 3$, $y < 2$, and $xy = 9$; NOT sufficient.
2. Given that $y^2 < 64$, then it easily follows that $-8 < y < 8$. Thus, we have $\frac{1}{2} < x < \frac{2}{3}$ and $-8 < y < 8$. We consider two cases, according to the sign of y. *Case 1*: Suppose that $-8 < y \le 0$. Since $x > 0$ and $y \le 0$, it follows that $xy \le 0 < 6$. *Case 2*: Suppose that $0 < y < 8$. Then xy is the product of two positive quantities. Since the product of two positive quantities is greatest when each of the quantities is greatest, it follows that $xy < \left(\frac{2}{3}\right)(8) = \frac{16}{3} < 6$.

Since $xy < 6$ in each case, and the two cases include all possible values of x and y, we have $xy < 6$; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS86602.01

101. What is the value of $\frac{x}{yz}$?

1. $x = \frac{y}{2}$ and $z = \frac{2x}{5}$.
2. $\frac{x}{z} = \frac{5}{2}$ and $\frac{1}{y} = \frac{1}{10}$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simplifying algebraic expressions

1. Given that $x = \frac{y}{2}$ and $z = \frac{2x}{5}$, it follows that $\frac{x}{yz} = \frac{x}{(2x)\left(\frac{2x}{5}\right)} = \frac{5}{4x}$, which will have different values for different nonzero values of x; NOT sufficient.
2. Given that $\frac{x}{z} = \frac{5}{2}$ and $\frac{1}{y} = \frac{1}{10}$, it follows that $\frac{x}{yz} = \frac{\frac{5z}{2}}{10z} = \frac{1}{4}$; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS47602.01

102. In a certain group of people, the average (arithmetic mean) weight of the males is 180 pounds and of the females, 120 pounds. What is the average weight of the people in the group?

1. The group contains twice as many females as males.
2. The group contains 10 more females than males.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Applied problems; Statistics

Let M and F, respectively, be the number of males and females in the group. Also, let $\sum M$ and $\sum F$, respectively, be the total weight, in pounds, of the males and females in the group. We are given that $\frac{\sum M}{M} = 180$ and $\frac{\sum F}{F} = 120$. What is the value of $\frac{\sum M + \sum F}{M + F}$?

1. Given that $F = 2M$ (equivalently, $M = \frac{1}{2}F$), it follows that $\frac{\sum M + \sum F}{M + F} = \frac{\sum M}{M + F} + \frac{\sum F}{M + F} = \frac{\sum M}{M + 2M} + \frac{\sum F}{\frac{1}{2}F + F} = \frac{1}{3}\left(\frac{\sum M}{M}\right) + \frac{2}{3}\left(\frac{\sum F}{F}\right) = \frac{1}{3}(180) + \frac{2}{3}(120)$; SUFFICIENT.

2. We are given that $F = M + 10$. If $M = 2$ and $F = 12$, then there are six times as many females as males, and hence the average weight of the people in the group will be strongly skewed toward the average weight of the females. However, if $M = 100{,}000$ and $F = 100{,}010$, then the ratio of females to males is close to 1, and hence the average weight of the people in the group will be close to the average of 120 and 180.

> **Tip**
>
> In data sufficiency problems it is often helpful to consider contrasting extreme scenarios when they exist. If a variable can be any positive real number, then consider the scenario when the variable is very close to 0 and the scenario when the variable is very large. If a quadrilateral can be any rectangle, then consider the scenario when the rectangle is a square and the scenario when the rectangle is very long with a small width.

Alternatively, $\frac{\sum M + \sum F}{M + F} = \frac{180M + 120(M + 10)}{M + (M + 10)} = \frac{150M + 600}{M + 5}$, and by long division (takes one step), this can be written as $150 - \frac{150}{M + 5}$, which can clearly vary when the value of M varies; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS57602.01

103. If $y = 2^{x+1}$, what is the value of $y - x$?

1. $2^{2x+2} = 64$
2. $y = 2^{2x-1}$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Equations; Exponents

1. Given that $2^{2x+2} = 64 = 2^6$, it follows that $2x + 2 = 6$. Therefore, $x = 2$ and $y - x = 2^{2+1} - 2 = 6$; SUFFICIENT.

2. From the given information we have $y = 2^{x+1}$ and from (2) we have $y = 2^{2x-1}$. Therefore, $2^{x+1} = 2^{2x-1}$, and hence $x + 1 = 2x - 1$. Solving for x gives $x = 2$, and thus $y - x = 2^{2+1} - 2 = 6$; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.

DS67602.01

104. If $x \neq 1$, is y equal to $x + 1$?

1. $\frac{y-2}{x-1} = 1$
2. $y^2 = (x + 1)^2$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Simplifying algebraic expressions

Determine if $y = x + 1$.

1. Given $\frac{y-2}{x-1} = 1$, then $y - 2 = x - 1$ and $y = x + 1$; SUFFICIENT.
2. Given $y^2 = (x + 1)^2$, then $y = x + 1$ or $y = -(x + 1)$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS18602.01

105. If $x + y + z > 0$, is $z > 1$?

1. $z > x + y + 1$
2. $x + y + 1 < 0$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Inequalities

Determine if $z > 1$ is true.

1. Given that $z > x + y + 1$, by adding z to both sides, it follows that $2z > x + y + z + 1$. Also, $x + y + z + 1 > 1$ because $x + y + z > 0$. Thus, $2z > 1$ and $z > \frac{1}{2}$. It is possible that $z > 1$ is true and it is possible that $z > 1$ is not true. For example, if $z = 1.1$ and $x = y = 0$, then $x + y + z > 0$ and $z > x + y + 1$ are both true, and $z > 1$ is true. However, if $z = 1$, $x = -0.5$ and $y = -0.25$, $x + y + z > 0$ and $z > x + y + 1$ are both true, and $z > 1$ is not true; NOT sufficient.
2. Given that $x + y + 1 < 0$, it follows that $1 < -x - y$. It is also given that $x + y + z > 0$, so $z > -x - y$ or $-x - y < z$. Combining $1 < -x - y$ and $-x - y < z$ gives $1 < z$ or $z > 1$; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

Geometry

DS35210.01

106. In the rectangular coordinate system, line k passes through the point $(n,-1)$. Is the slope of line k greater than zero?

1. Line k passes through the origin.
2. Line k passes through the point $(1,n + 2)$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry **Simple coordinate geometry**

1. The slope of a line through $(n,-1)$ and $(0,0)$ is $-\frac{1}{n}$, which is greater than zero if $n < 0$ and less than zero if $n > 0$; NOT sufficient.
2. Given that line k passes through the points $(n,-1)$ and $(1,n + 2)$, then the slope of line k (when it exists) is equal to $\frac{(n+2)-(-1)}{1-n} = \frac{n+3}{1-n}$. If $n = 0$, then the slope of line k is 3, which is positive. However, if $n = 2$, then the slope of line k is -5, which is negative; NOT sufficient.

Given (1) and (2), it follows that $-\frac{1}{n} = \frac{n+3}{1-n}$, which by cross-multiplying is equivalent to $(-1)(1 - n) = n(n + 3)$ when n is not equal to 0 or 1. This is a quadratic equation that can be rewritten as $n^2 + 2n + 1 = 0$, or $(n + 1)^2 = 0$. Therefore, $n = -1$ and the slope of line k is $-\frac{1}{n} = 1$, which is greater than zero.

The correct answer is C;
both statements together are sufficient.

DS88111.01

107. In quadrilateral $ABCD$, is angle BCD a right angle?

1. Angle ABC is a right angle.
2. Angle ADC is a right angle.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

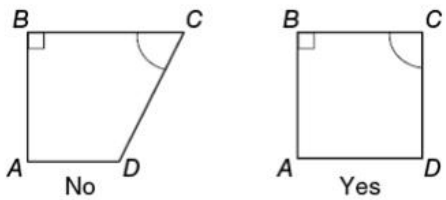
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

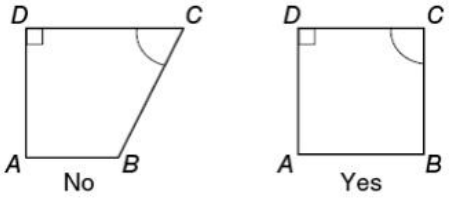
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry **Quadrilaterals**

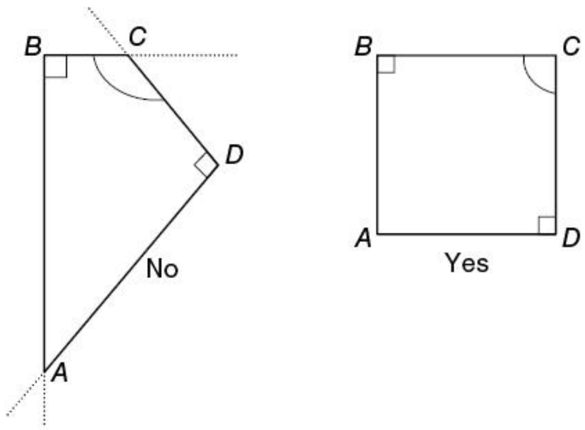
1. The figure below shows two possibilities for a quadrilateral $ABCD$ such that angle ABC is a right angle. One quadrilateral is such that angle BCD is not a right angle (i.e., the answer to the question can be NO), and the other quadrilateral is a square (i.e., the answer to the question can be YES); NOT sufficient.



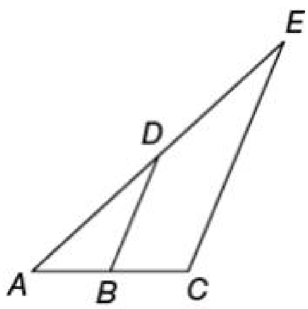
2. The figure below shows that relabeling the vertices of the examples used in (1) above will give an example such that angle *BCD* is not a right angle and an example such that angle *BCD* is a right angle; NOT sufficient.



Given (1) and (2), the figure below indicates how an example satisfying both (1) and (2) can be constructed such that angle *BCD* is not a right angle. A right angle is constructed with vertex *B* and another right angle, appropriately rotated with respect to the first right angle, is constructed with vertex *D*. The rays of these two angles intersect at points *A* and *C* to form a quadrilateral *ABCD* that satisfies both (1) and (2) and is such that angle *BCD* is not a right angle (i.e., the answer to the question can be NO). For completeness, a square is also shown, which satisfies both (1) and (2) and is such that angle *BCD* is a right angle (i.e., the answer to the question can be YES).



The correct answer is E;
both statements together are still not sufficient.



DS29831.01

108. In the figure above, *B* is on $\overline{AC}$, *D* is on $\overline{AE}$, $\overline{AB}$ has the same length as $\overline{BC}$, and $\angle ABD$ has the same measure as $\angle ACE$. What is the length of $\overline{DB}$?

1. The length of $\overline{EC}$ is 6.
2. The length of $\overline{DE}$ is 5.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

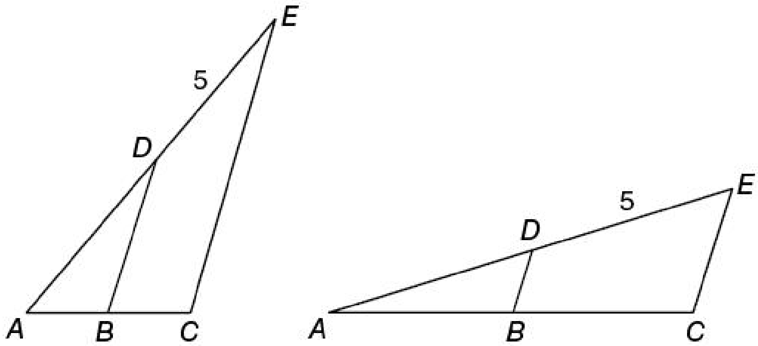
D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Triangles

The given information implies that ΔABD is similar to ΔACE, since $\angle ABD$ and $\angle ACE$ have the same measure and these two triangles share an angle at point A. Therefore, the lengths of corresponding sides of these two triangles are proportional. Using $AB = BC$, it follows that $AC = 2(AB)$, and hence the lengths of the sides of ΔACE are twice the lengths of the corresponding sides of ΔABD.

1. Given that $EC = 6$, it follows from the remarks above that $DB = \frac{1}{2}(6) = 3$; SUFFICIENT.
2. The figure below shows two possibilities satisfying the given information and $DE = 5$ that have different values for DB; NOT sufficient.



The correct answer is A;
statement 1 alone is sufficient.

DS92931.01

109. Sprinklers are being installed to water a lawn. Each sprinkler waters in a circle. Can the lawn be watered completely by 4 installed sprinklers?

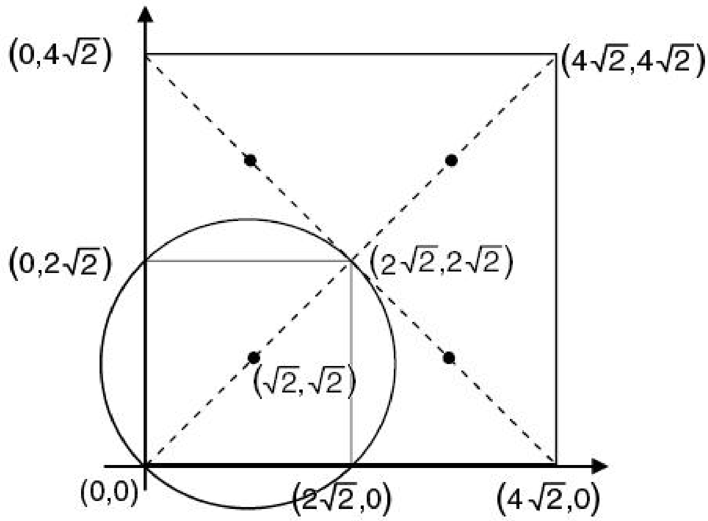
1. The lawn is rectangular and its area is 32 square yards.
2. Each sprinkler can completely water a circular area of lawn with a maximum radius of 2 yards.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Circles; Rectangles

1. No information is given about the area of the region that can be completely covered by four installed sprinklers; NOT sufficient.
2. No information is given about the area or the shape of the lawn; NOT sufficient.

Given (1) and (2), if the length of the rectangular lawn is sufficiently large, for example if the length is 32 yards and the width is 1 yard, then it is clear that the four sprinklers cannot completely water the lawn. However, if the lawn is in the shape of a square, then it is possible that four sprinklers can completely water the lawn. To see this, we first note that the side length of the square lawn is $\sqrt{32} = 4\sqrt{2}$ yards. To assist with the mathematical details, the figure below shows the square lawn positioned in the standard (x,y) coordinate plane so that the vertices of the lawn are located at (0,0), $(0, 4\sqrt{2})$, $(4\sqrt{2}, 4\sqrt{2})$, and $(4\sqrt{2}, 0)$. The two diagonals of the square, each of length 8, are shown as dashed segments, and the four sprinklers are at the four marked points located at the midpoints of the left and right halves of the diagonals. For example, one of the sprinklers is located at the point $(\sqrt{2}, \sqrt{2})$. Using the distance formula, it is straightforward to show that a circle centered at $(\sqrt{2}, \sqrt{2})$ with radius 2 passes through each of the points (0,0), $(0, 2\sqrt{2})$, $(2\sqrt{2}, 2\sqrt{2})$, and $(2\sqrt{2}, 0)$. Therefore, the interior of this circle covers the lower left square portion of the square lawn—that is, the square portion having vertices (0,0), $(0, 2\sqrt{2})$, $(2\sqrt{2}, 2\sqrt{2})$, and $(2\sqrt{2}, 0)$. Hence, the four sprinklers together, when located as described above, can completely water the square lawn. Therefore, it is possible that the lawn cannot be completely watered by the four sprinklers, and it is possible that the lawn can be completely watered by the four sprinklers.



The correct answer is E;
both statements together are still not sufficient.

DS18041.01

110. What is the length of the hypotenuse of ΔABC?

1. The lengths of the three sides of ΔABC are consecutive even integers.
2. The hypotenuse of ΔABC is 4 units longer than the shorter leg.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Pythagorean theorem

1. Let n, $n + 2$, and $n + 4$ be the consecutive even integers. Using the Pythagorean theorem, we have $n^2 + (n + 2)^2 = (n + 4)^2$. Because this is a quadratic equation that may have two solutions, we need to investigate further to determine whether there is a unique hypotenuse length.

$$\begin{aligned} n^2 + (n+2)^2 &= (n+4)^2 \\ n^2 + n^2 + 4n + 4 &= n^2 + 8n + 16 \\ n^2 - 4n - 12 &= 0 \\ (n-6)(n+2) &= 0 \end{aligned}$$

Therefore, $n = 6$ or $n = -2$. Since $n = -2$ corresponds to side lengths of -2, 0, and 2, we discard $n = -2$. Therefore $n = 6$, the hypotenuse has length $n + 4 = 10$; SUFFICIENT.

2. Let the side lengths be a, b, and $a + 4$. Using the Pythagorean theorem, we have $a^2 + b^2 = (a + 4)^2$. Expanding and solving for b in terms of a will facilitate our search for multiple hypotenuse length possibilities.

$$\begin{aligned} a^2 + b^2 &= (a+4)^2 \\ a^2 + b^2 &= a^2 + 8a + 16 \\ b^2 &= 8a + 16 \\ b &= 2\sqrt{2a+4} \end{aligned}$$

When $a = 1$, we obtain side lengths 1 and $2\sqrt{6}$, and hypotenuse length 5. When $a = 2$, we obtain side lengths 2 and $4\sqrt{2}$, and hypotenuse length 6; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS37571.01

111. Patricia purchased x meters of fencing. She originally intended to use all of the fencing to enclose a square region, but later decided to use all of the fencing to enclose a rectangular region with length y meters greater than its width. In square meters, what is the positive difference between the area of the square region and the area of the rectangular region?

1. $xy = 256$
2. $y = 4$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Rectangles; Perimeter

The square's perimeter is x meters, and thus the square has adjacent sides of length $\frac{x}{4}$ meters each. Since the rectangle's perimeter is also x meters, with adjacent side lengths that differ by y meters, it follows that the rectangle's length is $\left(\frac{x}{4}+\frac{y}{2}\right)$ meters (i.e., lengthen two opposite sides of the square by $\frac{y}{2}$ meters) and the rectangle's width is $\left(\frac{x}{4}-\frac{y}{2}\right)$ meters (i.e., shorten the two other opposite sides of the square by $\frac{y}{2}$ meters). Alternatively, letting L and W be the length and width, respectively and in meters, of the rectangle, then we can express each of L and W in terms of x and y by algebraically eliminating L and W from the equations $2L + 2W = x$ and $L = W + y$.

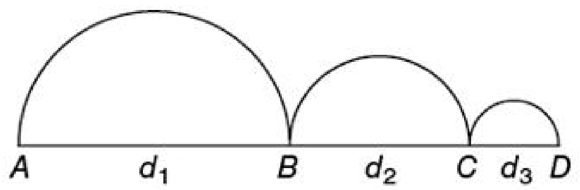
$2L + 2W = x$		given
$2(W + y) + 2W = x$		substitute $L = W + y$
$W = \frac{x}{4} - \frac{y}{2}$		solve for W
$L = \frac{x}{4} + \frac{y}{2}$		use $L = W + y$

Therefore, in square meters, the area of the square is $\left(\frac{x}{4}\right)^2$, the area of the rectangle is $\left(\frac{x}{4}+\frac{y}{2}\right)\left(\frac{x}{4}-\frac{y}{2}\right)=\left(\frac{x}{4}\right)^2-\left(\frac{y}{2}\right)^2$, and the positive difference between these two areas is $\left(\frac{y}{2}\right)$. Determine the value of $\left(\frac{y}{2}\right)^2$.

1. Given $xy = 256$, it is clearly not possible to determine the value of $\left(\frac{y}{2}\right)^2$; NOT sufficient.

2. Given $y = 4$, the value of $\left(\frac{y}{2}\right)^2$ is equal to 4; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.



DS45771.01

112. In the figure above, points A, B, C, and D are collinear and $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CD}$ are semicircles with diameters d_1 cm, d_2 cm, and d_3 cm, respectively. What is the sum of the lengths of $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CD}$, in centimeters?

1. d_1:d_2:d_3 is 3:2:1.
2. The length of $\overline{AD}$ is 48 cm.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry **Circles; Circumference**

Since the circumference of a semicircle is $\pi\left(\frac{\text{diameter}}{2}\right)$, it follows that $\widehat{AB}$ has length $\pi\left(\frac{d_1}{2}\right)$ cm, $\widehat{BC}$ has length $\pi\left(\frac{d_2}{2}\right)$ cm, and $\widehat{CD}$ has length $\pi\left(\frac{d_3}{2}\right)$ cm. Therefore, the sum of the lengths, in centimeters, of $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CD}$ is $\pi\left(\frac{d_1}{2}\right)+\pi\left(\frac{d_2}{2}\right)+\pi\left(\frac{d_3}{2}\right)=\frac{\pi}{2}(d_1+d_2+d_3)$. Determine the value of $\frac{\pi}{2}(d_1+d_2+d_3)$.

1. Given that d_1:d_2:d_3 is 3:2:1, it is not possible to determine the value of $\frac{\pi}{2}(d_1+d_2+d_3)$ because d_1, d_2, and d_3 could be 3, 2, and 1 ($d_1 + d_2 + d_3 = 6$) or d_1, d_2, and d_3 could be 6, 4, and 2 ($d_1 + d_2 + d_3 = 12$); NOT sufficient.

2. Given that $AD = 48$ and $AD = d_1 + d_2 + d_3$, it follows that $\frac{\pi}{2}(d_1+d_2+d_3)=\frac{\pi}{2}(48)$; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS16291.01

113. In the standard (x,y) coordinate plane, what is the slope of the line containing the distinct points P and Q ?

1. Both P and Q lie on the graph of $|x| + |y| = 1$.
2. Both P and Q lie on the graph of $|x + y| = 1$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Simple coordinate geometry

1. If $P = (1,0)$ and $Q = (0,1)$, then both P and Q lie on the graph of $|x| + |y| = 1$ and the slope of the line containing P and Q is -1. However, if $P = (1,0)$ and $Q = (-1,0)$, then both P and Q lie on the graph of $|x| + |y| = 1$ and the slope of the line containing P and Q is 0; NOT sufficient.
2. If $P = (1,0)$ and $Q = (0,1)$, then both P and Q lie on the graph of $|x + y| = 1$ and the slope of the line containing P and Q is -1. However, if $P = (1,0)$ and $Q = (-1,0)$, then both P and Q lie on the graph of $|x + y| = 1$ and the slope of the line containing P and Q is 0; NOT sufficient.

Taking (1) and (2) together is still not sufficient because the same examples used in (1) were also used in (2).

Although it is not necessary to visualize the graphs of $|x| + |y| = 1$ and $|x + y| = 1$ to solve this problem, some readers may be interested in their graphs. The graph of $|x| + |y| = 1$ is a square with vertices at the four points $(\pm1,0)$ and $(0,\pm1)$. This can be seen by graphing $x + y = 1$ in the first quadrant, which gives a line segment with endpoints $(1,0)$ and $(0,1)$, and then reflecting this line segment about one or both coordinate axes for the other quadrants (e.g., in the second quadrant, $x < 0$ and $y > 0$, and so $|x| + |y| = 1$ becomes $-x + y = 1$). The graph of $|x + y| = 1$ is the union of two lines, one with equation $x + y = 1$ and the other with equation $x + y = -1$.

The correct answer is E;
both statements together are still not sufficient.

DS61791.01

114. When opened and lying flat, a birthday card is in the shape of a regular hexagon. The card must be folded in half along 1 of its diagonals before being placed in an envelope for mailing. Assuming that the thickness of the folded card will not be an issue, will the birthday card fit inside a rectangular envelope that is 4 inches by 9 inches?

1. Each side of the regular hexagon is 4 inches long.
2. The area of the top surface (which is the same as the area of the bottom surface) of the folded birthday card is less than 36 square inches.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

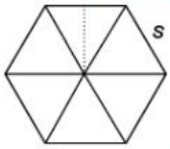
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

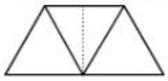
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Polygons



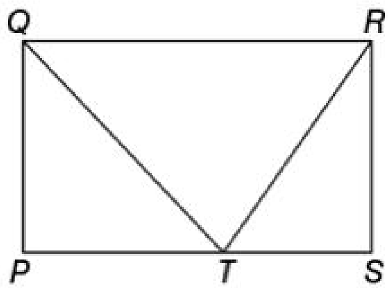
As shown in the figure above, a regular hexagon with sides of length s can be partitioned into six equilateral triangles. Using these triangles, it is possible to determine the length of each diagonal ($2s$), the height, shown as a dashed line, of each triangle $\left(\frac{s\sqrt{3}}{2}\right)$, the area of each triangular region $\left(\frac{s^2\sqrt{3}}{4}\right)$, and the area of the hexagonal region $\left(\frac{3s^2\sqrt{3}}{2}\right)$.

When the birthday card is folded in half along one of the diagonals it has the shape shown below.



1. Given $s = 4$, the maximum width of the birthday card is $2s = 8$, which is less than the width of the envelope, and its height is $\left(\frac{4\sqrt{3}}{2}\right) = 2\sqrt{3}$, which is less than the height of the envelope because $2\sqrt{3} < 2\sqrt{4} = 4$. Thus, the birthday card will fit in the envelope; SUFFICIENT.
2. Given that the surface area of the card when folded is less than 36 square inches, it follows that $\left(\frac{1}{2}\right)\left(\frac{3s^2\sqrt{3}}{2}\right) < 36$, which simplifies to $s < 4\sqrt[4]{3}$. If $s = 4$, then the birthday card will fit in the envelope, as shown in (1) above. However, if $s = 5$, then $s < 4\sqrt[4]{3}$ (note that $625 = 5^4 < (4\sqrt[4]{3})^4 = 768$), but the maximum width of the birthday card will be $2s = 10$, and the card will not fit in the envelope; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.



DS77302.01

115. In rectangular region *PQRS* above, *T* is a point on side *PS*. If *PS* = 4, what is the area of region *PQRS* ?

1. ΔQTR is equilateral.
2. Segments *PT* and *TS* have equal length.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

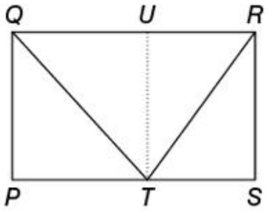
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry **Triangles**

It is given that *PQRS* is a rectangle and *PS* = 4. The area of *PQRS* can be determined if and only if *PQ* can be determined.

1. 

It is given that ΔQTR is equilateral. If $\overline{TU}$ is the height of ΔQTR as shown above, then ΔQUT is a 30–60–90° triangle with *UR* = 2. Using the ratios for 30–60–90 triangles, $TU = 2\sqrt{3}$. Since *PQ* = *TU*, the area of *PQRS* can be determined; SUFFICIENT.

2. Given that *PT* = *TS*, *PQ* could be any positive number. Thus, it is not possible to determine *PQ* and therefore not possible to determine the area of *PQRS*; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS58302.01

116. The top surface area of a square tabletop was changed so that one of the dimensions was reduced by 1 inch and the other dimension was increased by 2 inches. What was the surface area before these changes were made?

1. After the changes were made, the surface area was 70 square inches.
2. There was a 25 percent increase in one of the dimensions.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry **Quadrilaterals; First- and second-degree equations**

Letting *S* be the side length of the square tabletop, the surface area before the change was S^2, and the area after the change is $(S - 1)(S + 2)$. Determine S^2.

1. It is given that $(S - 1)(S + 2) = 70$. Solving this equation gives $S = -9$ and $S = 8$. Discarding $S = -9$, it follows that $S = 8$ and the surface area before the changes was 64 square inches; SUFFICIENT.
2. Since only one dimension was increased and that increase was 2 inches, it follows from (2) that $S + 2 = 1.25S$. Solving this equation gives $S = 8$. Therefore, the surface area before the changes was 64 square inches; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.

DS49302.01

117. If the lengths of the legs of a right triangle are integers, what is the area of the triangular region?

1. The length of one leg is $\frac{3}{4}$ the length of the other.
2. The length of the hypotenuse is 5.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Triangles

1. Let x represent one leg of the triangle. Then the other leg is $\frac{3}{4}x$ and the area is $\frac{3}{8}x^2$. Since the value of x can be any positive multiple of 4, the area cannot be determined; NOT sufficient.
2. Given that the lengths of the legs are the integers a and b and the length of the hypotenuse is 5, it follows that $a^2 + b^2 = 25$, where $1 \le a \le 4$ and $1 \le b \le 4$. The only pair of integers that meet these conditions are 3 and 4, so the area of the triangular region is 6; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS12402.01

118. If cubical blocks in a display are stacked one on top of the other on a flat surface, what is the volume of the stack of blocks in cubic centimeters?

1. The volume of the top block is 8 cubic centimeters.
2. The height of the stack of blocks is 10 centimeters.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Volume

1. It is given that the volume of the top cube in the stack is 8 cubic centimeters, from which it follows that the top block has edges of length 2 cm, but no information is given about the size of the other blocks in the stack or how many blocks the stack contains; NOT sufficient.
2. It is given that the height of the stack of blocks is 10 cm, but no information is given about the size of any of the blocks in the stack or how many blocks are in the stack.

Taking (1) and (2) together gives no information about the size of the blocks below the top block or how many blocks are in the stack. For example, there could be two blocks with edges of lengths 2 cm and 8 cm. The volume of the top block would be 8 cubic centimeters, the height of the stack would be 10 cm, and the volume of the stack of blocks would be 520 cubic centimeters. But there could also be three blocks with edges of lengths 2 cm, 3 cm, and 5 cm. The volume of the top block would be 8 cubic centimeters, the height of the stack would be 10 cm, and the volume of the stack of blocks would be 160 cubic centimeters.

The correct answer is E;
both statements together are still not sufficient.

> **Tip**
>
> Do not assume anything that is not explicitly stated in the problem. In this problem, it is tempting to assume that all of the blocks are identical, in which case there would be five blocks, each with height 2 cm to give the whole stack a height of 10 cm and a volume of 40 cubic centimeters. Under the assumption that all of the blocks are identical, the correct answer would be C.

DS53402.01

119. Is the perimeter of a certain rectangular garden greater than 50 meters?

1. The two shorter sides of the garden are each 15 meters long.
2. The length of the garden is 5 meters greater than the width of the garden.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

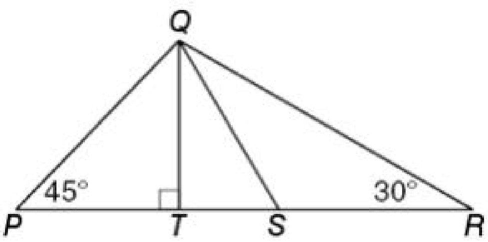
Geometry **Quadrilaterals; Perimeter**

Let W represent the length of the shorter side (that is, the width) of the rectangular garden, and let L represent the length of the longer side (that is, the length). The perimeter is then $2(L + W)$. Determine if $2(L + W) > 50$ is true, or equivalently, if $L + W > 25$ is true.

1. It is given that $W = 15$, from which it follows that $L > 15$. Then, $L + W > 15 + 15 > 25$; SUFFICIENT.
2. It is given that $L = W + 5$, from which it follows that $L + W = 2W + 5$. If, for example, $W = 1$, then $2W + 5$ is not greater than 25, but if $W = 11$, then $2W + 5$ is greater than 25; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

> **Tip**
>
> When determining whether a certain condition holds for an expression involving two variables, avoid assuming that the values of both variables must be known. In (1), knowing that L represents the length of the **longer** side and knowing the value of W, the length of the shorter side, is enough information to determine whether $L + W$ meets the given condition.



DS34402.01

120. In the figure above, what is the perimeter of ΔPQR ?

1. The length of segment PT is 2.
2. The length of segment RS is $\sqrt{3}$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

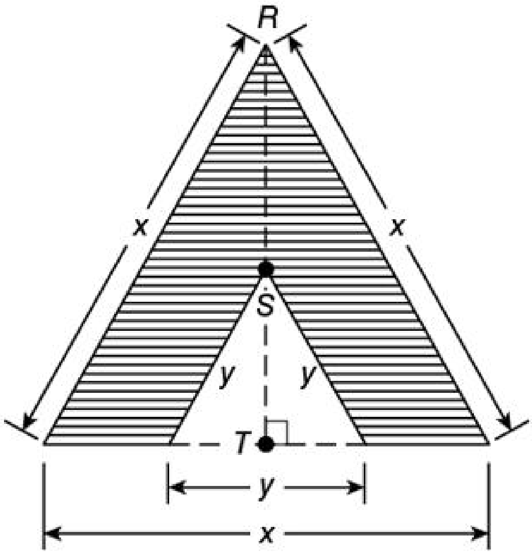
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Triangles; Perimeter

Determine the perimeter of ΔPQR by determining $PQ + QR + PR$.

1. It is given that $PT = 2$. Since ΔPTQ is a 45°–45°–90° triangle, it follows that $QT = 2$ and $PQ = 2\sqrt{2}$. Since ΔQTR is a 30°–60°–90° triangle and $QT = 2$, it follows that $QR = 4$ and $TR =$ $\frac{\sum M + \sum F}{M + F} = \frac{180M + 120(M + 10)}{M + (M + 10)} = \frac{150M + 600}{M + 5}$. $PT + TR = PR$, so PR, QR, and PQ are known and the perimeter of ΔPQR can be determined; SUFFICIENT.
2. It is given that $RS = \sqrt{3}$, but no information is given to determine TS. If, for example, $TS = \sqrt{3}$, then ΔQTR is a 30°–60°–90° triangle with $TS + SR = TR = 2\sqrt{3}$. It follows that $QT = 2$ and $QR = 4$. Also, ΔPTQ is a 45°–45°–90° triangle with $QT = 2$. It follows that $PT = 2$ (hence $PR = 2 + 2\sqrt{3}$) and $PQ = 2\sqrt{2}$, so the perimeter of the triangle is $2\sqrt{2} + 4 + (2 + 2\sqrt{3})$. However, if $TS = 2\sqrt{3}$, then ΔQTR is a 30°–60°–90° triangle with $TS + SR = TR = 3\sqrt{3}$. It follows that $QT = 3$, and $QR = 6$. Also, ΔPTQ is a 45°–45°–90° triangle with $QT = 3$. It follows that $PT = 3$ (hence $PR = 3 + 3\sqrt{3}$) and $PQ = 3\sqrt{2}$, so the perimeter of the triangle is $3\sqrt{2} + 6 + (3 + 3\sqrt{3})$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.



DS07402.01

121. In the figure above, the shaded region represents the front of an upright wooden frame around the entrance to an amusement park ride. If $RS = \frac{5\sqrt{3}}{2}$ meters, what is the area of the front of the frame?

1. $x = 9$ meters
2. $ST = 2\sqrt{3}$ meters

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Triangles

The front of the frame is in the shape of an equilateral triangle with sides of length x with an opening in the shape of an equilateral triangle with sides of length y, where $y < x$. The area of the front of the frame is the area of the larger triangle minus the area of the smaller triangle.

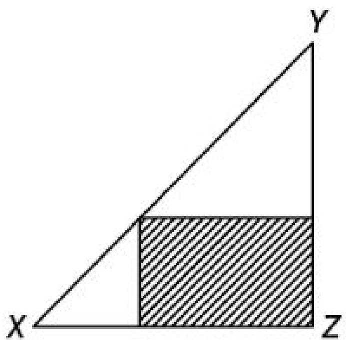
Note that the height of an equilateral triangle with sides of length s is given by $\frac{s\sqrt{3}}{2}$ and the area is given by $\frac{s^2\sqrt{3}}{4}$, both of which are easily derived using the ratios for the sides of a 30°−60°−90° triangle.

1. Given that $x = 9$, it follows that RT, the height of the large equilateral triangle, is $\frac{9\sqrt{3}}{2}$. Since $RS = \frac{5\sqrt{3}}{2}$ and $RT = RS + ST$, it follows that $ST = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$, where ST is the height of the small equilateral triangle.

   Therefore, $y = 4$. With the side lengths of both triangles known, the area of the front of the frame can be determined; SUFFICIENT.

2. Given that $ST = 2\sqrt{3}$, it follows that $y = 4$. Also, since $RT = RS + ST$ and $RS = \frac{5\sqrt{3}}{2}$, it follows that $RT = \frac{9\sqrt{3}}{2}$ and $x = 9$. With the side lengths of both triangles known, the area of the front of the frame can be determined; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.



DS05502.01

122. In the figure above, if the shaded region is rectangular, what is the length of XY?

1. The perimeter of the shaded region is 24.
2. The measure of $\angle XYZ$ is 45°.

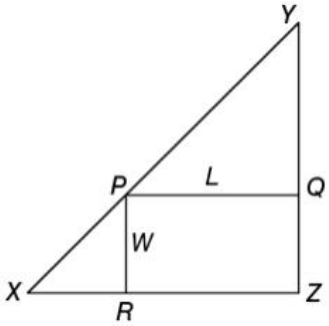
A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Triangles

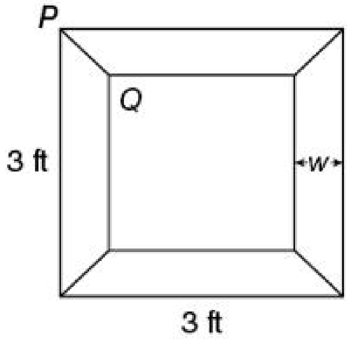
Determine XY.

1. Letting L and W be the length and width of the shaded rectangular region, then from (1), $L + W = 12$. This does not give enough information about ΔXZY to determine XY; NOT sufficient.
2. From (2), ΔXZY is a 45°−45°−90° triangle, so $XZ = YZ$ and $XY = XZ\sqrt{2}$, but no information is given to determine XZ; NOT sufficient.

Taking (1) and (2) together and using the figure below, from (2), ΔXRP is a 45°−45°−90° triangle, so $XR = PR = QZ = W$. Likewise ΔPQY is a 45°−45°−90° triangle, so $PQ = YQ = RZ = L$. It follows that $XZ = XR + RZ = W + L$ and $W + L = 12$ from (1). Likewise, $YZ = YQ + QZ = L + W = 12$. Since the length of the legs of ΔXZY are known, XY can be determined.



The correct answer is C;
both statements together are sufficient.



DS87602.01

123. The figure above shows the dimensions of a square picture frame that was constructed using four identical pieces of frame as shown. If w is the width of each piece of the frame, what is the area of the front surface of each piece? (1 ft = 12 inches)

1. $w = 3$ inches
2. $PQ = \sqrt{18}$ inches

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

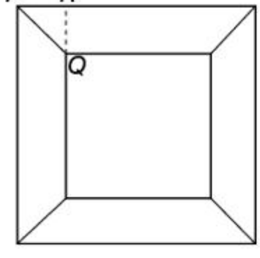
D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Quadrilaterals; Area

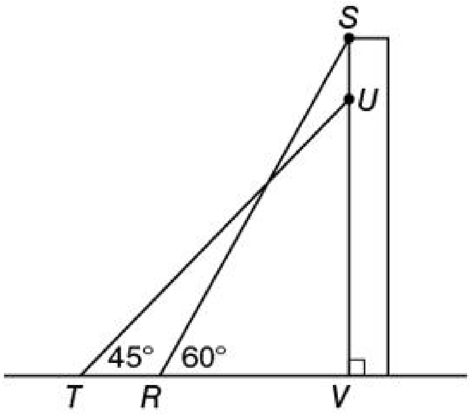
Determine the area of the surface of each of the four pieces of wood used to construct the square picture frame. In the following explanation, the larger square with one vertex labeled *P* will be referred to as Square *P* and the smaller square with one vertex labeled *Q* will be referred to as Square *Q*.

1. It is given that the width of each piece of wood is 3 inches. Then, the area one piece of wood, in square inches, is $\frac{\text{area of Square } Q - \text{area of Square } P}{4}$ or $\frac{36^2 - 30^2}{4}$; SUFFICIENT.



2. It is given that $PQ = \sqrt{18} = 3\sqrt{2}$ inches. Since the pieces of the frame are identical, each of the angles at P is 45° and ΔPRQ in the figure above is a 45°–45°–90° triangle. It follows that $RQ = 3$. This is the same information given in (1), which was shown to be sufficient; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.



DS48602.01

124. In the figure above, segments $\overline{RS}$ and $\overline{TU}$ represent two positions of the same ladder leaning against the side $\overline{SV}$ of a wall. The length of $\overline{TV}$ is how much greater than the length of $\overline{RV}$?

1. The length of $\overline{TU}$ is 10 meters.
2. The length of $\overline{RV}$ is 5 meters.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Geometry Triangles

Note that $TU = RS$ because they both represent the length of the same ladder. Determine $TV - RV$.

1. It is given that $TU = 10$. Since ΔTVU is a 45°–45°–90° triangle, it follows that $TV = 5\sqrt{2}$. Since $RS = TU = 10$ and ΔRVS is a 30°–60°–90° triangle, it follows that $RV = 5$. Therefore, $TV - RV$ can be determined; SUFFICIENT.
2. It is given that $RV = 5$. Then $RS = 10$ since ΔRVS is a 30°–60°–90° triangle. Since $RS = TU$, it follows that $TU = 10$. This is the same information as in (1); SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.

Rates/Ratios/Percent

DS67410.01

125. A large flower arrangement contains 3 types of flowers: carnations, lilies, and roses. Of all the flowers in the arrangement, $\frac{1}{2}$ are carnations, $\frac{1}{3}$ are lilies, and $\frac{1}{6}$ are roses. The total price of which of the 3 types of flowers in the arrangement is the greatest?

1. The prices per flower for carnations, lilies, and roses are in the ratio 1:3:4, respectively.
2. The price of one rose is \$0.75 more than the price of one carnation, and the price of one rose is \$0.25 more than the price of one lily.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Applied problems; Ratios**

Let T be the total number of flowers, and let \$$C$, \$$L$, and \$$R$ be the cost, respectively, of one carnation, one lily, and one rose.

1. We are given that $L = 3C$ (because C:L is 1:3) and $R = 4C$ (because C:R is 1:4). The table below shows the total price for each type of flower.

Flower	Number of flowers	Price per flower	Total price
Carnation	$\frac{1}{2}T$	C	$\frac{1}{2}TC$
Lily	$\frac{1}{3}T$	$3C$	TC
Rose	$\frac{1}{6}T$	$4C$	$\frac{2}{3}TC$

From the table it is clear that lilies have the greatest total cost; SUFFICIENT.

2. We are given that $R = 0.75 + C$ and $R = 0.25 + L$. To simplify matters, we can use these equations to express each of the variables C, L, and R in terms of a single fixed variable, for example, $C = R - \frac{3}{4}$ and $L = R - \frac{1}{4}$. This will allow us to replace all appearances of C, L, and R with appearances of R only, thereby reducing by two the number of variables that have to be dealt with. The table below shows, for two values of T and R, the total price for each type of flower.

Flower	Number of flowers	Price per flower	Total price	Total price: $T = 24, R = 1$	Total price: $T = 24, R = 10$
Carnation	$\frac{1}{2}T$	$R - \frac{3}{4}$	$\frac{1}{2}TR - \frac{3}{8}T$	3	**111**
Lily	$\frac{1}{3}T$	$R - \frac{1}{4}$	$\frac{1}{3}TR - \frac{1}{12}T$	**6**	78
Rose	$\frac{1}{6}T$	R	$\frac{1}{6}TR$	4	40

From the table it is clear that the type of flower having the greatest total cost can vary; NOT sufficient.

Tip

Consider the expressions under "Total price" in the previous table. Note that, for a fixed value of *T*, as the value of *R* increases without bound, the total price for carnation will eventually exceed the total price for each of the other two types of flowers. Therefore, for non-sufficiency of (2), it is only necessary to determine whether there exist values for *T* and *R* such that carnations do not have the greatest total price. This suggests trying a small value for *R*, for example $R = 1$. Also, note that $T = 24$ was chosen to avoid fractions in the computations—24 is divisible by both 8 and 12.

The correct answer is A;
statement 1 alone is sufficient.

DS34010.01

126. Town X has 50,000 residents, some of whom were born in Town X. What percent of the residents of Town X were born in Town X ?

1. Of the male residents of Town X, 40 percent were not born in Town X.
2. Of the female residents of Town X, 60 percent were born in Town X.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Applied problems

1. We are given that 40 percent of the male residents were NOT born in Town X, or equivalently, that 60 percent of the male residents were born in Town X. However, no information is given about the number of female residents or about the percent of female residents born in Town X. By considering extreme cases, it is easy to see that the percent of residents born in Town X cannot be determined. For example, if only 10 of the 50,000 residents were male and 0 percent of the female residents were born in Town X, then only 6 residents (i.e., close to 0 percent of the residents) would have been born in Town X. However, if only 10 of the 50,000 residents were male and 100 percent of the female residents were born in Town X, then 49,996 residents (i.e., close to 100 percent of the residents) would have been born in Town X; NOT sufficient.
2. We are given that 60 percent of the female residents were born in Town X. However, no information is given about the number of male residents or about the percent of male residents born in Town X. By considering extreme cases in the same manner that was done in (1), it is easy to see that the percent of residents born in Town X cannot be determined; NOT sufficient.

Given (1) and (2), it follows that 60 percent of the male residents and 60 percent of the female residents were born in Town X. Therefore, 60 percent of the residents were born in Town X.

The correct answer is C;
both statements together are sufficient.

DS29931.01

127. A bank account earned 2% annual interest, compounded daily, for as long as the balance was under $1,000, starting when the account was opened. Once the balance reached $1,000, the account earned 2.5% annual interest, compounded daily until the account was closed. No deposits or withdrawals were made. Was the total amount of interest earned at the 2% rate greater than the total amount earned at the 2.5% rate?

1. The account earned exactly $25 in interest at the 2.5% rate.
2. The account was open for exactly three years.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Applied problems

Let P_O, P_1, and P_2 be the initial balance, the balance after one year, and the balance after two years.

1. Since $25 is the exact amount of interest earned in one year by an initial amount of $1,000 earning 2.5 percent annual interest, compounded yearly, it follows that $25 is the total amount of interest earned in slightly less than one year by an initial amount of $1,000 earning 2.5 percent annual interest, compounded daily. However, the total amount of interest earned at the 2 percent rate could be less than $25 (for example, if P_O = $990, then the interest earned at the 2 percent rate is $10) and the total amount of interest earned at the 2 percent rate could be greater than $25 (for example, if P_O = $900, then the interest earned at the 2 percent rate is $100); NOT sufficient.
2. Given that the account was open for exactly three years, then the total amount of interest at the 2 percent rate could be less than the total amount of interest at the 2.5 percent rate (for example, if the balance reached $1,000 a few days after the account was open). On the other hand, the total amount of interest at the 2 percent rate could also be greater than the total amount of interest at the 2.5 percent rate (for example, if the balance reached $1,000 a few days before the account was closed); NOT sufficient.

Given (1) and (2), it follows that the account earned interest at the 2.5 percent rate for slightly less than one year and the account earned interest at the 2 percent rate for slightly more than two years. Therefore, the balances of P_1 and P_2 were reached while the account was earning interest at the 2 percent rate. Since $P_O(1.02) < P_1$ and $P_1(1.02) < P_2$ (compounding daily for one year produces a greater amount than compounding annually for one year), the values of P_O, P_1, and P_2 satisfy the following inequalities.

$P_O < P_O(1.02) < P_1 < P_1(1.02) < P_2 < 1{,}000$

Note that the difference $1{,}000 - P_O$ is the total amount of interest earned at the 2 percent rate. Thus, using (2), we wish to determine whether this difference is greater than 25. From $P_O(1.02) < P_1$ it follows that $P_O(1.02)^2 < P_1(1.02)$, and since $P_1(1.02) < 1{,}000$, we have $P_O(1.02)^2 < 1{,}000$. Therefore, $P_O < \frac{1{,}000}{(1.02)^2}$, from which we can conclude the following inequality.

$$1{,}000 - P_0 > 1{,}000 - \frac{1{,}000}{(1.02)^2}$$

Since $1{,}000 - \frac{1{,}000}{(1.02)^2} > 25$ (see below), it follows that $1{,}000 - Po > 25$ and hence the total amount of interest earned at the 2 percent rate is greater than the total amount of interest earned at the 2.5 percent rate.

One way to verify that $1{,}000 - \frac{1{,}000}{(1.02)^2} > 25$ is to verify that $1 - \frac{1}{(1.02)^2} > \frac{1}{40}$, or equivalently, verify that $\frac{1}{(1.02)^2} < \frac{39}{40}$, or $40 < 39(1.02)^2$. Now note that we can obtain this last inequality from $40 < 39(1.04)$ (because $39 + 39(0.04)$ is greater than $39 + 1$) and $1.04 < (1.02)^2$.

The correct answer is C;
both statements together are sufficient.

DS53541.01

128. A novelist pays her agent 15% of the royalties she receives from her novels. She pays her publicist 5% of the royalties, plus a yearly fee. Did the novelist pay more to her agent last year than she paid to her publicist?

1. The publicist's yearly fee is $2,000.
2. The novelist earned an average of $3,500 in royalties last year on each of her novels.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Applied problems

Let $\$R$ be the novelist's royalties last year, and let $\$Y$ be the yearly fee paid to the publicist. Determine whether $0.15R > 0.05R + Y$, or equivalently, whether $R > 10Y$.

1. No information is given that allows us to determine whether R is greater than $10Y = 10(2{,}000) = 20{,}000$; NOT sufficient.
2. No information is given that allows us to determine whether $3{,}500n$ is greater than $10Y$, where n is the number of novels; NOT sufficient.

Given (1) and (2) and letting n be the number of novels, we are to determine whether $3{,}500n > 20{,}000$. If $n = 1$, then the answer is NO. However, if $n = 10$, then the answer is YES.

The correct answer is E;
both statements together are still not sufficient.

Value/Order/Factors

DS85100.01

129. If x and z are integers, is $x + z^2$ odd?

1. x is odd and z is even.
2. $x - z$ is odd.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

1. We are given that x is odd and z is even. Therefore, z^2 is even and hence $x + z^2$ is odd, because an odd integer added to an even integer is an odd integer; SUFFICIENT.
2. We are given that $x - z$ is odd. Since there is not a readily apparent useful algebraic relation between $x - z$ and $x + z^2$, we consider all possible cases.

x	z	z^2	$x - z$	$x + z^2$
even	even	even	even	even
even	odd	odd	odd	odd
odd	even	even	odd	odd
odd	odd	odd	even	even

From the table it is clear that if $x - z$ is odd, then $x + z^2$ is odd; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.

×	a	b	c
a	d	e	f
b	e	g	h
c	f	h	j

DS95850.01

130. Each entry in the multiplication table above is an integer that is either positive, negative, or zero. What is the value of a ?

1. $h \neq 0$
2. $c = f$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

1. If a, b, c equal 1, 2, 3 in this order, then each entry will be an integer, $h \neq 0$, and $a = 1$. However, if a, b, c equal 2, 3, 4 in this order, then each entry will be an integer, $h \neq 0$, and $a = 2$. Hence, the value of a cannot be determined; NOT sufficient.
2. The assumption $c = f$ is equivalent to $ac = c$, or $(a - 1)c = 0$. Hence, $c = f$ is equivalent to $a = 1$ or $c = 0$. If a, b, c equal 1, 2, 3 in this order, then each entry will be an integer, $c = f$, and $a = 1$. However, if a, b, c equal −2, −1, 0 in this order, then each entry will be an integer, $c = f$, and $a = -2$. Hence, the value of a cannot be determined; NOT sufficient.

Given (1) and (2), then from (1) we have $bc = h \neq 0$, and hence $b \neq 0$ and $c \neq 0$. From (2) we have $a = 1$ or $c = 0$, but since $c \neq 0$, it follows that $a = 1$. Hence, the value of a can be determined.

The correct answer is C;
both statements together are sufficient.

DS36141.01

131. Given a positive number N, when N is rounded by a certain method (for convenience, call it Method Y), the result is 10^n if and only if n is an integer and $5 \times 10^{n-1} \leq N < 5 \times 10^n$. In a certain gas sample, there are, when rounded by Method Y, 10^{21} molecules of H_2 and also 10^{21} molecules of O_2. When rounded by Method Y, what is the combined number of H_2 and O_2 molecules in the gas sample?

1. The number of H_2 molecules and the number of O_2 molecules are each less than 3×10^{21}.
2. The number of H_2 molecules is more than twice the number of O_2 molecules.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Rounding**

Let H be the number of H_2 molecules and let O be the number of O_2 molecules. We are given that $0.5 \times 10^{21} \leq H < 5 \times 10^{21}$ and $0.5 \times 10^{21} \leq O < 5 \times 10^{21}$. When rounded by Method Y, what is the value of $H + O$?

1. If $H = 2 \times 10^{21}$ and $O = 2 \times 10^{21}$, then each of H and O equals 10^{21} when rounded by Method Y, each of H and O is less than 3×10^{21}, and $H + O = 4 \times 10^{21}$, which equals 10^{21} when rounded by Method Y. However, if $H = 2.6 \times 10^{21}$ and $O = 2.6 \times 10^{21}$, then each of H and O equals 10^{21} when rounded by Method Y, each of H and O is less than 3×10^{21}, and $H + O = 5.2 \times 10^{21}$, which equals 10^{22} when rounded by Method Y; NOT sufficient.
2. If $H = 2 \times 10^{21}$ and $O = 0.8 \times 10^{21}$, then each of H and O equals 10^{21} when rounded by Method Y, $H > 2 \times O$, and $H + O = 2.8 \times 10^{21}$, which equals 10^{21} when rounded by Method Y. However, if $H = 4.5 \times 10^{21}$ and $O = 2 \times 10^{21}$, then each of H and O equals 10^{21} when rounded by Method Y, $H > 2 \times O$, and $H + O = 6.5 \times 10^{21}$, which equals 10^{22} when rounded by Method Y; NOT sufficient.

Given (1) and (2), since $O \geq 0.5 \times 10^{21}$ (given information), it follows from statement (2) that $H > 1 \times 10^{21}$. Also, since $H < 3 \times 10^{21}$ (statement (1)), it follows from statement (2) that $O < 1.5 \times 10^{21}$. Thus, $1 \times 10^{21} < H < 3 \times 10^{21}$ and $0.5 \times 10^{21} \leq O < 1.5 \times 10^{21}$, and hence $1.5 \times 10^{21} < H + O < 4.5 \times 10^{21}$. Therefore, the value of $H + O$ equals 10^{21} when rounded by Method Y.

The correct answer is C;
both statements together are sufficient.

DS05541.01

132. If x is a positive integer, how many positive integers less than x are divisors of x ?

1. x^2 is divisible by exactly 4 positive integers less than x^2.
2. $2x$ is divisible by exactly 3 positive integers less than $2x$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Properties of integers**

Tip

For problems that involve how many divisors an unspecified integer has, it is sometimes useful to consider separate cases based on the number of repeated prime factors and the number of distinct prime factors the integer has. For example, let p, q, and r be distinct prime numbers. Then the factors of p^3 are 1, p, q, and p^2; the factors of p^2 are 1, p, q, r, p^2, pr, qr, and pqr; the factors of p^3 are 1, p, p^2, and p^3; the factors of p^2q are 1, p, p^2, q, pq, and p^2q.

1. If x has at least two prime factors, say p and q, then among the factors of x^2 are p, q, pq, p^2, q^2, p^2q, and pq^2, each of which is less than x^2 (because $x^2 \geq p^2q^2$). Thus, x cannot have at least two prime factors, otherwise, x^2 would have more than four divisors less than x^2. Therefore, x has the form $x = p^n$ for some prime number p and positive integer n. There are $2n$ divisors of $x^2 = (p^n)^2 = p^{2n}$ that are less than x^2, namely 1, p, p^2, p^3, ..., p^{2n-2}, and p^{2n-1}. Statement (1) implies that $2n = 4$, and hence $n = 2$. It follows that $x = p^2$ for some prime number p, and so x has exactly two divisors less than x, namely 1 and p. Alternatively, the last part of this argument can be accomplished in a more concrete way by separately considering the number of prime factors of p, p^2, p^3, etc.; SUFFICIENT.
2. Probably the simplest approach is to individually consider the divisors of $2x$ that are less than $2x$ for various values of x. If $x = 1$, then $2x = 2$ has one such divisor, namely 1. If $x = 2$, then $2x = 4$ has two such divisors, namely 1 and 2. If $x = 3$, then $2x = 6$ has three such divisors, namely 1, 2, and 3. If $x = 4$, then $2x = 8$ has three such divisors, namely 1, 2, and 4. At this point we have two integers satisfying statement (2), $x = 3$ and $x = 4$. Since $x = 3$ has one divisor less than $x = 3$ and $x = 4$ has two divisors less than $x = 4$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS33551.01

133. If m and n are positive integers, is n even?

1. $m(m + 2) + 1 = mn$
2. $m(m + n)$ is odd.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

1. Given that $m(m + 2) + 1 = mn$, then m cannot be even, since if m were even, then we would have an odd integer, namely $m(m + 2) + 1$, equal to an even integer, namely mn. Therefore, m is odd. Hence, $m(m + 2)$ is odd, being the product of two odd integers, and thus $m(m + 2) + 1$ is even. Since $m(m + 2) + 1 = mn$, it follows that mn is even, and since m is odd, it follows that n is even; SUFFICIENT.

Alternatively, the table below shows that $m(m + 2) + 1 = mn$ is only possible when m is odd and n is even.

m	n	$m(m + 2) + 1$	mn
even	even	odd	even
even	odd	odd	even
odd	even	**even**	**even**
odd	odd	even	odd

2. Since $m(m + n)$ is odd, it follows that m is odd and $m + n$ is odd. Therefore, $n = (m + n) - m$ is a difference of two odd integers and hence n is even; SUFFICIENT.

The correct answer is D;
each statement alone is sufficient.

DS65291.01

134. If m and n are positive integers, what is the value of $\frac{3}{m} + \frac{n}{4}$?

(1) $mn = 12$

(2) $\frac{3}{m}$ is in lowest terms and $\frac{n}{4}$ is in lowest terms.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Fractions

1. If $m = 1$ and $n = 12$, then m and n are positive integers, $mn = 12$, and $\frac{3}{m}+\frac{n}{4} = 6$. However, if $m = 3$ and $n = 4$, then m and n are positive integers, $mn = 12$, and $\frac{3}{m}+\frac{n}{4} = 2$; NOT sufficient.

2. If $m = 2$ and $n = 1$, then m and n are positive integers, $\frac{3}{m}$ and $\frac{n}{4}$ are in lowest terms, and $\frac{3}{m}+\frac{n}{4}=\frac{3}{2}+\frac{1}{4}$. However, if $m = 2$ and $n = 3$, then m and n are positive integers, $\frac{3}{m}$ and $\frac{n}{4}$ are in lowest terms, and $\frac{3}{m}+\frac{n}{4}=\frac{3}{2}+\frac{3}{4}$; NOT sufficient.

Given (1) and (2), the table below shows that only one possibility exists for the values of m and n, and hence there is only one possible value of $\frac{3}{m}+\frac{n}{4}$.

(m,n)	$\frac{3}{m}$ lowest terms?	$\frac{n}{4}$ lowest terms?
(1,12)	$\frac{3}{1}$, YES	$\frac{12}{4}$, NO
(2,6)	$\frac{3}{2}$, YES	$\frac{6}{4}$, NO
(3,4)	$\frac{3}{3}$, NO	$\frac{4}{4}$, NO
(4,3)	$\frac{3}{4}$, **YES**	$\frac{3}{4}$, **YES**
(6,2)	$\frac{3}{6}$, NO	$\frac{3}{4}$, NO
(12,1)	$\frac{3}{12}$, NO	$\frac{1}{4}$, YES

The correct answer is C;
both statements together are sufficient.

DS21891.01

135. The first four digits of the six-digit initial password for a shopper's card at a certain grocery store is the customer's birthday in day-month digit form. For example, 15 August corresponds to 1508 and 5 March corresponds to 0503. The 5th digit of the initial password is the units digit of seven times the sum of the first and third digits, and the 6th digit of the initial password is the units digit of three times the sum of the second and fourth digits. What month, and what day of that month, was a customer born whose initial password ends in 16 ?

1. The customer's initial password begins with 21, and its fourth digit is 1.
2. The sum of the first and third digits of the customer's initial password is 3, and its second digit is 1.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Computation with integers

Let d_1, d_2, m_1, and m_2, respectively, represent the first four digits of the customer's initial password. Then, the entire password has the form $d_1d_2m_1m_2 16$.

Because the maximum number of days per month is 31 and the number of months in a year is 12, d_1 can be only 0, 1, 2, or 3 and m_1 can be only 0 or 1. The following summarizes the possible values for d_1, d_2, m_1, and m_2.

d_1	d_2	m_1	m_2
0	1–9	0	1–9
1	0–9	1	0, 1, 2
2	0–9		
3	0, 1		

It is given that the fifth digit, which is 1, is the units digit of $7(d_1 + m_1)$. The only relevant multiple of 7 with units digit 1 is (7)(3) = 21, from which it follows that $d_1 + m_1 = 3$. Considering the restrictions on the values of the digits, then $d_1 = 2$ and $m_1 = 1$ or $d_1 = 3$ and $m_1 = 0$. Also, it is given that the sixth digit, which is 6, is the units digit of $3(d_2 + m_2)$. The only relevant multiples of 3 with units digit 6 are (3)(2) = 6 and (3)(12) = 36, from which it follows that $d_2 + m_2 = 2$ or $d_2 + m_2 = 12$.

Considering the restrictions on the values of the digits, if $d_2 + m_2 = 2$, then the only possibilities are $d_2 = 0$ and $m_2 = 2$ or $d_2 = 1$ and $m_2 = 1$ or $d_2 = 2$ and $m_2 = 0$. If $d_2 + m_2 = 12$, each of d_2 and m_2 is at least 3 because if either of the digits is less than 3, then the sum of the two digits cannot be 12. But if $d_1 = 2$ and $m_1 = 1$, which is one of the possibilities for d_1 and m_1 above, then m_2 can be only 0, 1, or 2; and if $d_1 = 3$ and $m_1 = 0$, which is the other possibility above for d_1 and m_1, then d_2 can be only 0 or 1. The table below summarizes the first four digits of the passwords that meet all conditions thus far.

First digit d_1	Second digit d_2	Third digit m_1	Fourth digit m_2
2	0	1	2
2	1	1	1
2	2	1	0
3	0	0	2
3	1	0	1
3	2	0	0

1. It is given that the customer's password begins with 21 and the fourth digit is 1. In the table above, only one possible password meets these conditions, so the first four digits of the password are 2111 and the customer's birthday is the 21st day of November; SUFFICIENT.
2. It is given that $d_1 + m_1 = 3$ and $d_2 = 1$. In the table above, the possibilities for the first four digits of the customer's password, where $d_2 = 1$, are 3101 and 2111, so the customer's birthday could be the 31st day of January or the 21st day of November; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS38302.01

136. If K is a positive integer less than 10 and $N = 4{,}321 + K$, what is the value of K?

1. N is divisible by 3.
2. N is divisible by 7.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Computation with integers**

1. Dividing 4,321 by 3 gives a quotient of 1,440 and a remainder of 1, so 4,321 = 3(1,440) + 1. It follows that $N = [3(1,440) + 1] + K = 3(1,440) + (1 + K)$. It is given that N is divisible by 3, from which it follows that $1 + K$ must be a multiple of 3. Therefore K can be 2, 5, or 8 since $K < 10$.

   Alternatively, a number is divisible by 3 if and only if the sum of its digits is divisible by 3. If $K \neq 9$, the sum of the digits of $N = 4,321 + K$ is $4 + 3 + 2 + 1 + K = 10 + K = 1 + K$, which is divisible by 3 when $K = 2$, 5, or 8; NOT sufficient.

2. Dividing 4,321 by 7 gives a quotient of 617 and a remainder of 2, so 4,321 = 7(617) + 2. It follows that $N = [7(617) + 2] + K = 7(617) + (2 + K)$. It is given that N is divisible by 7 from which it follows that $2 + K$ must be a multiple of 7. Thus, $K = 5$ since $K < 10$; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS99302.01

137. If s is an integer, is 24 a divisor of s ?

1. Each of the numbers 3 and 8 is a divisor of s.
2. Each of the numbers 4 and 6 is a divisor of s.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Properties of integers**

1. If each of the numbers 3 and 8 is a divisor of s, then using the prime factorization of 8 gives $s = 2^3 \times 3 \times q = 24q$, for some positive integer q. Thus, 24 is a divisor of s; SUFFICIENT.
2. If each of the numbers 4 and 6 is a divisor of s, then s could be 24 and it follows that 24 is a divisor of s. On the other hand, s could be 12 because 4 and 6 are both divisors of 12 and 24 is not a divisor of s; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

Tip

If the integer n is divisible by each of the integers a and b and the greatest common factor of a and b is 1, then n is divisible by ab. However, if the greatest common divisor of a and b is greater than 1, then n may or may not be divisible by ab.

DS32402.01

138. $n = 2^4 \cdot 3^2 \cdot 5^2$ and positive integer d is a divisor of n. Is $d > \sqrt{n}$?

1. d is divisible by 10.
2. d is divisible by 36.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

Given $n = 2^4 \cdot 3^2 \cdot 5^2$, then $\sqrt{n} = (2^2)(3)(5) = 60 = (2^2)(3)(5) = 60$. If d is a divisor of n, determine whether $d > 60$ is true.

1. It is given that d is divisible by 10. If $d = 10$, then d is divisible by 10 and d is a divisor of n since $n = (10)(2^3 \cdot 3^2 \cdot 5)$, but $d > 60$ is not true. However, if $d = 80$, then d is divisible by 10, d is a divisor of n since $n = (80)(3^2 \cdot 5)$, and $d > 60$ is true; NOT sufficient.
2. It is given that d is divisible by 36. If $d = 36$, then d is divisible by 36 and d is a divisor of n since $n = (36)(2^2 \cdot 5^2)$, but $d > 60$ is not true. However, if $d = 72$, then d is divisible by 36, d is a divisor of n since $n = (72)(2 \cdot 5^2)$, and $d > 80$ is true; NOT sufficient.

Taking (1) and (2) together, it follows that d is divisible by $2^2 \cdot 3^2 \cdot 5 = 180$ and every multiple of 180 is greater than 60.

The correct answer is C;
both statements together are sufficient.

DS52402.01

139. Exactly 3 deposits have been made in a savings account and the amounts of the deposits are 3 consecutive integer multiples of \$7. If the sum of the deposits is between \$120 and \$170, what is the amount of each of the deposits?

1. The amount of one of the deposits is \$49.
2. The amount of one of the deposits is \$63.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

If k represents the least of the multiples of 7, then the three deposits, in dollars, are represented by $7k$, $7(k + 1)$, and $7(k + 2)$. The sum, in dollars, of the deposits is $21k + 21$, where $120 < 21k + 21 < 170$. It follows that the value of the integer k is 5, 6, or 7. If $k = 5$, then the deposits could be 35, 42, 49, with a sum of 126, which is between 120 and 170. If $k = 6$, then the deposits could be 42, 49, 56 with a sum of 147, which is between 120 and 170. If $k = 7$, then the deposits could be 49, 56, 63 with a sum of 168, which is between 120 and 170.

1. It is given that one of the deposits, in dollars, is 49. Since 49 is one of the amounts for each value of k in the remarks above, the amounts of the three deposits cannot be determined; NOT sufficient.
2. It is given that one of the deposits, in dollars, is 63. Since 63 occurs for exactly one value of k in the remarks above, the amounts, in dollars, of the deposits are 49, 56, and 63; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS44402.01

140. If x, y, and d are integers and d is odd, are both x and y divisible by d?

1. $x + y$ is divisible by d.
2. $x - y$ is divisible by d.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

Determine whether both of the integers x and y are divisible by the odd integer d.

1. It is given that $x + y$ is divisible by d. It is possible that both x and y are divisible by d, and it is possible that they are not both divisible by d. For example, if $x = 4$, $y = 2$, and $d = 3$, then $4 + 2$ is divisible by 3, but neither 4 nor 2 is divisible by 3. On the other hand, if $x = 3$, $y = 6$, and $d = 3$, then $3 + 6$ is divisible by 3, and both 3 and 6 are divisible by 3; NOT sufficient.
2. It is given that $x - y$ is divisible by d. It is possible that both x and y are divisible by d, and it is possible that they are not both divisible by d. For example, if $x = 4$, $y = -2$, and $d = 3$, then $4 - (-2)$ is divisible by 3, but neither 4 nor -2 is divisible by 3. On the other hand, if $x = 3$, $y = -6$, and $d = 3$, then $3 - (-6)$ is divisible by 3, and both 3 and -6 are divisible by 3; NOT sufficient.

Taking (1) and (2) together, $x + y$ is divisible by d, so $\frac{x+y}{d}$ is an integer and $x - y$ is divisible by d, so $\frac{x-y}{d}$ is an integer. It follows that $\frac{x+y}{d} + \frac{x-y}{d} = \frac{2x}{d}$ is an integer and $\frac{x}{d}$ is an integer because d is odd. Similarly, $\frac{x+y}{d} - \frac{x-y}{d} = \frac{2y}{d}$ is an integer and $\frac{y}{d}$ is an integer because d is odd.

The correct answer is C;
both statements together are sufficient.

DS06402.01

141. If x and y are integers, is $xy + 1$ divisible by 3 ?

1. When x is divided by 3, the remainder is 1.
2. When y is divided by 9, the remainder is 8.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

Determine whether $xy + 1$ is divisible by 3, where x and y are integers.

1. It is given that the remainder is 1 when x is divided by 3. It follows that $x = 3q + 1$ for some integer q. So, $xy + 1 = (3q + 1)y + 1$. If $y = 2$, then $xy + 1 = 6q + 3$, which is divisible by 3. However, if $y = 1$, then $xy + 1 = 3q + 2$, which is not divisible by 3; NOT sufficient.
2. It is given that the remainder is 8 when y is divided by 9. It follows that $y = 9r + 8$ for some integer r. So, $xy + 1 = (9r + 8)x + 1$. If $x = 1$, then $xy + 1 = 9r + 9$, which is divisible by 3. However, if $x = 2$, then $xy + 1 = 18r + 17$, which is not divisible by 3; NOT sufficient.

Taking (1) and (2) together gives $x = 3q + 1$ and $y = 9r + 8$, from which it follows that $xy + 1 = (3q + 1)(9r + 8) + 1 = 27qr + 9r + 24q + 9 = 3(9qr + 3r + 8q + 3)$, which is divisible by 3.

The correct answer is C;
both statements together are sufficient.

DS00502.01

142. If x and y are integers between 10 and 99, inclusive, is $\frac{x-y}{9}$ an integer?

1. x and y have the same two digits, but in reverse order.
2. The tens' digit of x is 2 more than the units' digit, and the tens' digit of y is 2 less than the units' digit.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Properties of integers

Determine whether $\frac{x-y}{9}$ is an integer, where x and y are 2-digit integers.

1. From (1), if $x = 10a + b$, then $y = 10b + a$. It follows that $x - y = 9(a - b)$, so $\frac{x-y}{9}$ is an integer; SUFFICIENT.
2. From (2), if $x = 10a + b$, then $a = b + 2$. If $y = 10c + d$, then $c = d - 2$. It is possible that $\frac{x-y}{9}$ is an integer (for example, if $x = 75$ and $y = 57$), and it is possible that $\frac{x-y}{9}$ is not an integer (for example, if $x = 75$ and $y = 46$); NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS85502.01

143. If b is the product of three consecutive positive integers c, $c + 1$, and $c + 2$, is b a multiple of 24 ?

1. b is a multiple of 8.
2. c is odd.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

Since $24 = 2^3 \times 3$, and 1 is the only common factor of 2 and 3, any positive integer that is a multiple of 24 must be a multiple of both $2^3 = 8$ and 3. Furthermore, the product of any three consecutive positive integers is a multiple of 3. This can be shown as follows. In $b = c(c + 1)(c + 2)$, when the positive integer c is divided by 3, the remainder must be 0, 1, or 2. If the remainder is 0, then c itself is a multiple of 3. If the remainder is 1, then $c = 3q + 1$ for some positive integer q and $c + 2 = 3q + 3 = 3(q + 1)$ is a multiple of 3. If the remainder is 2, then $c = 3r + 2$ for some positive integer r and $c + 1 = 3r + 3 = 3(r + 1)$ is a multiple of 3. In all cases, $b = c(c + 1)(c + 2)$ is a multiple of 3.

1. It is given that b is a multiple of 8. It was shown above that b is a multiple of 3, so b is a multiple of 24; SUFFICIENT.
2. It is given that c is odd. If $c = 3$, then $b = (3)(4)(5) = 60$, which is not a multiple of 24. If $c = 7$, then $b = (7)(8)(9) = (24)(7)(3)$, which is a multiple of 24; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS17602.01

144. If ⊛ denotes a mathematical operation, does $x ⊛ y = y ⊛ x$ for all x and y ?

1. For all x and y, $x ⊛ y = 2(x^2 + y^2)$.
2. For all y, $0 ⊛ y = 2y^2$.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
D. EACH statement ALONE is sufficient.
E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra Functions

1. For all x and y, $x \circledcirc y = 2(x^2 + y^2)$ is equal to $y \circledcirc x = 2(y^2 + x^2)$; SUFFICIENT.

2. If $x \circledcirc y = 2(x^2 + y^2)$ for all x and y, then $0 \circledcirc y = 2y^2$ for all y and $x \circledcirc y = y \circledcirc x$ for all x and y. However, if $x \circledcirc y = 2(x^3 + y^2)$ for all x and y, then $0 \circledcirc y = 2y^2$ for all y, but $1 \circledcirc 2 = 2(1^3) + 2(2^2) = 10$ and $2 \circledcirc 1 = 2(2^3) + 2(1^2) = 18$; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS37602.01

145. If n is an integer, is $\frac{n}{15}$ an integer?

1. $\frac{3n}{15}$ is an integer.
2. $\frac{8n}{15}$ is an integer.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic Properties of integers

1. We are given that $\frac{3n}{15} = \frac{n}{5}$ is an integer. If $n = 15$, then $\frac{n}{5}$ is an integer and $\frac{n}{15}$ is an integer. However, if $n = 5$, then $\frac{n}{5}$ is an integer and $\frac{n}{15}$ is not an integer; NOT sufficient.

2. We are given that $\frac{8n}{15} = k$, where k is an integer. Since $8n = 15k$, it follows that both 3 and 5 are factors of $8n$. Therefore, the prime factorization of $8n = 2^3 \times n$ includes at least one factor of 3 and at least one factor of 5, and it is clear that each of these factors must be among the prime factors of n. Thus, both 3 and 5 are factors of n, and hence n is divisible by 15.

Alternatively, since 15 divides $8n$, and 8 and 15 are relatively prime, then it follows that 15 divides n; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS97602.01

146. If $1 < d < 2$, is the tenths digit of the decimal representation of d equal to 9 ?

1. $d + 0.01 < 2$
2. $d + 0.05 > 2$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Place value**

Determine if the tenths digit of d, where $1 < d < 2$, is 9.

1. Given $d + 0.01 < 2$, it follows that $d < 1.99$. It is possible that the tenths digit of d is 9 (for example, $1.98 < 1.99$, and the tenths digit of 1.98 is 9), and it is possible that the tenths digit of d is not 9 (for example, $1.88 < 1.99$, and the tenths digit of 1.98 is 8); NOT sufficient.
2. Given $d + 0.05 > 2$, it follows that $d > 1.95$. Then $1.95 < d < 2$, since $d < 2$, so the tenths digit of d is 9; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.

DS08602.01

147. The 9 participants in a race were divided into 3 teams with 3 runners on each team. A team was awarded $6 - n$ points if one of its runners finished in nth place, where $1 \le n \le 5$. If all of the runners finished the race and if there were no ties, was each team awarded at least 1 point?

1. No team was awarded more than a total of 6 points.
2. No pair of teammates finished in consecutive places among the top five places.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Operations with integers**

Determine whether each team was awarded at least 1 point.

1. It is given that no team was awarded more than 6 points. Since there were no ties, one of the nine runners had to have finished in first place. Say this runner was on Team A, and Team A was awarded 5 points. Since Team A was awarded at most 6 points, the best finish for one of the two other runners on Team A could be fifth place, leaving second place to a runner on one of the other teams. Say a runner on Team B finished in second place, and Team B was awarded 4 points. Since Team B was awarded at most 6 points, the best finish for one of the two other runners on Team B could be fourth place, leaving third place to a runner on the only team remaining, which would then be awarded 3 points. Thus, each team was awarded at least 1 point; SUFFICIENT.
2. Given that no pair of teammates finished in consecutive places, it is possible that each team was awarded at least 1 point and it is also possible that at least one team was not awarded at least 1 point. For example, if the three runners on Team A placed first, third, and fifth and two runners on Team B placed second and fourth, then no pair of teammates finished in consecutive places, and Team C was awarded 0 points. On the other hand, if the three runners on Team A placed first, third, and fifth, a runner on Team B placed second, and a runner on Team C placed fourth, then no pair of teammates finished in consecutive places and each team was awarded at least 1 point; NOT sufficient.

The correct answer is A;
statement 1 alone is sufficient.

DS38602.01

148. Can the positive integer n be written as the sum of two different positive prime numbers?

1. n is greater than 3.
2. n is odd.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Arithmetic **Properties of integers**

Determine if the positive integer n can be written as the sum of two different positive prime numbers.

1. It is given that $n > 3$. If $n = 5$, then $n = 2 + 3$ and 2 and 3 are different positive prime numbers. However, $n = 11$, then n can be written as the following sums of two different positive numbers: $1 + 10$, $2 + 9$, $3 + 8$, $4 + 7$, and $5 + 6$. In no case are the addends both prime; NOT sufficient.
2. It is given that n is odd. The values of n in the examples used to show that (1) is not sufficient also satisfy (2); NOT sufficient.

Taken together, (1) and (2) are not sufficient because the same examples used to show that (1) is not sufficient also show that (2) is not sufficient.

The correct answer is E;
both statements together are still not sufficient.

DS73402.01

149. Is x an integer?

1. x^2 is an integer.
2. $\frac{x}{2}$ is not an integer.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Properties of integers**

Determine if x is an integer.

1. It is given that x^2 is an integer. If, for example, $x^2 = 49$, then x is an integer. However, if $x^2 = 5$, then x is not an integer; NOT sufficient.
2. It is given that $\frac{x}{2}$ is not an integer. If, for example, $x = 7$, then $\frac{x}{2}$ is not an integer, but x is an integer. However, if $x = \sqrt{5}$, then $\frac{x}{2}$ is not an integer and neither is x; NOT sufficient.

Taken together, (1) and (2) are not sufficient because the same examples used to show that (1) is not sufficient also show that (2) is not sufficient.

The correct answer is E;
both statements together are still not sufficient.

DS46402.01

150. If b is an integer, is $\sqrt{a^2 + b^2}$ an integer?

1. $a^2 + b^2$ is an integer.
2. $a^2 - 3b^2 = 0$

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

Algebra **Operations on radical expressions**

Given that b is an integer, determine if $\sqrt{a^2+b^2}$ is an integer.

1. It is given that $a^2 + b^2$ is an integer. If $a = 3$ and $b = 4$, then $\sqrt{a^2+b^2} = 5$, which is an integer. However, if $a = 1$ and $b = 2$, then $\sqrt{a^2+b^2} = \sqrt{5}$, which is not an integer; NOT sufficient.

2. It is given that $a^2 - 3b^2 = 0$, from which it follows that $a^2 = 3b^2$. Then, $\sqrt{a^2+b^2} = \sqrt{3b^2+b^2} = \sqrt{4b^2} = 2|b|$, which is an integer; SUFFICIENT.

The correct answer is B;
statement 2 alone is sufficient.