数学冲刺

GMAT

第二章
整数的性质

本节课授课要点

2.1
奇数与偶数

2.2
因数、质数与合数

2.3
质因数

2.4
整除和余数

2.5
连续数

2.1 奇数与偶数

- 奇数与偶数（odd and even number）
- 包括负数和0
- 奇数+奇数=偶数　奇数X奇数=奇数
- 偶数+偶数=偶数　奇数X偶数=偶数
- 奇数+偶数=奇数　偶数X偶数=偶数
- 加号可以换成减号，乘号不一定能换成除号。
- 结果为偶，两数奇偶性相同；结果为奇，两数奇偶性不同。
- N个整数相加减：奇数的个数为奇数个，和或者差为奇数；
- N个整数相乘： 有偶则偶，无偶则奇。
- 如果n为正整数，一个整数的n次方和这个整数的奇偶性相同。
- 奇数的0次方是奇数，偶数的0次方还是奇数

2.1 奇数与偶数

If $a^2 + b^2 = c^2$, where a,b,c are integers, which of the following cannot be a value of a+b+c?

A 2

B 1

C -2

D 4

E 6

2.1 奇数与偶数

If x and y are integers, is y an even integer?

(1) $2y-x=x^2-y^2$

(2) x is an odd integer.

2.1 奇数与偶数

- n is an integer, is np even?
- (1) n is an even integer
- (2) p is an even integer

本节课授课要点

2.1
奇数与偶数

2.2
因数、质数与合数

2.3
质因数

2.4
整除和余数

2.5
连续数

2.2 因数、质数与合数

因数（factor divisor）

一个整数可以写成其他整数相乘的形式，则这个其他整数就是这个数的因数。

a=b *c

6=2*3

2.2 因数、质数与合数

质数（prime number）与合数（composite number）

- 若某正整数只有1和它自己两个正因数，则为质数。
- 若某正整数除了1和它自己之外还有其他正因数，则为合数。
- 20以内质数有哪些?
- 2 3 5 7 11 13 17 19
- 注意：1 既不是质数，也不是合数。
- 2 是最重要的质数，唯一一个既是偶数又是质数的数。

2.2 因数、质数与合数

十个不同质数的和是一个奇数，请问这些质数中一定有下列哪个数？

A. 2
B. 3
C. 5
D. 7
E. 11

2.2 因数、质数与合数

Two different primes may be said to "rhyme" around and integer if they are the same distance from the integer on the number line. For instance, 3 and 7 rhyme around 5. What integer between 1 and 20, inclusive, has the greatest number of distinct rhyming primes around it?

A. 12

B. 15

C. 17

D. 18

E. 20

2.2 因数、质数与合数

- 12: 2 3 5 7 11
- 22 21 19 17 13
- 15: 2 3 5 7 11 13
- 28 27 25 23 19 17
- 17: 2 3 5 7 11 13
- 32 31 29 27 23 21
- 18: 2 3 5 7 11 13 17
- 34 33 31 29 25 23 19
- 20: 2 3 5 7 11 13 17 19
- 38 37 35 33 29 27 23 21

本节课授课要点

2.1
奇数与偶数

2.2
因数、质数与合数

2.3
质因数

2.4
整除和余数

2.5
连续数

2.3 质因数

质因数（prime factor）

- 一个数所有正因数中的质数为这个数的质因数。
- 12=2*6=2*2*3
- 任意一个整数可以写成其质因数相乘的形式，这种思想对于解题十分重要

2.3 质因数

If x is the product of the positive integers from 1 to 8, inclusive, and i, k, m and p are positive integers such that $x= 2^i\ 3^k\ 5^m\ 7^p$, then i+ k + m + p=

A.4 B.7 C.8 D.11 E.12

2.3 质因数

The product of the units digit, the tens digit, and the hundreds digit of the positive integer m is 96.

What is the units digit of m?

(1) m is odd.

(2) The hundreds digit of m is 8.

2.3 质因数

质因数对倍数关系判断的帮助

- 若A的质因数的种类是B的子集(subset)，且对每种质因数来说，A的个数≤B的个数则B是A的倍数(multiple)，A是B的因数.
- 如果A具有B不具备的质因数，或者相同质因数的数量超过B，则A不可能是B的因数，B也就不可能是A的倍数。
- $A=mn^2$ $B=m^2n^2$

2.3 质因数

A school administrator will assign each student in a group of n students to one of m classroom. If $3<m<13<n$, is it possible to assign each of the n students to one of the m classrooms so that each classroom has the same number of students assigned to it?

(1) It is possible to assign each of 3n students to one of m classrooms so that each classroom has the same number of students assigned to it.

(2) It is possible to assign each of 13n students to one of m classroom so that each classroom has the same number of students assigned to it.

2.3 质因数

In the decimal representation of x, where $0<x<1$, is the tenths digit of x nonzero?

(1) 16x is an integer.

(2) 8x is an integer.

A. Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.

B. Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.

C. BOTH statement TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

D. EACH statement ALONE is sufficient.

E. Statements (1) and (2) TOGETHER are NOT sufficient.

2.3 质因数

If the integer n has exactly three positive divisors, including 1 and n, how many positive divisors does n^2 have?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 9

2.3 质因数

Is the integer n odd?

(1)n is divisible by 3.

(2)2n is divisible by twice as many positive integers as n.

2.3 质因数

$X=a^m b^n$ 是一个完全平方数，则m，n都是偶数

If n is a positive integer and n^2 is divisible by 72, then the largest positive integer that must divide n is

(A)6

(B) 12

(C)24

(D)36

(E) 48

本节课授课要点

2.1
奇数与偶数

2.2
因数、质数与合数

2.3
质因数

2.4
整除和余数

2.5
连续数

2.4 整除和余数

- M除以n余数为r
- M=qn+r
- M=a+b+c
- 若a,b，c是n的倍数，M是n的倍数。
- 若只有a，b是n的倍数， m/n的余数就是c/n的余数。
- 若M，a，b是n的倍数，c是n的倍数
- 判断整除与余数用分数形式表示更直观

2.4 整除和余数

- if a,b,and c are integers , is the number 3(a+b)-c divisible by 3?
- (1)a+b is divisible by 3
- (2) c is divisible by 3.

2.4 整除和余数

What is the remainder when the positive integer n is divided by the positive integer k, where k>1?

(1) $n=(k+1)^3$

(2) k=5

A. Statement (1)ALONE is sufficient ,but statement (2) alone is not sufficient .

B. Statement (2)ALONE is sufficient ,but statement (1) alone is not sufficient .

C.BOTH statement TOGETHER are sufficient, but NEITHER statement ALONE is sufficient .

D.EACH statement ALONE is sufficient .

E.Statements (1) and (2) TOGETHER are NOT suffiicient .

2.4 整除和余数

If q is positive integer less than 17 and r is the remainder when 17 is divided by q, what is the value of r ?

(1)$q>10$

(2)$q=2^k$, where k is a positive integer .

A. Statement (1)ALONE is sufficient ,but statement (2) alone is not sufficient .

B. Statement (2)ALONE is sufficient ,but statement (1) alone is not sufficient .

C.BOTH statement TOGETHER are sufficient, but NEITHER statement ALONE is sufficient .

D.EACH statement ALONE is sufficient .

E.Statements (1) and (2) TOGETHER are NOT suffiicient .

2.4 整除和余数

二项式展开定理在余数问题中的运用

- $(a+b)^n=a^n+c_1a^{n-1}b+c_2a^{n-2}b^2+\ldots\ldots+b^n$
- What is the remainder when 9^{12} is divided by 7?

2.4 整除和余数

- 一个数是否能够被5整除，只要看它的最后一位（是0或5）。
- 一个数是否能够被4整除，只要看它的后两位（是否是4的倍数）。
- 一个数是否能够被8整除，只要看它的后三位（是否是8的倍数）。
- 一个数能否被3整除，取决于各位之和能否被3整除。
- 一个数能否被9整除，也取决于各位之和能否被9整除。

2.4 整除和余数

If n and m are positive integers, what is the remainder when 3^(4n+2+m) is divided by 10 ?

(1)n=2

(2)m=1

A. Statement (1)ALONE is sufficient ,but statement (2) alone is not sufficient .

B. Statement (2)ALONE is sufficient ,but statement (1) alone is not sufficient .

C.BOTH statement TOGETHER are sufficient, but NEITHER statement ALONE is sufficient .

D.EACH statement ALONE is sufficient .

E.Statements (1) and (2) TOGETHER are NOT suffiicient .

2.4 整除和余数

- 若m/n小数部分为x，则余数为nx
- If s and t are positive integers such that s/t=64.12 ,which of the following could be the
- remainder when s is divided by t?
- (A) 2 (B) 4 (C) 8 (D) 20 (E) 45

本节课授课要点

2.1
奇数与偶数

2.2
因数、质数与合数

2.3
质因数

2.4
整除和余数

2.5
连续数

2.5 连续数

consecutive numbers 连续数

- 连续数的平均数是第一个和最后一个的平均值

The average(arithmetic mean)of the integers from 200 to 400,inclusive, is how much greater than the average of the integers from 50 to 100, inclusive?

A.150
B.175
C.200
D.225
E.300

2.5 连续数

连续的n个整数中，有且只有1个是n的倍数

If n is an integer greater than 6, which of the following must be divisible by 3?

(A) n(n+1)(n-4)
(B) n(n+2)(n-1)
(C) n(n+3)(n-5)
(D) n(n+4)(n-2)
(E) n(n+5)(n-6)

2.5 连续数

连续的偶数中，有且只有一个是4的倍数

If n is a positive integer and r is remainder when (n-1)(n+1)is divided by 24,what is the value of r ?

(1) 2 is not a factor of n.

(2) 3 is not a factor of n.

2.5 连续数

If an integer n is to be chosen at random from the integers 1 to 96,inclusive, what is the probability that $n(n+1)(n+2)$ will be divisible by 8?

A. $\frac{1}{4}$ B. $\frac{3}{8}$ C. $\frac{1}{2}$ D. $\frac{5}{8}$ E. $\frac{3}{4}$

2.5 连续数

- N个连续的整数除以m的所得的余数也是连续的，若N≥M，则余数以m为周期循环，分别是1,2,…m-1,0。

- 1，2，3，4，5，6，7除以4余数为1,2,3,0,1,2,3

2.5 连续数

Seven different numbers are selected from the integers 1to 100, and each numbers is divided by 7.What is the sum of the remainders?

(1)The range of the seven remainder is 6.

(2)The seven numbers selected are consecutive integers.

2.5 连续数

- 连续的两个整数互质

- What is the greatest common factor of the positive integers j and k?
  (1) k = j + 1
  (2) jk is divisible by 5.

2.5 连续数

For every positive even integer n, the function h(n) is defined to be the product of all the even integers from 2 to n,inclusive .If p is the smallest prime factor of h(100) +1,then p is

A. between 2 and 10

B. between 10 and 20

C. between 20 and 30

D. between 30 and 40

E. greater than 40

回顾本节课授课要点

2.1
奇数与偶数

2.2
因数、质数与合数

2.3
质因数

2.4
整除和余数

2.5
连续数

预告下节课授课要点

3.1
代数核心知识及重点题目

THANK YOU