Lecture Four

（代数计算）

本节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

指数运算 (Rules of Exponents)

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^m \times b^m = (a \times b)^m$$

$$a^m \div b^m = (a \div b)^m$$

1. $\dfrac{(8)^2(3)^3(2)^4}{(96)^2} = ?$

(A) 3

(B) 6

(C) 9

(D) 12

(E) 18

2. If $P = 2^{64}$ and $P^P = 2^K$, then k =

(A) 70

(B) 128

(C) 256

(D) 2^{64}

(E) 2^{70}

3. The function f is defined for each positive three-digit integer n by $f(n) = 2^x3^y5^z$, where x, y and z are the hundreds, tens, and units digits of n, respectively. If m and v are three-digit positive integers such that f(m) = 9f(v), then m – v = ?

(A)8
(B) 9
(C)18
(D)20
(E) 80

本节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

解方程 (Equations)

一元二次方程 $a\,x^2 + b\,x + c = 0$

标准根的公式为：$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. A square playground has the same area as a rectangular playground that is 30 meters longer but 20 meters narrower. What is the length, in meters, of a side of the square playground?

(A) $10\sqrt{5}$

(B) $10\sqrt{6}$

(C) 25

(D) 50

(E) 60

2. A certain theater has 100 balcony seats. For every $2 increase in the price of a balcony seat above $10, 5 fewer seats will be sold. If all the balcony seats are sold when the price of each seat is $10, which of the following could be the price of a balcony seat if the revenue from the sale of balcony seats is $1,360 ?

(A) $12
(B) $14
(C) $16
(D) $17
(E) $18

3. If the sum of two positive integers is 24 and the difference of their squares is 48, what is the product of the two integers?

(A) 108
(B) 119
(C) 128
(D) 135
(E) 143

4. If $x - \sqrt{10} = \sqrt{x} + 10$, then $x =$

5. What is Steve's annual salary and Maria's annual salary?

(1) The combined total of the annual salaries of Steve and Maria is $80,000.

(2) If Steve were to receive a 10 percent increase in annual salary and Maria an 10 percent increase, their combined annual salaries would be $88,000.

本节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

对已有的不等式两边取倒数或负数，不等号通常要改变方向

对$\sqrt[4]{x},\sqrt[3]{x},\sqrt{x},x,x^2,x^3,x^4$等函数的性质有一定的认识.

在 x,x^2,x^3 几个函数的比较大小中，对x的取值范围要有清醒的分段意识：

$x<-1$，　$-1<x<0$，　$0<x<1$，　$x>1$

绝对值：|x| 恒非负

1. If $x > 0.9$, which of the following could be the value of x ?

(A) $\sqrt{0.81}$

(B) $\sqrt{0.9}$

(C) (0.9)

(D) $(0.9)\ (0.9)$

(E) $1 - \sqrt{0.01}$

2. If $x \neq 0$, is $|x| < 1$?

(1) $x^2 < 1$

(2) $|x| < \frac{1}{x}$

3. Which of the following inequalities has a solution set that, when graphed in the number line, is a single line segment of finite length?

(A) $x^4 \geq 16$

(B) $x^3 \leq 27$

(C) $x^2 \geq 16$

(D) $2 \leq |x| \leq 5$

(E) $2 \leq 3x + 4 \leq 6$

本节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

题目会定义新的算符和运算法则，模仿运算

1. For all real numbers v, the operation v* is defined by the equation v* = v − v/3. If (v*)* = 8, then v =

(A) 15

(B) 18

(C) 21

(D) 24

(E) 27

2. If the operation $\odot$ is defined for all integers a and b by $a \odot b = a + b - ab$, which of the following statements must be true for all integers a, b, and c ?

I. $a \odot b = b \odot a$

II. $a \odot 0 = a$

III. $(a \odot b) \odot c = a \odot (b \odot c)$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II and III

3. If $\odot$ denotes one of two arithmetic operations, addition or multiplication, and if k is an integer, what is the value of $3 \odot k$?

(1) $2 \odot k = 3$

(2) $1 \odot 0 = k$

本节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

1.等差数列 (Arithmetic Sequence)

2.等比数列 (Geometric Sequence)

1. How many integers between 234 and 567, inclusive, are the multiples of 5 ?

(A) 62

(B) 65

(C) 67

(D) 69

(E) 71

2. If the sum of 7 consecutive integers is 434, then the greatest of the 7 integers is

(A) 65

(B) 66

(C) 67

(D) 68

(E) 69

3. In the sequence 1, 2, 4, 8, 16, 32, ... , each term after the first is twice the previous term. What is the sum of the 16^{th}, 17^{th}, and 18^{th} terms in the sequence?

(A) 2^{18}

(B) $3(2^{17})$

(C) $7(2^{16})$

(D) $3(2^{16})$

(E) $7(2^{15})$

4. In a certain sequence, the first term is 1, and each successive term is 1 more than the reciprocal of the term that immediately precedes it. What is the fifth term of the sequence?

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{8}{5}$ (D) $\frac{5}{3}$ (E) $\frac{9}{2}$

5. The sequence $a_1, a_2, \ldots, a_n, \ldots$ is such that $a_n = a_{n-1} - a_{n-2}$ for all positive integers $n > 2$. If $a_1 = -1$ and $a_2 = 1$, what is the sum of the first 1000 terms in the sequence?

(A) 0
(B) 3
(C) 750
(D) 1000
(E) 3000

回顾本节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

预告下节课授课要点

初等几何

- 三角形与四边形
- 平行线
- 圆
- 立体几何
- 直角坐标系

The End