Lecture Three

(基本数论)

本节课授课要点

基本数论

- 奇偶数
- 因数与质因数
- 最大公约数与最小公倍数
- 余数
- 小数、分数与科学计数法
- 比率与比例

奇数与偶数 (Odd and Even Numbers)

奇数 + 奇数 = 偶数

偶数 + 偶数 = 偶数

奇数 + 偶数 = 奇数

奇数 × 奇数 = 奇数

奇数 × 偶数 = 偶数

偶数 × 偶数 = 偶数

多个整数之和为奇数——其中包含奇数个奇数.

多个整数之和为偶数——其中包含偶数个奇数.

多个整数之积为奇数——全部都是奇数.

多个整数之积为偶数——其中包含至少一个偶数.

1. If x and y are integers and xy^2 is a positive odd integer, which of the following must be true?

I. xy is positive.
II. xy is odd.
III. x + y is even.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

2. Is x an even integer?

(1)x is the square of an integer.

(2)x is the cube of an integer.

3. If a and b are positive integers such that a –b and a /b are both even integers, which of the following must be an odd integer?

(A) a /2

(B) b /2

(C) (a + b) /2

(D) (a + 2) /2

(E) (b + 2) /2

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因数与质因数 (Factors and Prime Factors)

1. If $y=x+x^{(n+1)}+x^{(n+2)}+x^{(n+3)}$, and if $x=-1$, and n is the sum of the first 404 prime numbers, then y =

(A) –2

(B) –1

(C) 0

(D) 1

(E) 2

2. If y is the smallest positive integer such that 3,150 multiplied by y is the square of an integer, then y must be

(A) 2
(B) 5
(C) 6
(D) 7
(E) 14

3. If positive integer x is a multiple of 6 and positive integer y is a multiple of 14, is xy a multiple of 105 ?

(1) x is a multiple of 9.

(2)y is a multiple of 25.

4. How many different prime numbers are factors of the positive integer n ?

(1)Four different prime numbers are factors of 2n.

(2)Four different prime numbers are factors of n^2.

5. n is a factor of the product of all the odd integers from 99 to 199, inclusive. If $n=5^k$, then the greatest possible value of k is

(A) 10

(B) 12

(C) 13

(D) 15

(E) 20

6. How many factors does 360 have?

(A) 24

(B) 36

(C) 48

(D) 120

(E) 360

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最大公约数与最小公倍数

(Greatest Common Divisors and Least Common Multiples)

1. If n is a positive integer and the greatest common divisor of $(n-1)!$, $(n+1)!$, and $(n+3)!$ is 120, then n =

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

两个数的最大公约数与最小公倍数的求解方法：

(1) 将两个数分别各自分解质因数；

(2) 每一个质数，取较小的指数，相乘得到最大公约数；

每一个质数，取较大的指数，相乘得到最小公倍数.

2. If M is the least common multiple of 90, 196, and 300, which of the following is NOT a factor of M ?

(A) 600
(B) 700
(C) 900
(D) 2,100
(E) 4,900

3. The greatest common divisor of a and b is 21, and the least common multiple of a and b is 126, where a and b are positive integers, what is the sum of a and b ?

(A) 105
(B) 147
(C) 150
(D) 105 or 147
(E) 105 or 150

4. Three sorts of juices are served at a party. Every 2 guests share a bottle of apple juice, every 3 guests share a bottle of lemon juice, and every 4 guests share a bottle of orange juice. If 65 bottles of juices are drunk off finally, how many guests are at this party?

(A) 12
(B) 24
(C) 36
(D) 48
(E) 60

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余数 (Remainders)

1. When 20 is divided by the positive integer k, the remainder is k – 2, which of the following is a possible value of k ?

(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

2. What is the sum of the remainders when the first 40 positive integers are divided by 6 ?

(A) 96
(B) 100
(C) 120
(D) 132
(E) 136

3. What is the remainder when the positive integer x is divided by 8 ?

(1) When x is divided by 12, the remainder is 5.

(2) When x is divided by 18, the remainder is 11.

4. If n is a positive integer, what is the remainder when $3^{8n+3} + 2$ is divided by 5?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

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小数、分数与科学计数法

(Decimals, Fractions, and Scientific Notation)

识别各位数字名称”7654.321”，其中：

“7”: thousands

“6”: hundreds

“5”: tens

“4”: units (or ones)

“.” : decimal point

“3”: tenths

“2”: hundredths

“1”: thousandths

1. 3.2□△6

If □ and △ each represent single digits in the decimal above, what digit does □ represent?

(1) When the decimal is rounded to the nearest tenth, 3.2 is the result.

(2)When the decimal is rounded to the nearest hundredth, 3.24 is the result.

2. If x is 0.abc, where a, b, and c are the tenths, hundredths and thousandths digits of x, respectively, is x greater than 2/3 ?

(1) $a+b>14$.

(2) $a+c>15$.

3. Any decimal that has only a finite number of nonzero digits is a terminating decimal. For example, 24, 0.82, and 5.096 are three terminating decimals. If r and s are positive integers and the ratio is expressed as a decimal, is a terminating decimal?

(1) $90 < r < 100$

(2) $s = 4$

4. Which of the following fractions has a decimal equivalent that is a terminating decimal?

(A) $\frac{10}{189}$ (B) $\frac{15}{196}$ (C) $\frac{16}{225}$ (D) $\frac{25}{144}$ (E) $\frac{39}{128}$

5. Of the following which best approximates

$$\frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)}$$

(A) 2.00

(B) 2.40

(C) 2.43

(D) 2.50

(E) 3.43

6. What is the least number of digits (including repetitions) needed to express 10^{100} in decimal notation?

(A) 4

(B) 100

(C) 101

(D) 1,000

(E) 1,001

7. If $10^{50} - 74$ is written as an integer in base decimal notation, what is the sum of the digits in that integer?

(A) 424

(B) 431

(C) 440

(D) 449

(E) 456

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比率与比例 (Ratios and Proportions)

the ratio of A to B 表示为 A : B .

There is twice as much A as B 表示为 A = 2B .

1. A certain fraction is equivalent to 2/5 . If the numerator of the fraction is increased by 4 and the denominator is doubled, the new fraction is equivalent to 1/3. What is the sum of the numerator and denominator of the original fraction?

(A) 21

(B) 26

(C) 28

(D) 35

(E) 49

2. If an automobile average 22.5 miles per gallon of gasoline, approximately how many kilometers per liter of gasoline did the automobile average? (1 mile = 1.6 kilometers and 1 gallon = 3.8 liters, both rounded to the nearest tenth.)

(A) 3.7

(B) 9.5

(C) 31.4

(D) 53.4

(E) 136.8

3. A merchant purchased a jacket for $60 and then determined a selling price that equaled the purchase price of the jacket plus a markup that was 25 percent of the selling price. During a sale, the merchant discounted the selling price by 20 percent and sold the jacket. What was the merchant's gross profit on this sale?

(A) $0

(B) $3

(C) $4

(D) $12

(E) $15

4. In a certain formula, p is directly proportional to s and inversely proportional to r. If p = 1 when r = 0.5 and s = 2, what is the value of p in terms of r and s ?

(A) s/r

(B) r/4s

(C) s/4r

(D) r/s

(E) 4r/s

5. A certain quantity is measured on two different scales, the R-scale and the S-scale, that are related linearly. Measurements on the R-scale of 6 and 24 correspond to measurements on the S-scale of 30 and 60, respectively. What measurement on the R-scale corresponds to a measurement of 100 on the S-scale?

(A) 20

(B) 36

(C) 48

(D) 60

(E) 84

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预告下节课授课要点

代数计算

- 指数运算
- 解方程
- 不等式
- 符号运算
- 数列

The End