Lecture Five

(初等几何)

本节课授课要点

初等几何

- 三角形与四边形
- 平行线
- 圆
- 立体几何
- 直角坐标系

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三角形与四边形 (Triangles and Quadrilaterals)

三角形的某些性质：

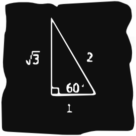
三角形内角和为180°

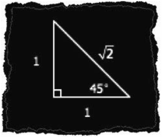
三角形两边之和大于第三边，两边之差小于第三边.

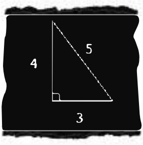
三角形中，较大角的对边也较大.

勾股定理：$(\text{直角边}a)^2 + (\text{直角边}b)^2 = (\text{斜边}c)^2$

要对以下形状的直角三角形要特别熟悉：







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三角形：面积 = 底 × 高

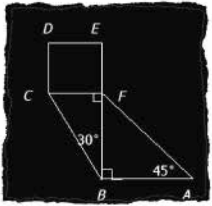
矩形 (Rectangles)：面积 = 长 × 宽

周长 = 2 × (长 + 宽)

正方形 (Squares)：面积 = 边长^2

周长 = 4 × 边长

1. In the figure above, square CDEF has area 4. What is the area of $\triangle ABF$?



(A) $2\sqrt{2}$

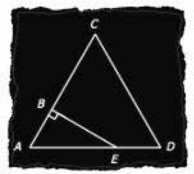
(B) $2\sqrt{3}$

(C) 4

(D) $3\sqrt{3}$

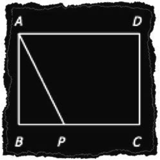
(E) 6

2. If each side of △ACD above has length 3 and if AB has length 1, what is the area of region BCDE ?



(A) $\frac{9}{4}$ (B) $\frac{7}{4}\sqrt{3}$ (C) $\frac{9}{4}\sqrt{3}$ (D) $\frac{7}{2}\sqrt{3}$ (E) $6+\sqrt{3}$

3. In the rectangle above, AB = 6, AD = 8. If point P is selected from segment BC at random, what is the probability that the length of segment AP is less than ?



(A) $\frac{3}{8}$ (B) $\frac{\sqrt{19}}{8}$ (C) $\frac{3}{5}$ (D) $\frac{3}{4}$ (E) $\frac{3\sqrt{5}}{8}$

4. A ladder 25 feet long is leaning against a wall that is perpendicular to level ground. The bottom of the ladder is 7 feet from the base of the wall. If the top of the ladder slips down 4 feet, how many feet will the bottom of the ladder slip?

(A) 4
(B) 5
(C) 8
(D) 9
(E) 15

5. Is quadrilateral Q a square?

(1) The sides of Q have the same length.

(2) The diagonals of Q have the same length.

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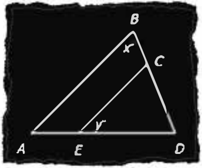
平行线 (Parallel lines)

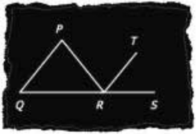
$$\text{两直线平行} \Leftrightarrow \begin{cases} \text{同位角相等} \\ \text{内错角相等} \\ \text{同旁内角互补} \end{cases}$$

1. In the figure above, if AB // CE, CE = DE, and y = 45, then x =

(A) 45
(B) 60
(C) 67.5
(D) 112.5
(E) 135



2. In the figure above, QRS is a straight line and line TR bisects $\angle PRS$. Is it true that lines TR and PQ are parallel?



(1) PQ = PR
(2) QR = PR

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圆 (Circles)

半径为 r 的圆： 面积 = πr^2

周长 = $2\pi r$

角度为 x°的圆弧： 弧长 = $2\pi r \frac{x}{360}$

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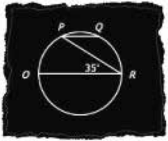
同一段圆弧所对圆心角是圆周角的两倍

1. In the circle above, PQ is parallel to diameter OR, and OR has length 18. What is the length of minor arc PQ ?



(A) 2π (B) $\frac{9\pi}{4}$ (C) $\frac{7\pi}{2}$ (D) $\frac{9\pi}{2}$ (E) 3π

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长方体 (Rectangular Solids)： 体积 = 长 × 宽 × 高

正方体 (Cubes)： 体积 = 边长^3

圆柱 (Cylinders)： 体积 = π × 底面半径^2 × 高

1. What is the volume of a certain rectangular solid?

(1) Two adjacent faces of the solid have areas 15 and 24, respectively.

(2) Each of two opposite faces of the solid has area 40.

2. A grocer is storing small cereal boxes in large cartons that measure 25 inches by 42 inches by 60 inches. If the measurement of each small cereal box is 7 inches by 6 inches by 5 inches, then what is the maximum number of small cereal boxes that can be placed in each large carton?

(A) 25
(B) 210
(C) 252
(D) 300
(E) 420

3. The inside dimensions of a rectangular wooden box are 6 inches 8 inches by 10 inches. A cylindrical canister is to be placed inside the box so that it stands upright when the closed box rests on one of its six faces. Of all such canisters that could be used, what is the radius, in inches, of the one that has maximum volume?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 8

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平面直角坐标几何 (Plane Rectangular Coordinate Geometry)

平面直角坐标上两点间距离为：$\sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2}$

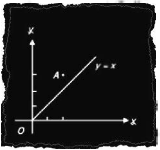
斜截式：$y = kx + b$ 其中，k 为斜率 (Slope)

b为y轴截距 (Intercept)

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

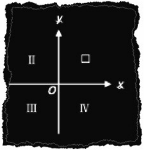
若两直线垂直，其斜率乘积为 -1

1. In the rectangular coordinate system above, the line is perpendicular bisector of segment AB (not shown), and the x-axis is the perpendicular bisector of segment BC (not shown). If the coordinates of point A are (2, 3), what are the coordinates of point C ?



(A) (–3, –2)
(B) (–3, 2)
(C) (2, –3)
(D) (3, –2)
(E) (2, 3)

2. In the rectangular coordinate system shown above, does the line k (not shown) intersect quadrant I?



(1) The x-intercept of k is negative.

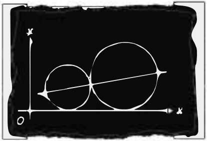
(2) The slope of k is positive.

3. In the xy-plane, does the point (4, 12) lie on line k ?

(1) The point (1, 7) lies on line k.

(2) The point (–2, 2) lies on line k.

4. In the rectangular coordinate system above, both of two tangent circles are tangent to the x-axis. If the radii of the two circles are 4 and 6, respectively, what is the slope of the line on which two centers lie?



(A) $\frac{1}{2\sqrt{6}}$ (B) $\frac{1}{3\sqrt{2}}$ (C) $\frac{1}{3}$ (D) $\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2}$

5. An isosceles triangle lies on the rectangular coordinate plane, the coordinates of point A are (0, 0), and the coordinates of point B are (3, 1), point C could lie at one of 6 positions such that (1, 3), (–1, 3), (–3, 1), (–1, –3), (1, –3), (3, –1). How many lengths of side BC are possible?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

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预告下节课授课要点

文字应用题

- 工作问题
- 利息问题
- 集合问题（文氏题）
- 集合问题（表格法）
- 排列组合问题
- 概率问题
- 描述统计学

The End