

趴趴 GMAT 数学

内部资料



PART 1 统计

知识点总结

1. 算数平均数 (arithmetic mean)

$$E = \frac{1}{n} \sum_{i=1}^n a_i$$

$$\frac{2}{\frac{1}{a} + \frac{1}{b}} \leq \sqrt{ab} \leq \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$$

当 $a, b > 0$ 时, 下式成立, 当 $a=b$ 时取等号。

$$\frac{2}{\frac{1}{a} + \frac{1}{b}} \text{ 调和平均, } \sqrt{ab} \text{ 几何平均, } \frac{a+b}{2} \text{ 算术平均, } \sqrt{\frac{a^2+b^2}{2}} \text{ 加权平均或平方平均。}$$

2. 期望 (expectation)

期望 (expectation) 在 GMAT 数学中等于算数平均数。通常用大写的 E 来表示。

$$E = \frac{1}{n} \sum_{i=1}^n a_i$$

3. 偏差 (deviation)

偏差 d 为数列中的该项(如 a_i 项)与该数列的算数平均数的差值。

$$\text{偏差 } d_i = a_i - E$$

4. 方差 (variance)

$$D = \frac{1}{n} \sum_{i=1}^n (a_i - E)^2$$

5. 标准差 (standard deviation)

$$\sigma = \sqrt{D}$$



6. 中间数 (median)

先排序, 后取中。E.G 一个数列{78, 2, 56, 5, 3}, 求它的中间数时, 应该先排序变成{2, 3, 5, 56, 78}, 然后取中为 5。如果数列含有偶数个数, 取中间两个数, 然后取这两个数的算术平均数。E.G 一个数列{78, 2, 56, 5}, 求它的中间数时, 应该先排序变成{2, 5, 56, 78}, 然后取 5 和 56 的平均数 30.5。

7. 众数 (mode)

数列中出现次数最多的数。比如说一个数列{1, 1, 2, 3}, 它的众数是 1。

8. 范围 (range)

数列中最大数与最小数的差。

练习题

1. Last year the range of the annual salaries of the 100 employees at Company X was \$30,000. If the annual salary of each of the 100 employees this year is 10 percent greater than it was last year, what is the range of the annual salaries of the 100 employees this year?

- (A) \$27,000
- (B) \$30,000
- (C) \$33,000
- (D) \$36,000
- (E) \$63,000

【答案】C

【思路】Last year the range of the annual salaries of the 100 employees at Company X = 30,000

假设最大annual salaries=50,000, 最小annual salaries=20,000

又今年比去年annual salary涨10% 表示the range of the annual salaries of the 100 employees this year=50,000x(1.1)-20,000x(1.1)=33,000

2. On July 1 of last year, the total number of employees at Company E was decreased by 10 percent. Without any change in the salaries of the remaining employees, the average (arithmetic mean) employee salary was 10 percent more after the decrease in number of employees than before the decrease. The total of the combined salaries of all of the employees at Company E after July 1 last year was what percent of that before July 1 last year?

- (A) 90%
- (B) 99%

- (C) 100%
(D) 101%
(E) 110%

【答案】B

【思路】

设原本人数 X ，平均薪资 $Y \Rightarrow$ 总薪资为 XY

之后人数变为 $0.9X$ ，平均薪资变成 $1.1Y \Rightarrow$ 总薪资变成 $0.99XY$ 故为99%

3. A certain builder is selling five homes for the following prices: \$157,000, \$168,000, \$195,000, \$235,000, and \$256,000. If the price of the most expensive home is increased \$10,000, which of the following statements best describes the change in the mean and the median of the home prices?

- (A) The mean and the median will remain unchanged.
(B) The mean will remain unchanged but the median will increase.
(C) The mean will increase but the median will remain unchanged.
(D) The mean and the median will increase by the same amount.
(E) The mean and the median will increase by different amounts.

【答案】C

【思路】

\$157,000, \$168,000, \$195,000, \$235,000, and \$256,000 最高价增加\$10,000

故\$157,000, \$168,000, \$195,000, \$235,000, and \$266,000

中位数不变

平均数改变

4. If a certain sample of data has a mean of 20.0 and a standard deviation of 3.0, which of the following values is more than 2.5 standard deviations from the mean?

- (A) 12.0
(B) 13.5
(C) 17.0
(D) 23.5
(E) 26.5

【答案】A

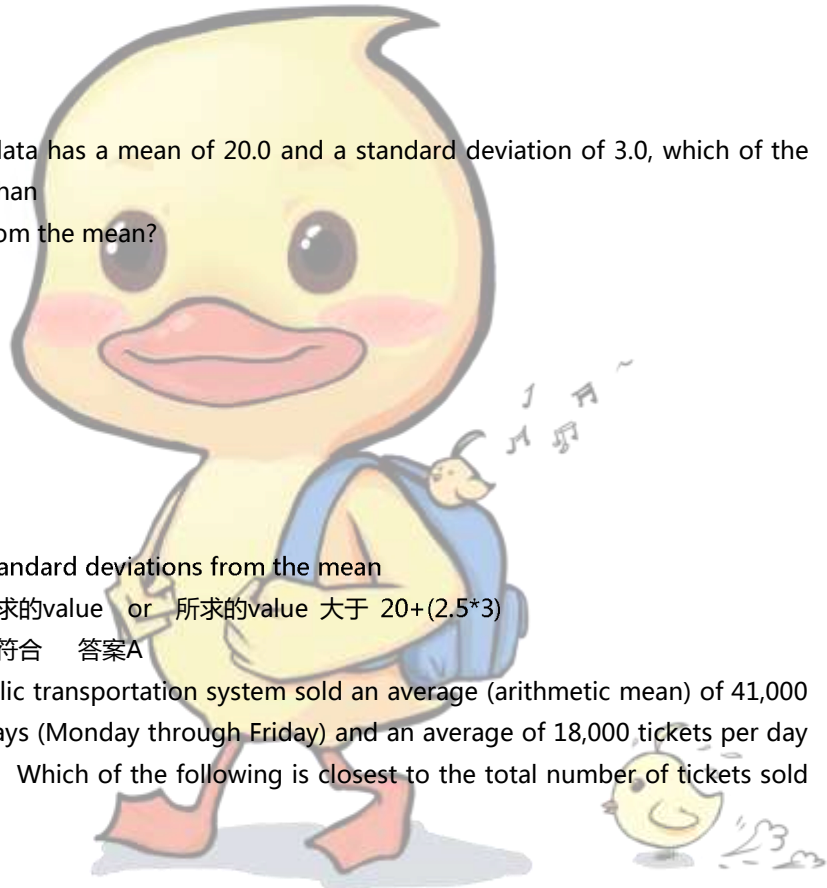
【思路】 more than 2.5 standard deviations from the mean

$20 - (2.5 \times 3)$ 大于所求的value or 所求的value 大于 $20 + (2.5 \times 3)$

A~E答案中只有A符合 答案A

5. Last year, a certain public transportation system sold an average (arithmetic mean) of 41,000 tickets per day on weekdays (Monday through Friday) and an average of 18,000 tickets per day on Saturday and Sunday. Which of the following is closest to the total number of tickets sold last year?

- (A) 1 million



- (B) 1.25 million
(C) 10 million
(D) 12.5 million
(E) 125 million

【答案】 D

【思路】 算术平均数的基本概念

平均在周一至周五票的销售量 41,000 张 → 周一到周五总销售量 $41,000 \times 5 = 820,000$ ---设(A)

平均在周末票的销售量 18,000 张 → 周末总销售量 $18,000 \times 2 = 36,000$ ----设(B)

$$\frac{A+B}{7} = \text{平均每日的销售量} \rightarrow \frac{820,000 + 360,000}{7} = 34,427$$

一年销售量 $\rightarrow 34,427 \times 365(\text{天}) = 12,565,855 \rightarrow$ 答案D (12.5 million)

6. The residents of Town X participated in a survey to determine the number of hours per week each resident spent watching television. The distribution of the results of the survey had a mean of 21 hours and a standard deviation of 6 hours. The number of hours that Pat, a resident of Town X, watched television last week was between 1 and 2 standard deviations below the mean. Which of the following could be the number of hours that Pat watched television last week?

- (A) 30
(B) 20
(C) 18
(D) 12
(E) 6

【答案】 D

【思路】 平均数与标准差的概念

题目的数据: 平均数(mean): 21 hrs 标准差(standard deviation): 6 hrs, 由Pat 上星期看电视的时间在1~2个标准差之下可得知(between 1 and 2 standard deviations below the mean), 时间范围应该坐落在9hrs~15hrs之间 答案D 符合算出来的要求

7. Three boxes of supplies have an average (arithmetic mean) weight of 7 kilograms and a median weight of 9 kilograms. What is the maximum possible weight, in kilograms, of the lightest box?

- (A) 1
(B) 2
(C) 3
(D) 4
(E) 5

【答案】 C

【思路】 中位数要为9 算数平均数为7 总和为 $7 \times 3 = 21$

组合型式为 1 9 11



2 9 10

3 9 9 若4 8 9则9就不是中位数了 就最轻盒子当中的最大=3

8. Is z equal to the median of the three positive integers x , y , and z ?

(1) $x < y + z$

(2) $y = z$

【答案】B

【思路】代数字推法

(1) $3 < 4 + 5$ 4为median,但4不知是y或z,所以无法判断,不充分

(2) 344或 445 不管x大于y,z或小于y,z都可确认z为median ,充分

9.If the average (arithmetic mean) of four different numbers is 30, how many of the numbers are greater than 30?

(1) None of the four numbers is greater than 60.

(2) Two of the four numbers are 9 and 10, respectively.

【答案】C

【思路】 $\frac{a+b+c+d}{4} = 30$, 问几个数超过30?

(1) $\frac{10+20+30+60}{4} = 30$; $\frac{20+25+35+40}{4} = 30$ -----insufficient

(2) $\frac{9+10+30+71}{4} = 30$; $\frac{9+10+35+66}{4} = 30$ ----- insufficient

(1)(2) $\frac{9+10+41+60}{4} = 30$ ----- sufficient

10.If the average (arithmetic mean) of the five numbers x , 7, 2, 16, and 11 is equal to the median of the five numbers, what is the value of x ?

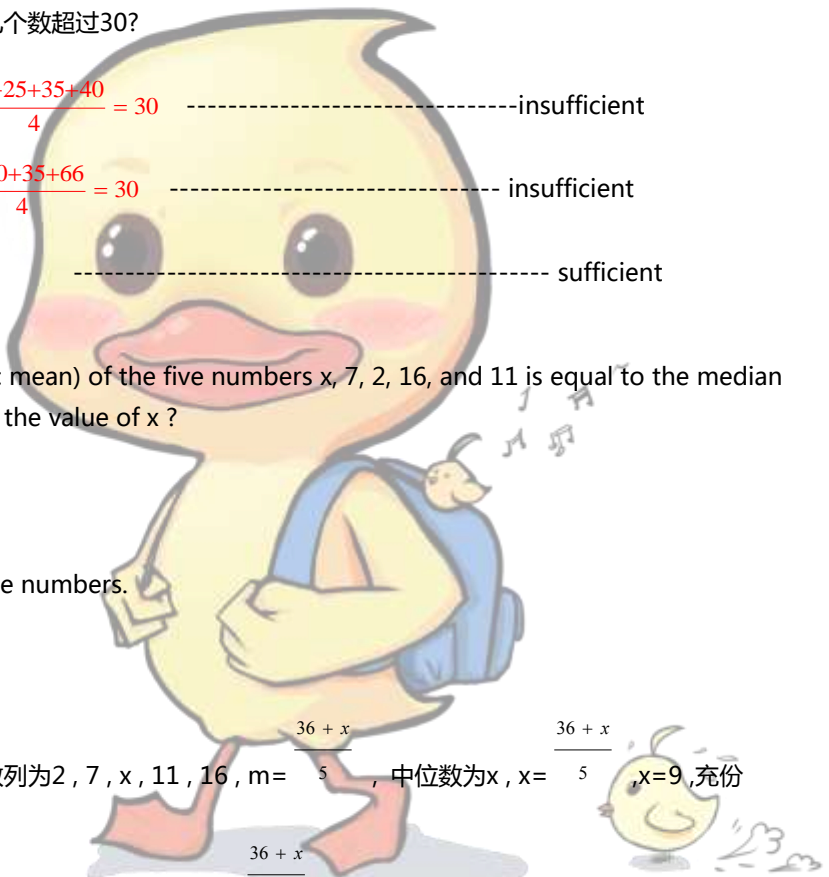
(1) $7 < x < 11$

(2) x is the median of the five numbers.

【答案】D

【思路】(1) $7 < x < 11$, 则数列为2, 7, x , 11, 16, $m = \frac{36+x}{5}$, 中位数为 x , $x = \frac{36+x}{5}$, $x=9$, 充份

(2) x 为中位数, 数列为 2, 7, x , 11, 16, $x = \frac{36+x}{5}$, $x=9$, 充份 (1) (2)皆充份 ,故选D



难题冲刺

1. A set of 15 different integers has a median of 25 and a range of 25. What is the greatest possible integer that could be in this set?

- (A) 32
- (B) 37
- (C) 40
- (D) 43
- (E) 50

【答案】 D

【思路】

定义=最大值-最小值

题目问说哪个最有可能是最大值,所以从最大的先推推看

E) Max: 50, range 25 代表 Min = 50 - 25 = 25 ==> 可是25又是中位数,所以不可能!!

D) Max: 43, range 25 代表 Min = 43 - 25 = 18

列举验证一下: 18 19 20 21 22 23 24 25 43

总共15个,所以中位数左边应该有7个,刚好25也是中位数,所以成立!!

[另解] 因为range 25, 各选项间距为 => (7~32)(12~37)(15~40)(18~43)(25~50)

由于15个不同的数字,此外,中位数为25=>只有D选项符合

2. During an experiment, some water was removed from each of 6 water tanks. If the standard deviation of the volumes of water in the tanks at the beginning of the experiment was 10 gallons, what was the standard deviation of the volumes of water in the tanks at the end of the experiment?

(1) For each tank, 30 percent of the volume of water that was in the tank at the beginning of the experiment was removed during the experiment.

(2) The average (arithmetic mean) volume of water in the tanks at the end of the experiment was 63 gallons.

【答案】 A

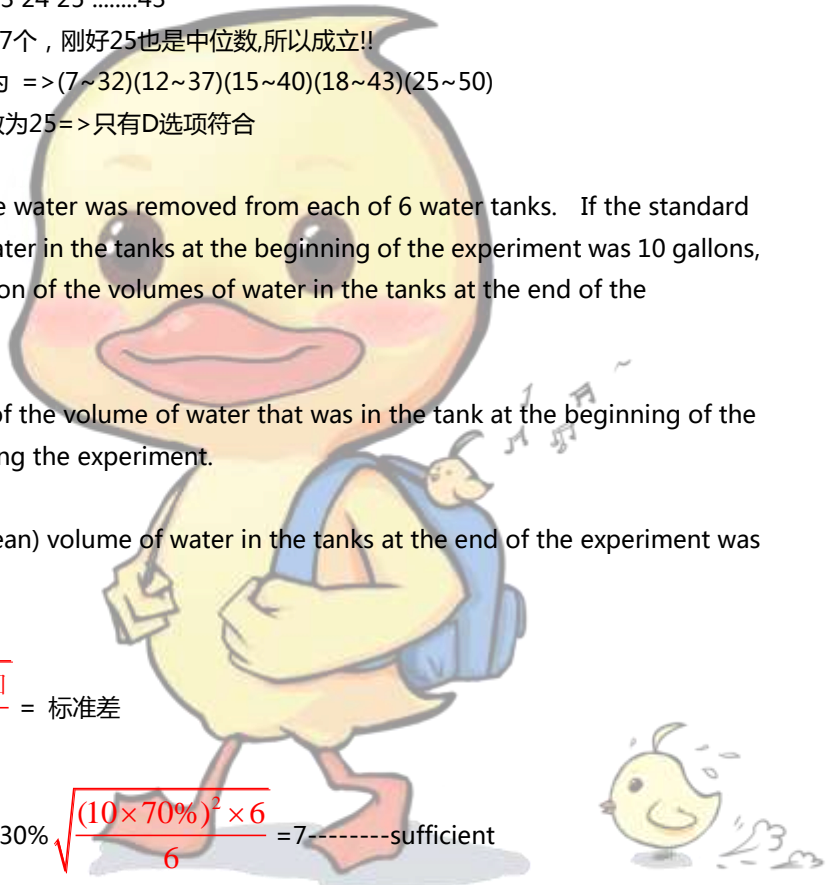
【思路】 $\sqrt{\frac{(\text{各项} - \text{mean})^2 \text{ 和}}{\text{项数}}} = \text{标准差}$

(1) $\sqrt{\frac{(10)^2 \times 6}{6}} = 10$, 每项移走30% $\sqrt{\frac{(10 \times 70\%)^2 \times 6}{6}} = 7$ -----sufficient

(2) 只知 mean, 不知各tank移走多少水 -----insufficient

P.S 项数标准差,若同时增加或减少某数,例如加5,则标准差不变

项数标准差,若同时增加或减少某比例,例如5%,则标准差会等比增加或减少



3. On Jane's credit card account, the average daily balance for a 30-day billing cycle is the average (arithmetic mean) of the daily balances at the end of each of the 30 days. At the beginning of a certain 30-day billing cycle, Jane's credit card account had a balance of \$600. Jane made a payment of \$300 on the account during the billing cycle. If no other amounts were added to or subtracted from the account during the billing cycle, what was the average daily balance on Jane's account for the billing cycle?

- (1) Jane's payment was credited on the 21st day of the billing cycle.
- (2) The average daily balance through the 25th day of the billing cycle was \$540.

【答案】D

【思路】平均值 = $\frac{d1 + \dots + d30}{30}$ (每天的余额平均, 昨天的期末于今天的期初余额), 求平均值。已知: $d1 = 600$, 其中 $dn = -300$ 。且其中无任何金额变动。

(1) 300元是在第21天中付出, 所以可得知1~20天每日余额是600, 21~30每日余额是300即可求总平均值
←选项(1)成立

(2) $\frac{d1 + \dots + d25}{25} = 540$ 且300是在第25天以前付掉(因期初为600), 第26~30天每日余额为540, 即可求总平均值, ←选项(2)成立

4. Each employee on a certain task force is either a manager or a director. What percent of the employees on the task force are directors?

(1) The average (arithmetic mean) salary of the managers on the task force is \$5,000 less than the average salary of all employees on the task force.

(2) The average (arithmetic mean) salary of the directors on the task force is \$15,000 greater than the average salary of all employees on the task force.

【答案】C

【思路】令 MANAGER 人数 = X, Director 人数 = Y M = Manger 平均工资, D = Director 平均工资

(1) $M + 5000 = (MX + DY)/(X + Y)$

(2) $D - 15000 = (MX + DY)/(X + Y)$

(1) + (2) $M + 5000 = (MX + DY)/(X + Y)$ $D - 15000 = (MX + DY)/(X + Y)$ 则先求出

$M + 5000 = D - 15000$ $M = D - 20000$ 代入第一个方程式,

$(D - 20000) + 5000 = [(D - 20000)X + DY]/(X + Y) \Rightarrow D - 15000 = (DX - 20000X + DY)/(X + Y)$

$\Rightarrow DX + DY - 15000Y - 15000X = DX - 20000X + DY \Rightarrow -15000Y - 15000X = -20000X$

$\Rightarrow 3Y + 3X = 4X \Rightarrow 3(X + Y) = 4X$ $X/(X + Y) = 3/4 = 75\% \dots \text{sufficient}$

5. List S and list T each contain 5 positive integers, and for each list the average (arithmetic mean) of the integers in the list is 40. If the integers 30, 40, and 50 are in both lists, is the standard deviation of the integers in list S greater than the standard deviation of the integers in list T?

- (1) The integer 25 is in list S
- (2) The integer 45 is in list T.

【答案】C

【思路】

(1) Since 30, 40 and 50 are already in S, therefore the 5th no. is $5 * 40 - (30 + 40 + 50 + 25) = 55$
Insufficient. This provides complete information for only List S.

(2) Since 30, 40 and 50 are already in T, therefore the 5th no. is $5 * 40 - (30 + 40 + 50 + 45) = 35$
Insufficient. This provides information for only List T.

(1)+(2) $S = \{25, 30, 40, 50, 55\}$ and $T = \{30, 35, 40, 45, 50\}$

No need to solve further.

Now, you can see that the difference between Mean and extreme values in S is greater than the difference between Mean and extreme values in T. Therefore, S.D for S is greater than S.D of T.

PART 2 方程与方程组

知识点总结

1. 一元二次方程

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

十字交叉法

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x_1 = 3, x_2 = -1$$

2. 二元一次方程组

二元一次方程组只需在解题过程中消去一个元素即可

注：并不是所有的二元一次方程组都有唯一解，还可能存在方程组有无穷多解的情况

E.G. $3x + y = 5$

该方程就有无穷多解

$$6x + 2y = 10$$

3. 二元二次方程组

对二元二次方程组的考察一般为一个方程为一次，一个方程为二次。所以只需将一次方程带



入二次方程即可：

$$a_1x + b_1y = c_1$$

$$a_2x^2 + b_2x + a_3y^2 + b_3y = c_2$$

4. 不等式

如果不等式两边同时乘以或者除以一个负数，这时不等式的方向发生变化。

如果不等式两边同时乘以或者除以一个正数，这时不等式的方向不发生变化。

若 $a > 0, a > 0$, 则 $b > 0$

若 $a > b, c > 0$, 则 $ac > bc$

若 $a > b, c < 0$, 则 $ac < bc$ (注意 c 的符号的影响)

若 $|x - a| < b$, 则 $-b < x - a < b$, 反之亦然，即两者等价。

若 $|x - a| > b$, 则 $x - a > b$ 或 $x - a < -b$

练习题

1. In a certain game, the units of currency of three countries are the crown, the shield, and the pound, respectively. If 2 crowns equal 3 shields and 3 shields equal 4 pounds, how many crowns equal 18 pounds?

- (A) 9
- (B) 16
- (C) 18
- (D) 32
- (E) 36

【答案】A

【思路】 $2c = 3s, 3s = 4p$

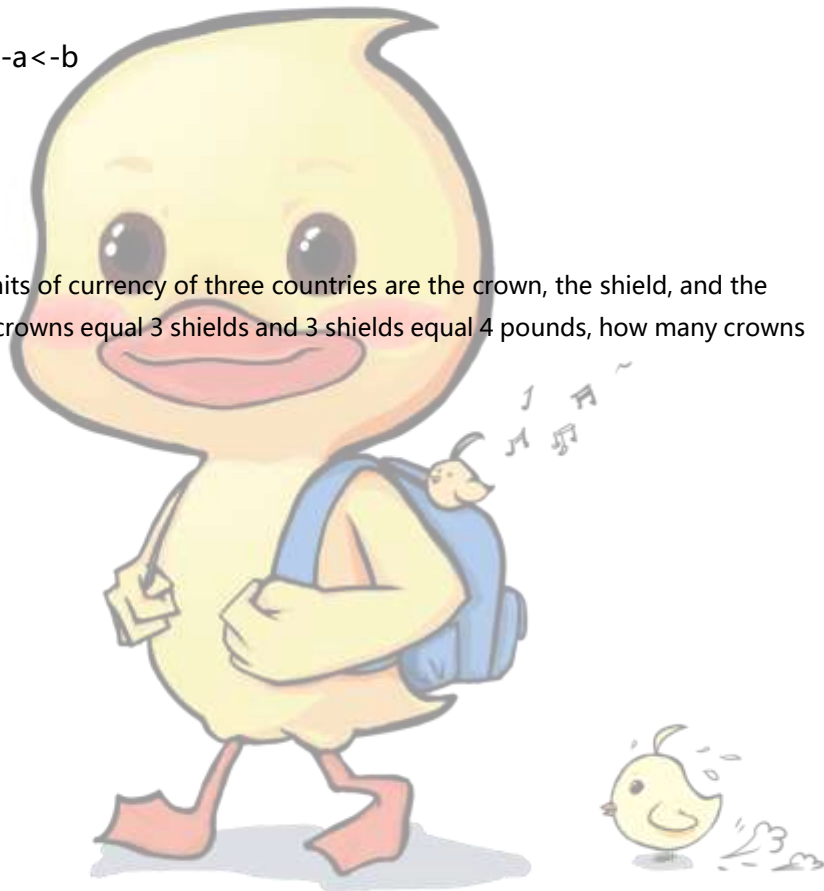
$$4p = 18, p = 4.5$$

$$3s = 18, s = 6$$

$$\text{代入 } 2c = 3s, c = 9$$

2. 38, 69, 22, 73, 31, 47, 13, 82

Which of the following numbers is greater than three-fourths of the numbers but less than one-fourth of the numbers in the list above?



- (A) 56
(B) 68
(C) 69
(D) 71
(E) 73

【答案】D

【思路】將上述的sequence由小到大重新排列 13,22,31,38,47,69,73,82

假设题目要求的number为X,X大于数列的 $\frac{3}{4}$, 小于数列的 $\frac{1}{4}$

数列共有8个数, $\frac{3}{4} \times 8 = 6$, $\frac{1}{4} \times 8 = 2$ so, $69 < X < 73$, 答案为71(D)

3.If $x^4 + y^4 = 100$, then the greatest possible value of x is between

- (A) 0 and 3
(B) 3 and 6
(C) 6 and 9
(D) 9 and 12
(E) 12 and 15

【答案】B

【思路】

$$3^4 = 81$$

$$\text{又 } 3.1^4 < 100$$

所以 the greatest possible value of x is between 3 and 6

4.A certain city with a population of 132,000 is to be divided into 11 voting districts, and no district is to have a population that is more than 10 percent greater than the population of any other district. What is the minimum possible population that the least populated district could have?

- (A) 10,700
(B) 10,800
(C) 10,900
(D) 11,000
(E) 11,100

【答案】D

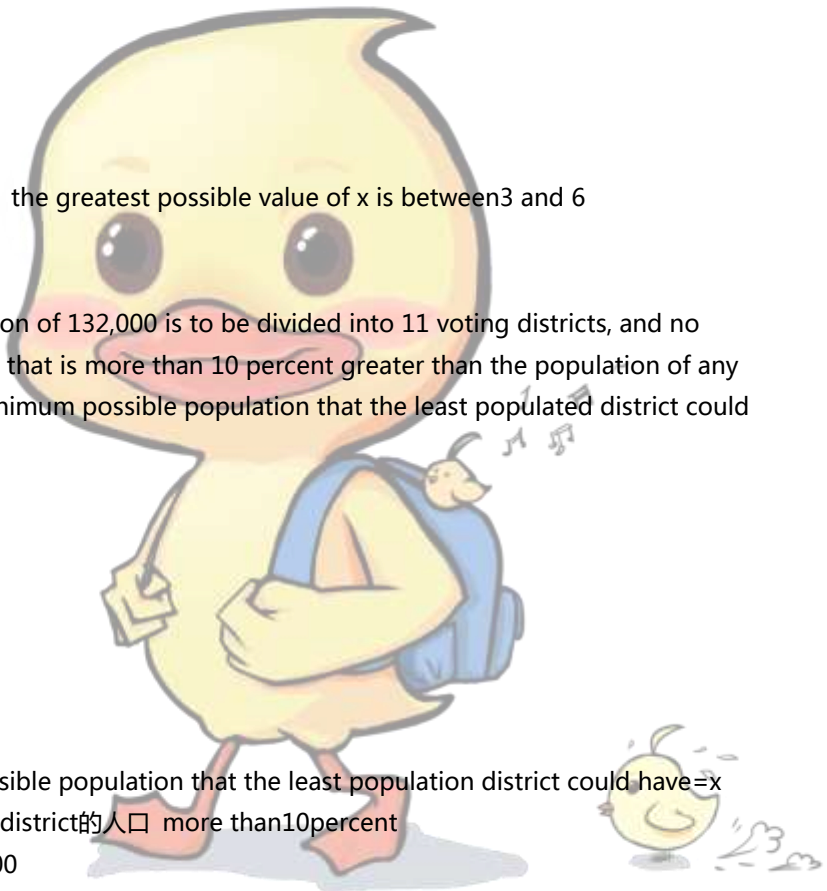
【思路】假设the minimum possible population that the least population district could have= x

且假设其他10个voting district的人口 more than 10 percent

$$x + (x + 0.1x) \times 10 = 132,000$$

$$x = 11,000$$

5. A company has two types of machines, type R and type S. Operating at a constant rate, a



machine of type R does a certain job in 36 hours and a machine of type S does the same job in 18 hours. If the company used the same number of each type of machine to do the job in 2 hours, how many machines of type R were used?

- (A) 3
- (B) 4
- (C) 6
- (D) 9
- (E) 12

【答案】c

【思路】

R 36小时完成 可得一小时完成 $\frac{1}{36}$

S 18小时完成表示一小时完成 $\frac{1}{18}$

设R S 各有X台 又题目写 2小时完成 表示一小时完成 $\frac{1}{2}$

$$X \left(\frac{1}{36} + \frac{1}{18} \right) = \frac{1}{2} \quad X=6$$

6. If x is a negative integer, which of the following expressions has the LEAST value?

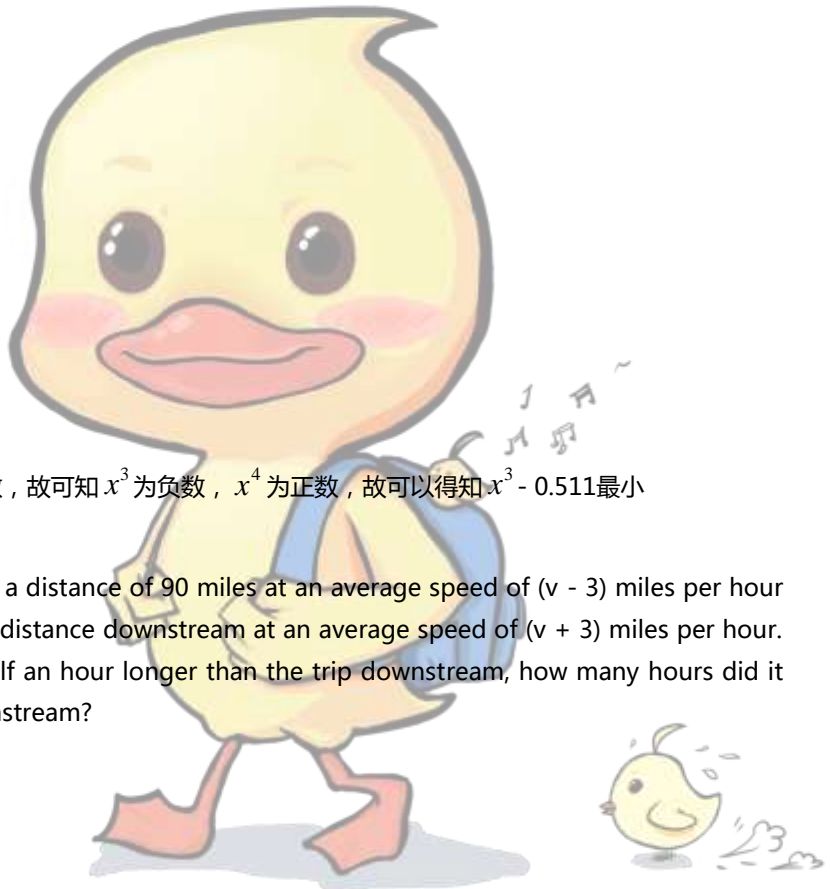
- (A) $x^2 - 0.50$
- (B) $x^2 - 0.51$
- (C) $x^3 - 0.502$
- (D) $x^3 - 0.511$
- (E) $x^4 - 0.512$

【答案】D

【思路】求最小值：X为负整数，故可知 x^3 为负数， x^4 为正数，故可以得知 $x^3 - 0.511$ 最小

7. A boat traveled upstream a distance of 90 miles at an average speed of $(v - 3)$ miles per hour and then traveled the same distance downstream at an average speed of $(v + 3)$ miles per hour. If the trip upstream took half an hour longer than the trip downstream, how many hours did it take the boat to travel downstream?

- (A) 2.5
- (B) 2.4
- (C) 2.3
- (D) 2.2
- (E) 2.1



【答案】 A

【思路】 速率与距离的问题

$$90/(v-3) - 1/2 = 90/(v+3)$$

全部乘上 $(v+3)(v-3)$

$$90(v+3) - 1/2(v+3)(v-3) = 90(v-3)$$

$$90v + 270 = 90v - 270 + 1/2(v^2 - 3^2) \leq \text{把} 90v \text{化简}$$

$$540 = 1/2(v^2 - 3^2)$$

$$1080 = v^2 - 9$$

$$1089 = v^2$$

因式分解, 抓因数 $1089 = 3^2 \times 11^2$; 得33

so travel downstream took the ship $90/33+3$ to finish the trip, finally we can get 2.5 hours

8. For which of the following functions f is $f(x) = f(1-x)$ for all x ?

(A) $f(x) = 1 - x$

(B) $f(x) = 1 - x^2$

(C) $f(x) = x^2 - (1-x)^2$

(D) $f(x) = (x^2)(1-x)^2$

(E) $f(x) = x/(1-x)$

【答案】 D

【思路】 $f(1)=(f(1)-1)=f(0)$ 要找出代1或0出来的数都相等

A $f(1)=1-1=0$, $f(0)=1-0=1$ that is to say, $f(1)$ 不相等 $f(0)$

B $f(1)=1-1^2=0$, but $f(0)=1-0=1$ $f(1)$ 不相等 $f(0)$

C $f(1)=1$, but $f(0)=-1$ $f(1)$ 不相等 $f(0)$

D $f(1)=0$, $f(0)=0$ $f(1)$ 相等 $f(0)$

9. The cost of delivery for an order of desk chairs was \$10.00 for the first chair, and \$1.00 for each additional chair in the order. If an office manager placed an order for n desk chairs, is $n > 24$?

(1) The delivery cost for the order totaled more than \$30.00.

(2) The average (arithmetic mean) delivery cost per chair of the n chairs was \$1.36.

【答案】 B

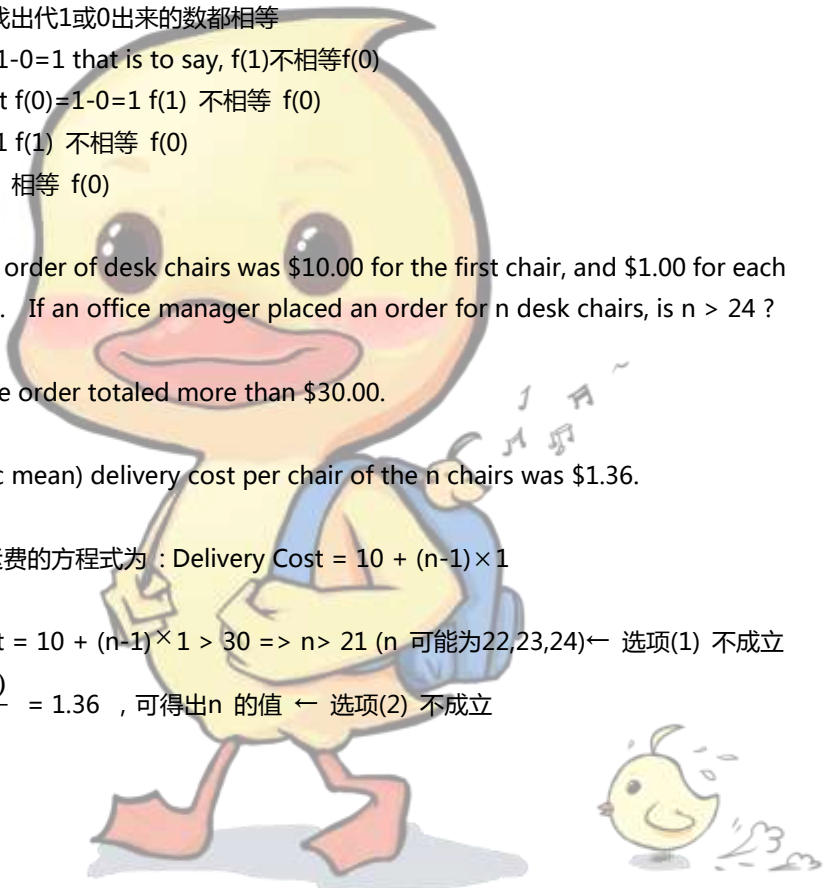
【思路】 根据题目可知计算运费的方程式为: $\text{Delivery Cost} = 10 + (n-1) \times 1$

(1) 套入方程式 $\text{Delivery Cost} = 10 + (n-1) \times 1 > 30 \Rightarrow n > 21$ (n 可能为22,23,24) ← 选项(1) 不成立

(2) 套入方程式, $\frac{10+(n-1)}{n} = 1.36$, 可得出 n 的值 ← 选项(2) 不成立

10. Is $\frac{1}{a-b} < b-a$?

(1) $a < b$.



$$(2) 1 < |a - b|.$$

【答案】A

【思路】在GMAT数学中，所有的未知数都只有唯一解，无法解出唯一解的条件，不算作充分条件。

(1) 可知a-b是负数，b-a是正数，因此 $\frac{1}{a-b} < b - a$ ，条件充分。

(2) 可知 $a-b > 1$ 或 $a-b < -1$ 。

若 $a-b > 1$ ： $b-a < -1$ ，因此 $\frac{1}{a-b} > b - a$ 。

若 $a-b < -1$ ： $b-a > 1$ ，因此 $\frac{1}{a-b} < b - a$ 。

没有唯一解答，因此条件不充分。

难题冲刺

1.If $xy = 1$, what is the value of $2^{(x+y)^2} / 2^{(x-y)^2}$?

- (A) 2
- (B) 4
- (C) 8
- (D) 16
- (E) 32

【答案】D

【思路】根据 $\frac{x^r}{x^s} = x^{r-s}$ ，所以 $\frac{2^{(x+y)^2}}{2^{(x-y)^2}} = \frac{2^{(x^2+2xy+y^2)}}{2^{(x^2-2xy+y^2)}} = 2^{x^2+2xy+y^2-x^2+2xy-y^2} = 2^{4xy}$

$xy=1$,故the value= $2^4 = 16$

2.Are at least 10 percent of the people in Country X who are 65 years old or older employed?

(1) In Country X, 11.3 percent of the population is 65 years old or older.

(2) In Country X, of the population 65 years old or older, 20 percent of the men and 10 percent of the women are employed.

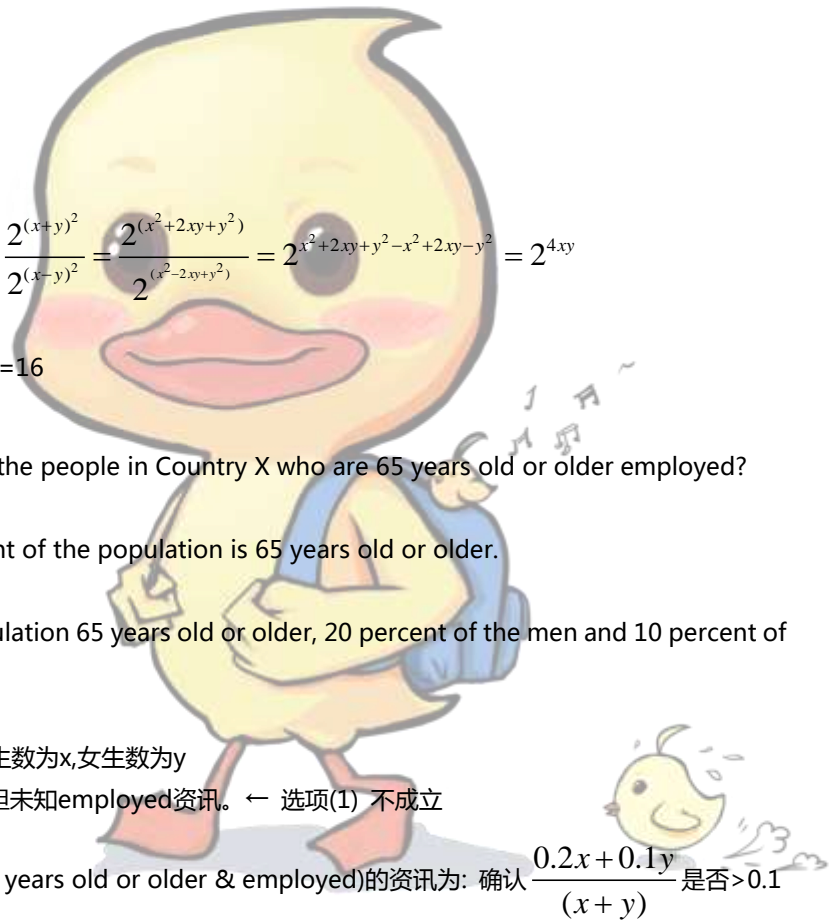
【答案】B

【思路】假设65岁(含)以上男生数为x,女生数为y

(1) 由此可得知 $x+y=11.3$ ，但未知employed资讯。← 选项(1) 不成立

(2) 由此可得知题目所确认(65 years old or older & employed)的资讯为: 确认 $\frac{0.2x+0.1y}{(x+y)}$ 是否 > 0.1

$$\frac{0.2x+0.1y}{(x+y)} = \frac{0.2x+0.2y-0.1y}{(x+y)} = 0.2-0.1\frac{y}{(x+y)} > 0.1?$$



$\Rightarrow 2 > 1 + \frac{y}{(x+y)}$ 由此式可得知 2 一定大于 $1 + \frac{y}{(x+y)}$ 因 $\frac{y}{(x+y)}$ 一定小于 1 (在 x, y 皆为正整数的情形下) 所以由此式可推得符合条件 1.65 years old or older & 2. Employed 的比率一定大于 10% ← 选项(2) 成立

3. Pat bought 5 pounds of apples. How many pounds of pears could Pat have bought for the same amount of money?

(1) One pound of pears costs \$0.50 more than one pound of apples.

(2) One pound of pears costs $1\frac{1}{2}$ times as much as one pound of apples.

【答案】 B

【思路】 P =Pear price, Q_p =Pears 重量

A =Apple price, Q_A =Apples 重量=5 pounds

Price \times Q = TP

TP/price=Q

$Q_p \times P = Q_A \times A = TP$

(1) $P = A(1) + 0.5$

$Q_p \times P = Q_A \times A$

$Q_p \times (A + 0.5) = 5 \times A$

$$Q_p = \frac{5 \times A}{A + 0.5}$$

上式无法将未知数 A 消除, 无法求得 Q_p , 故 insufficient

(2) $P = 1.5A$

$Q_p \times P = Q_A \times A$

$Q_p \times 1.5A = 5 \times A$

$$Q_p = \frac{5 \times A}{1.5A} = \frac{5}{1.5} \text{ sufficient}$$

4. In the sequence of positive numbers x_1, x_2, x_3, \dots , what is the value of x_1 ?

(1) $x_i = \frac{x_{i-1}}{2}$ for all integers $i > 1$

(2) $x_5 = \frac{x_4}{x_4+1}$

【答案】 C

【思路】

(1) $x_i = \frac{x_{i-1}}{2}$ insufficient

(2) $x_5 = \frac{x_4}{x_4+1}$ we can just find that relation between x_5 and x_4 , insufficient

(1)+(2) 聯立即可求得答案



$$x_4/2 = x_4/(x_4+1)$$

$$x_4^2 - x_4 = 0$$

$$x_4(x_4-1)=0$$

$x_4 = 0, 1$ and x_4 is positive $i > 1$, then $x_4 = 1$

substituting values > 1 in $x_i = \frac{x_{i-1}}{2}$

$$x_5 = x_4/2 = 1/2$$

$$x_4 = x_3/2 \rightarrow x_3 = x_4 \times 2 = 2$$

$$x_3 = x_2/2 \rightarrow x_2 = x_3 \times 2 = 4$$

$$x_2 = x_1/2 \rightarrow x_1 = x_2 \times 2 = 8$$

$$x_1 = 2(2(2x_4)) = 8 \text{ Answer: C}$$

5. Is $\frac{1}{p} > \frac{r}{r^2+2}$

(1) $p = r$

(2) $r > 0$

【答案】C

【思路】将题目变成 ?

(1) $p=r$ 分子分母都无法知道正负数,不充分

(2) 不知道 P 也无法知道是否大于 0,不充分

(1)+(2) 结合(1)(2)改写式子为

$$\frac{r^2+2-r^2}{r(r^2+2)} > 0, \text{分子只剩2(正号),分母也都是正号,所以相除确定大于零,充分!}$$

PART 3 数列与集合

知识点总结

1. 等差数列

$$a_n = a_1 + (n-1)d$$

$$s_n = (a_1 + a_n)n/2$$

$$n = (a_n - a_1)/d + 1$$

2. 等比数列

$$a_n = a_1 q^{n-1}$$



$$s_n = a_1 \cdot \frac{1-q^n}{1-q}$$

当 $|q| < 1$ 时,

$$s_\infty = \frac{a_1}{1-q}$$

3. 集合

无重复元素的序列 (或数列) 就是集合。

$$A \cap A = A$$

$$A \cap B = B \cap A$$

$$A \cap B \cap C = A \cap (B \cap C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

练习题

1. If each term in the sum $a_1 + a_2 + \dots + a_n$ is either 7 or 77 and the sum equals 350, which of the following could be equal to n ?

- (A) 38
- (B) 39
- (C) 40
- (D) 41
- (E) 42

【答案】C

【思路】

$a_1 + a_2 + \dots + a_n$ 有 X 个 7 有 Y 个 77

$$7X + 77Y = 350$$

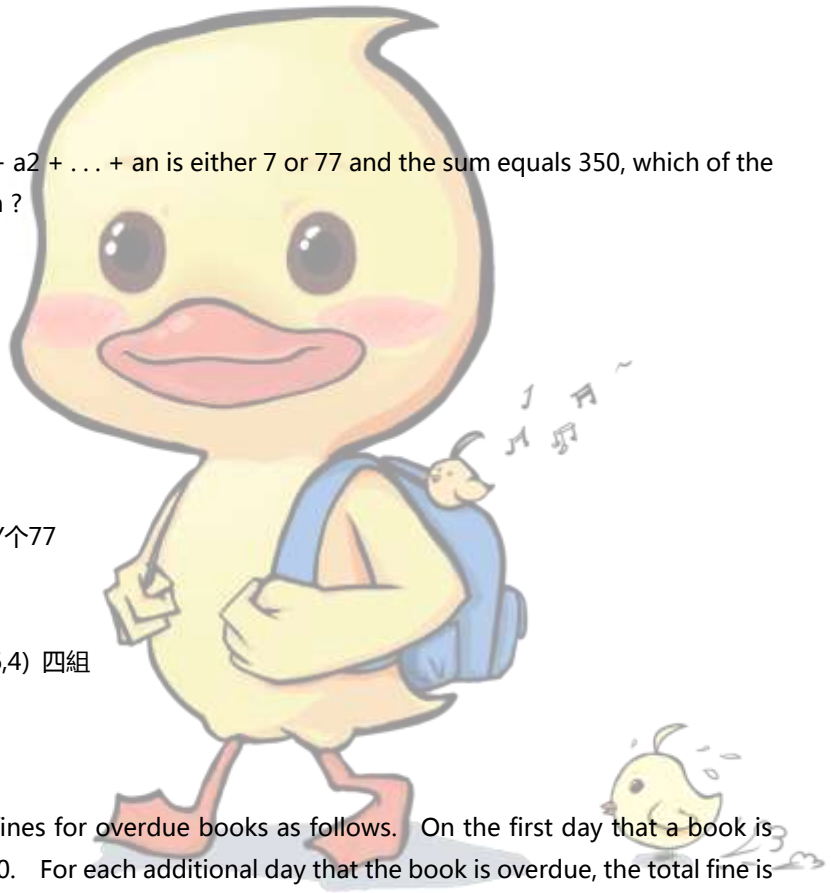
$$X + 11Y = 50$$

(X, Y) 有 (39, 1) (28, 2) (17, 3) (6, 4) 四组

$X + Y = N$ 故可得 40 30 20 10

答案为 C

2. A certain library assesses fines for overdue books as follows. On the first day that a book is overdue, the total fine is \$0.10. For each additional day that the book is overdue, the total fine is either increased by \$0.30 or doubled, whichever results in the lesser amount. What is the total fine for a book on the fourth day it is overdue?



- (A) \$0.60
 (B) \$0.70
 (C) \$0.80
 (D) \$0.90
 (E) \$1.00

【答案】B

【思路】这题非常的tricky，注意关键字whichever results in the lesser amount，所以两种方法都可以使用，只要是最便宜的即可。

1st : \$0.1

2nd: $\min\{0.1+0.3, 0.1*2\} = 0.2$

3rd: $\min\{0.2+0.3, 0.2*2\} = 0.4$

4th: $\min\{0.4+0.3, 0.4*2\} = 0.7$

所以答案是(B)

3. Of the students who eat in a certain cafeteria, each student either likes or dislikes lima beans and each student either likes or dislikes brussels sprouts. Of these students, $\frac{2}{3}$ dislike lima beans; and of those who dislike lima beans, $\frac{3}{5}$ also dislike brussels sprouts. How many of the students like brussels sprouts but dislike lima beans?

(1) 120 students eat in the cafeteria.

(2) 40 of the students like lima beans.

【答案】D

【思路】

	beans	~ beans	
Sprouts	a	b	
~ sprouts	c	d	
		$\frac{2}{3}t$	

注: ~代表dislike

设t为total students

设like beans & like sprouts人数 为a

设dislike beans & like sprouts人数为b

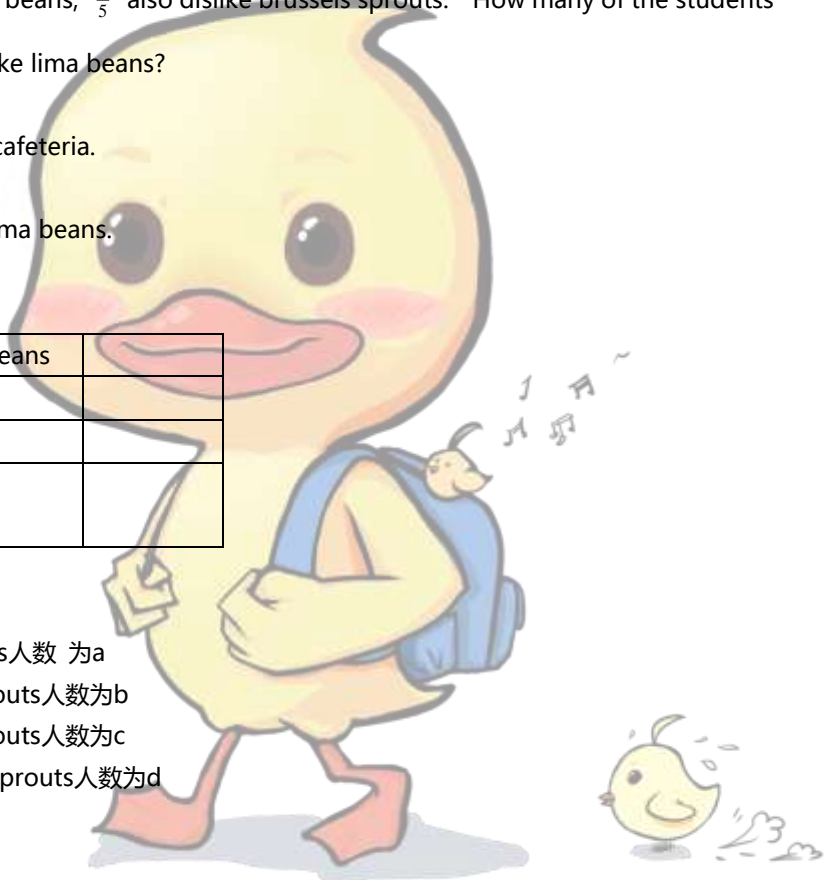
设like beans & dislike sprouts人数为c

设dislike beans & dislike sprouts人数为d

已知 $d = \frac{2}{3}t \times \frac{3}{5} = \frac{2}{5}t$

题目问: b=?

(1) $t = 120$, 得知 $b+d=80$ 且 $d=50$, 所以得出 $b=30$, 充分



(2) $a+c = 40 = \frac{1}{3}t$ 得出 $t=120$ 同(1)也可以算出 b , 充分

4. Of the 800 students at a certain college, 250 students live on campus and are more than 20 years old. How many of the

800 students live on campus and are 20 years old or less?

(1) 640 students at the college are more than 20 years old.

(2) 60 students at the college are 20 years old or less and live off campus.

【答案】C

【思路】

	campus	~ campus	
>20	250	B	
≤20	C	D	
			Total:800

设live off campus & old>20人数为b

设live campus & old ≤20人数为c

设live off campus & old ≤20人数为d

题目问 $c=?$

(1) $250 + b = 640$ 可知 b ,但无法知道 c ,不充分

(2) $d=60$ c 无从得知,不充分

(1)+(2)

由(1)得知 $250 + b = 640$,所以可以算出 $c+d = 160$

由(2)得知 $d = 60$ 代回 $c+d = 160$,所以 C 可以算出答案,充分!

5. Six countries in a certain region sent a total of 75 representatives to an international congress, and no two countries sent the same number of representatives. Of the six countries, if Country A sent the second greatest number of representatives, did Country A send at least 10 representatives?

(1) One of the six countries sent 41 representatives to the congress.

(2) Country A sent fewer than 12 representatives to the congress.

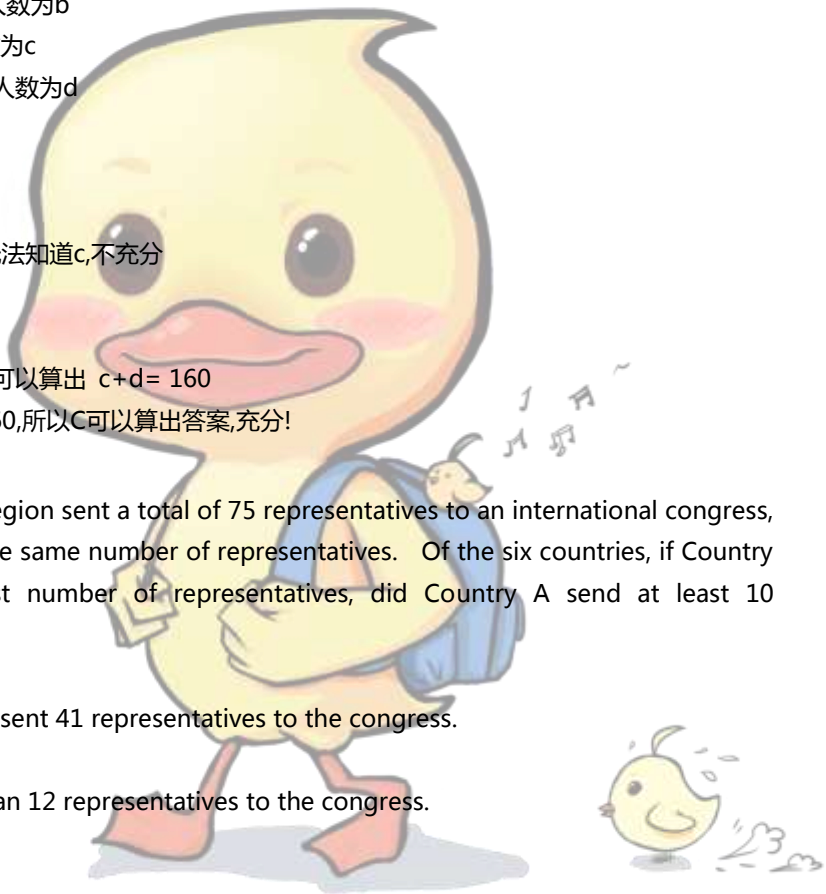
【答案】E

【思路】

(1)当有一个地区是41, 表示剩下的5个区域 $=75-41=34$

故41的那个区域就会是greatest

当 $A=9$ 时,这34可以分配给剩下的区域 EX: (9,8,7,6,4) 符合题目要求 但 A 小于10



当A=11时则可分配成(11,8,7,6,2) 符合题目要求 但A至少等于10

所以(1)不充分

(2)可由上面的举例发现A小于12不成立 因为等于9和等于11都可以 所以(2)对

(1)+(2) 没有资料可以预示其余国家的人数比例,所以资料不充分

6.In a certain conference room each row of chairs has the same number of chairs, and the number of rows is 1 less than the number of chairs in a row. How many chairs are in a row?

(1) There is a total of 72 chairs.

(2) After 1 chair is removed from the last row, there is a total of 17 chairs in the last 2 rows.

【答案】D

【思路】在说,有一个房间里面放了椅子, 每一排有K张椅子, 然后有(k-1)排, 问你说, 总共有多少椅子呢? 也就是要求k(k-1)

(1) 总共有72张椅子。可列示为 $k(k-1)=72$ 只有唯一正数解。

(2) 当一张椅子移开以后, 最后两排的椅子数变成 17, 所以也可以知道, 原来的时候, 一排有 $(17+1)/2=9$ 张椅子。充分

7.If the terms of a sequence are $t_1, t_2, t_3, \dots, t_n$, what is the value of n ?

(1) The sum of the n terms is 3,124.

(2) The average (arithmetic mean) of the n terms is 4.

【答案】C

【思路】

(1)由此可得知 $t_1 + \dots + t_n = 3124$, 但无其他资讯求得 n ←选项(1)不成立

(2)由此可得知 $\frac{t_1 + \dots + t_n}{n} = 4$, 但无其他资讯求得 n 。←选项(2)不成立

(1) + (2) 可联立解两方程式, 因此可求出 n ←选项(1)+(2)成立

8. The sum of the first 50 positive even integers is 2,550. What is the sum of the even integers from 102 to 200, inclusive?

(A) 5,100

(B) 7,550

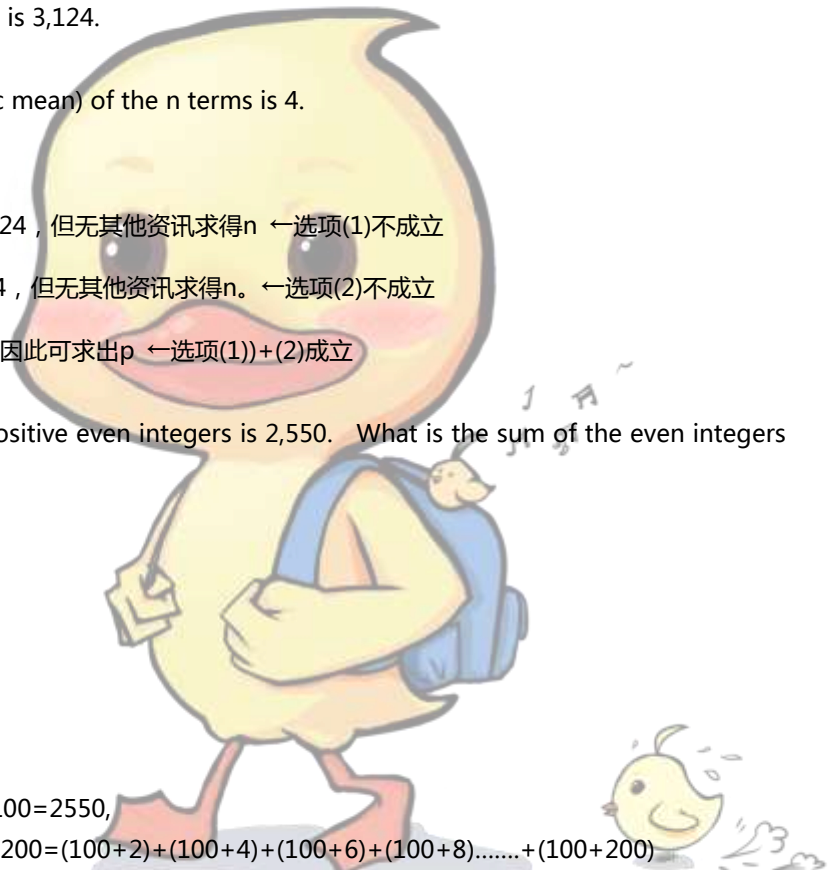
(C) 10,100

(D) 15,500

(E) 20,100

【答案】B

【思路】 $2+4+6+8+10+\dots+100=2550$,
 $102+104+106+108+110+\dots+200=(100+2)+(100+4)+(100+6)+(100+8)+\dots+(100+200)$
 $=50 \times 100 + (2+4+6+8+\dots+100)$
 $=5000+2550=7550$



$$9. 2 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 =$$

- (A) 2^9
 (B) 2^{10}
 (C) 2^{16}
 (D) 2^{35}
 (E) 2^{37}

【答案】A

【思路】

a_0, a_1, \dots, a_n

where $a_0 \times r = a_1, a_1 \times r = a_2, \dots$

then

$$S_n = \frac{a_0(1-r^{n+1})}{(1-r)} \quad \text{if } r < 1$$

or

$$S_n = \frac{a_0(r^{n+1}-1)}{(r-1)} \quad \text{if } r > 1$$

所以：

$$\frac{a(r^n - 1)}{(r - 1)}$$

a 首項 r 公比

$$2 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$$

$$= 2 + [2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8]$$

=

$$= 2 + \frac{2(2^8 - 1)}{(2 - 1)}$$

$$= 2 + 2(2^8 - 1)$$

$$= 2 + 2^9 - 2$$

$$= 2^9$$



10. Five pieces of wood have an average (arithmetic mean) length of 124 centimeters and a median length of 140 centimeters. What is the maximum possible length, in centimeters, of the shortest piece of wood?

- (A) 90
(B) 100
(C) 110
(D) 130
(E) 140

【答案】B

【思路】假设五根木头由低到高依序为A,B,C,D,E

$$A+B+C+D+E=124*5=620$$

$$C=140 \text{ 所以 } A+B+D+E=620-140=480$$

$$\text{又C是中数, 所以 } D+E \geq 140*2=280$$

$$\text{故 } A+B \leq 480-280=200$$

所以A最大的可能为A=B=100

难题冲刺

1. $a_1, a_2, a_3, \dots, a_{15}$ In the sequence shown, $a_n = a_{n-1} + k$, where $2 \leq n \leq 15$ and k is a nonzero constant. How many of the terms in the sequence are greater than 10?

$$(1) a_1 = 24$$

$$(2) a_8 = 10$$

【答案】B

【思路】In the sequence shown, $a_n = a_{n-1} + k$, where $2 \leq n \leq 15$ and k is a nonzero constant. How many of the terms in the sequence are greater than 10?

$$(1) a_1 = 24$$

$$a_2 = a_1 + k \Rightarrow a_2 = 24 + k$$

$$a_3 = a_2 + k \Rightarrow a_3 = 24 + 2k$$

$$a_4 = a_3 + k \Rightarrow a_4 = 24 + 3k$$

不知到k到底是positive or negative 也不知到 k的大小, 通通会影响>10的个数 insufficient

比方说若 $k=-15$, 那么全都<10

$$(2) a_8 = 10$$

$$a_7 = a_6 + k \Rightarrow a_6 = 10 - 2k$$

$$a_8 = a_7 + k \Rightarrow a_7 = 10 - k$$

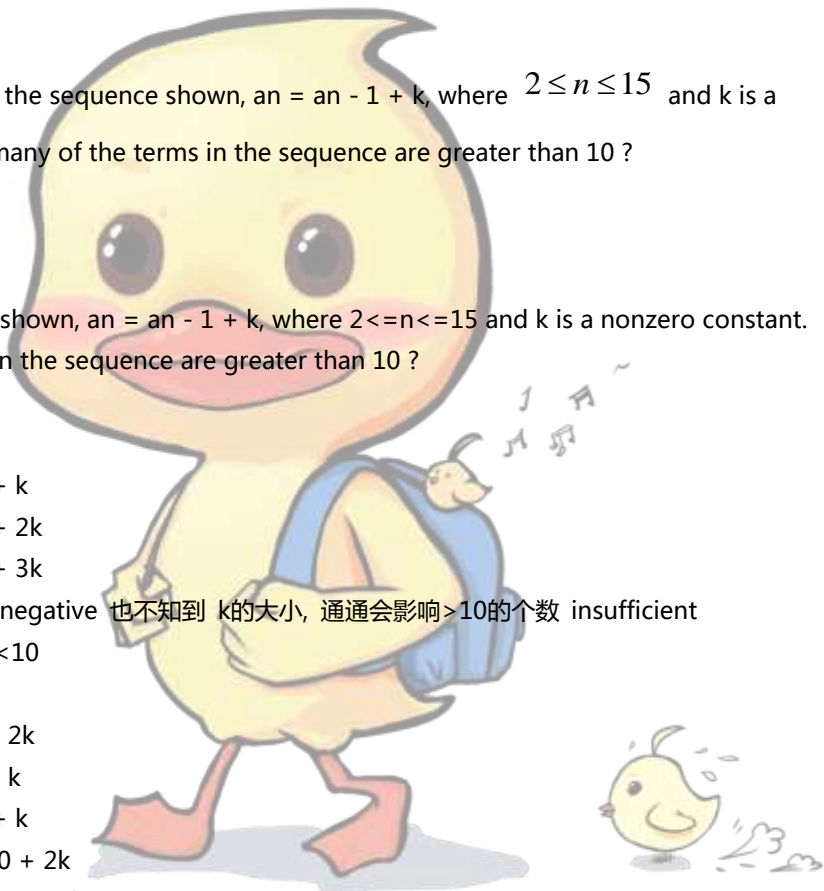
$$a_9 = a_8 + k \Rightarrow a_9 = 10 + k$$

$$a_{10} = a_9 + k \Rightarrow a_{10} = 10 + 2k$$

$$a_{11} = a_{10} + k \Rightarrow a_{11} = 10 + 3k$$

由于已知的数据是10, k不管大小, 都会影响10变成大于10或小于10, 可以推论

由此可见假若 $k>0$, 那么从 $a_9 \sim a_{15}$ 有七个大于10; 反之, 假若 $k<0$, $a_8 \sim a_2$ 有七个大于10;充分



2. Set S consists of five consecutive integers, and set T consists of seven consecutive integers. Is the median of the numbers in set S equal to the median of the numbers in set T?

(1) The median of the numbers in set S is 0.

(2) The sum of the numbers in set S is equal to the sum of the numbers in set T.

【答案】C

【思路】

设 S 里面有五个连续整数, $n-2, n-1, n, n+1, n+2$

设 T 里面有七个连续整数, $x-3, x-2, x-1, x, x+1, x+2, x+3$

中位数刚好就是 n 和 x

(1) 只知 S 不知 T, 无法确定中位数是否一样, 不充分

(2) S 的总和为 $5n$; T 的总和为 $7x$, $5n=7x$ $(n,x)=(0,0),(7,5),(14,10)\dots$ 太多, 不充分

(1)+(2) S 的中位数是 0, 则根据第二个条件 $(n,x)=(0,0)$, 确定中位数一样, 充分

3. If $\frac{2}{5}$ of the students at College C are business majors, what is the number of female students at College C?

(1) $\frac{2}{5}$ of the male students at College C are business majors.

(2) 200 of the female students at College C are business majors.

【答案】C

【思路】

(1) 只知道男学生 business major at College C 的比例, 没有 males 和 females 之间的关系, 无法得知 the number of female students at College C, insufficient

(2) of the students at College C are business majors 包含 females and males students, (2) 仅提供女学生内有 200 个人 business major, 无法知道 male 和 female 的比例关系, insufficient

(1)+(2) of the students at College C are business majors = (males + females)

(males + females) = x male + 200

x females = 200, female students at College C = 500

4. The sequence $a_1, a_2, a_3, \dots, a_n$ of n integers is such that $a_k = k$ if k is odd and $a_k = -a_{k-1}$ if k is even. Is the sum of the terms in the sequence positive?

(1) n is odd.

(2) a_n is positive.

【答案】D

【思路】当 k 奇数时, $a_k = k$ 当 k 偶数时, $a_k = -a_{k-1}$

If $k = \text{odd}$	$a_k = k$	$a_1 = 1$	$a_3 = 3$	$a_5 = 5$	$a_7 = 7$
If $k = \text{even}$	$a_k = -a_{k-1}$	$a_2 = -1$	$a_4 = -3$	$a_6 = -5$	$a_8 = -7$

(1) If n is odd:

$$\text{sum}(a_k) = (a_1 - a_1) + (a_3 - a_3) + \dots + (a_{n-2} - a_{n-2}) + a_n = a_n$$

见上表 $a_{\text{odd}} = \text{even}$, 数列总和 = 第 n 个数 (因为前面两两一组总和为 0), 所以总和一定为正, sufficient

(2) If $a(n)$ is positive, then n has to be odd, 第 n 个数为正, 也就是 n is odd, 结果跟(1)一样, 总和一定为正, sufficient

5. Of the 60 animals on a certain farm, $\frac{2}{3}$ are either pigs or cows. How many of the animals are cows?

(1) The farm has more than twice as many cows as it has pigs.

(2) The farm has more than 12 pigs.

【答案】C

【思路】由系统资讯可得知一方程式: $x+y=40$ (x 为 pig, y 为 cows)

(1) 由此可得知 $y > 2x$, 加上原本资讯 $x+y=40$ 可得知 $(x,y)=(11,29)$ 或 $(12,28)$ 或 $(13,27) \Rightarrow$ 无法得知为哪一组合 \leftarrow 选项(1)不成立

(2) 由此可得知 $x > 12$, 加上原本资讯 $x+y=40$ 可得知 $(x,y)=(13,27)\dots(39,1) \Rightarrow$ 无法得知为哪一组合 \leftarrow 选项(2)不成立

(1) + (2) 由此可得知 $y > 2x, x > 12$, 且 $x+y=40 \Rightarrow (x,y)$ 的组合中只有 $(13,27)$ 符合所有需求 \leftarrow 选项(1)+(2)成立

PART 4 排列组合与概率

知识点总结

1. 排列与组合

$$P_m^n = m! / (m-n)!$$

从 m 个人中挑出 n 个人进行排列的可能数

$$C_m^n = m! / n!(m-n)!$$

从 m 个人中挑出 n 个人进行组合的可能数

$$C_m^n = C_m^{m-n}$$

(1) 加法原理

某件事由两种方法来完成, 第一种方法可由 m 种方法完成, 第二种方法可由 n 种方

法完成, 则这件事可由 $m+n$ 种方法来完成。

(2) 乘法原理

某件事由两个步骤来完成, 第一个步骤可由 m 种方法完成, 第二个步骤可由 n 种方



法完成，则这件事可由 $m \times n$ 种方法来完成。

2. 概率

第一步： 概率基本原理（古典定义） $P(A) = \frac{A \text{ 所包含的基本事件数}}{\text{基本事件总数}}$ 。

第二步： 使用加法或乘法原则

3. 条件概率

条件概率公式：

$$P(A|B) = \frac{P(AB)}{P(B)}$$

公式中 $P(AB)$ 为事件 AB 的联合概率， $P(A|B)$ 为条件概率，表示在 B 条件下 A 的概率， $P(B)$ 为事件 B 的概率。

练习题

1. A flower arrangement consists of 30 roses, each of which is either white or red. If a rose is to be selected at random from the flower arrangement, the probability that the rose selected will be white is twice the probability that it will be red. How many white roses are in the flower arrangement?

- (A) 5
- (B) 10
- (C) 15
- (D) 20
- (E) 25

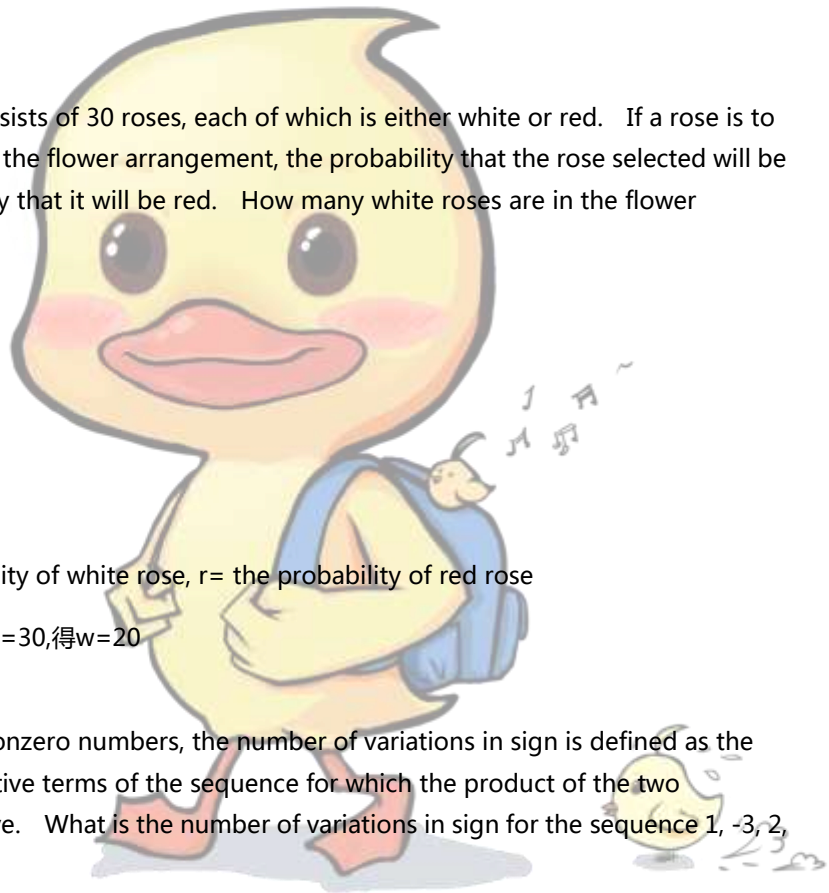
【答案】 D

【思路】 假设 w = the probability of white rose, r = the probability of red rose

$$\frac{w}{30} = 2 \times \frac{r}{30} \text{ 又 } w+r=30, \text{ 得 } w=20$$

2. For a finite sequence of nonzero numbers, the number of variations in sign is defined as the number of pairs of consecutive terms of the sequence for which the product of the two consecutive terms is negative. What is the number of variations in sign for the sequence 1, -3, 2, 5, -4, -6 ?

- (A) One
- (B) Two



(C) Three

(D) Four

(E) Five

【答案】C

【思路】题目What is the number of variations in sign for the sequence,所以题目已经给sequence,不用再排序,variation:当下一个数字正负号与自己不同时,则算一次 故(1,-3)(-3,2)(5,-4),共三次

3. There are 10 children in a company's day-care center, and a pair of children is to be selected to play a game. At most, how many different pairs are possible?

(A) 100

(B) 90

(C) 50

(D) 45

(E) 25

【答案】D

【思路】

$$C(10,2) = \frac{10!}{8!2!} = 45$$

4. In a certain deck of cards, each card has a positive integer written on it. In a multiplication game, a child draws a card and multiplies the integer on the card by the next larger integer. If each possible product is between 15 and 200, then the least and greatest integers on the cards could be

(A) 3 and 15

(B) 3 and 20

(C) 4 and 13

(D) 4 and 14

(E) 5 and 14

【答案】C

【思路】题意应该是抽一张牌就要乘上这张牌和其下一个号码

所以 $3 \times (3+1) = 12$, $4 \times (4+1) = 20$, $13 \times (13+1) = 182$, $14 \times (14+1) = 210$. 应该是4与13

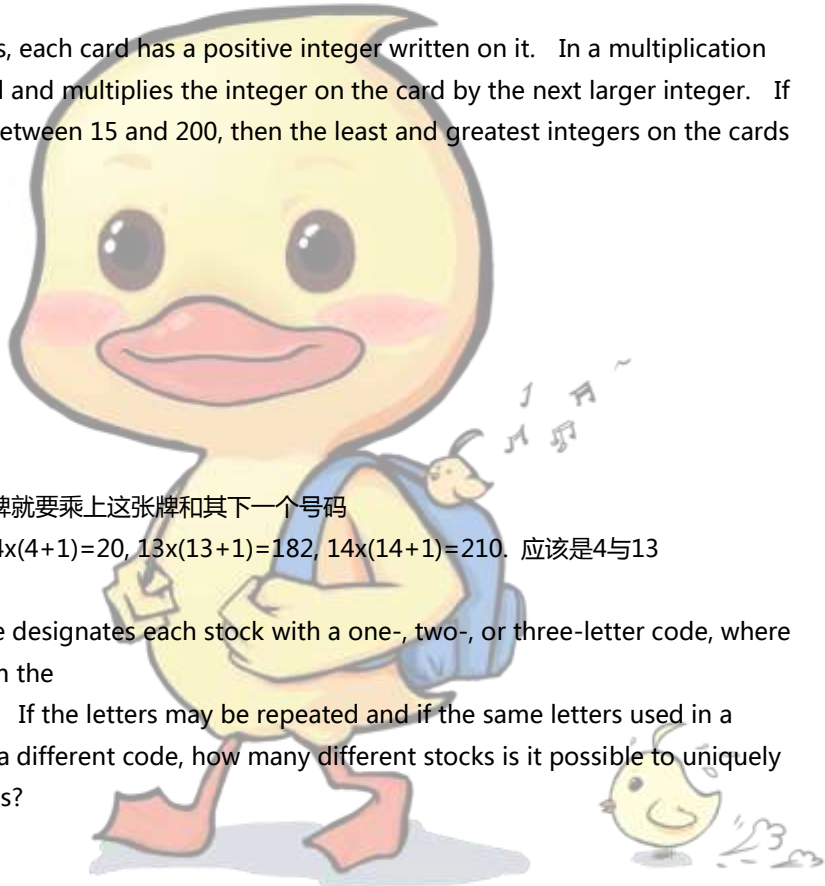
5. A certain stock exchange designates each stock with a one-, two-, or three-letter code, where each letter is selected from the

26 letters of the alphabet. If the letters may be repeated and if the same letters used in a different order constitute a different code, how many different stocks is it possible to uniquely designate with these codes?

(A) 2,951

(B) 8,125

(C) 15,600



(D) 16,302

(E) 18,278

【答案】E

【思路】题目有前提If the letters may be repeated and if the same letters used in a different order constitute a different code,

所以one letter-26种

two letters-26x26种

three letters-26x26x26种

total=18,278

6.A committee of three people is to be chosen from four married couples. What is the number of different committees that can be chosen if two people who are married to each other cannot both serve on the committee?

(A) 16

(B) 24

(C) 26

(D) 30

(E) 32

【答案】E

【思路】<作法1> committee中不能有couple的组合即 "全部的组合-committee中有couple的组合"

$$C(8,3)-4*6$$

$$=56-24$$

$$=32$$

4*6代表将couple视为一个人，有4个couple，另外一个committee则为其其他六个人还一个)

<作法2> $\frac{8 \times 6 \times 4}{3!} = 32$ OOO → 一个萝卜一个坑，求组合!! (3!是因为 abc acb bac bca cba cab 是同一种组合)

7. A certain company has 18 equally qualified applicants for 4 open positions. How many different groups of 4 applicants can be chosen by the company to fill the positions if the order of selection does not matter?

(A) 18

(B) 72

(C) 180

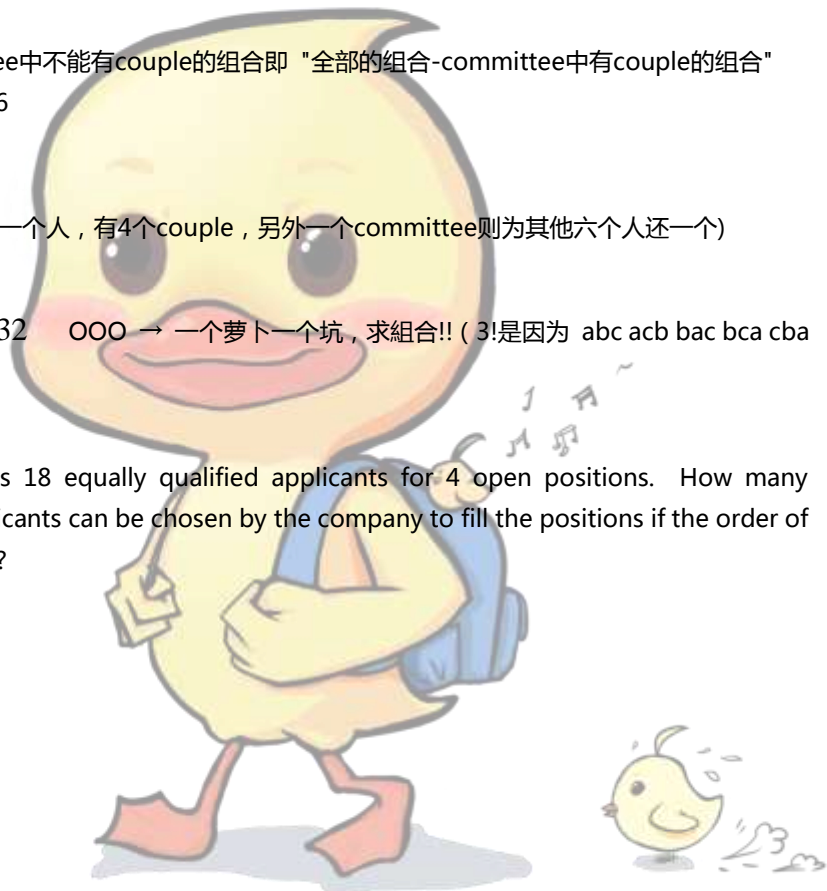
(D) 1,260

(E) 3,060

【答案】E

【思路】

$$\frac{18!}{4!(18-4)!} = 3060$$



8. A certain law firm consists of 4 senior partners and 6 junior partners. How many different groups of 3 partners can be formed in which at least one member of the group is a senior partner? (Two groups are considered different if at least one group member is different.)

- (A) 48
- (B) 100
- (C) 120
- (D) 288
- (E) 600

【答案】B

【思路】

团体里至少有一位senior組合=团体所有組合 - 团体里皆是三位junior的組合

$$\frac{10!}{3!(10-3)!} - \frac{6!}{3!(6-3)!} = 100$$

9. At a dinner party, 5 people are to be seated around a circular table. Two seating arrangements are considered different only when the positions of the people are different relative to each other. What is the total number of different possible seating arrangements for the group?

- (A) 5
- (B) 10
- (C) 24
- (D) 32
- (E) 120

【答案】C

【思路】若为一直线排法 为5！

但此题目为圆形五个人排圈圈...

ABCDE

BCDEA

CDEAB

DEABC

EABCD

都是一样

故排圈圈时就是5!/5=4!

除以五是为了消除每次相邻位置改变时都会有重复的情况

10. Of the 12 temporary employees in a certain company, 4 will be hired as permanent employees. If 5 of the 12 temporary employees are women, how many of the possible groups of 4 temporary employees consist of 3 women and 1 man?

- (A) 22
- (B) 35
- (C) 56
- (D) 70



(E) 105

【答案】 D

【思路】排列组合问题

这题看似复杂，其实就是C几取几的问题

12个temporary employees里面包含了7男 5女

题目问说 how many of the possible groups of 4 temporary employees consist of 3 women and 1 man?

<在四个temporary employees里面要有3个女人1个男人的可能为多少组>

因为是组合，每个男女都是一个独立的个体，故 $C_3^5 \times C_1^7 = 70 \rightarrow D$ **难题冲刺**

1.To furnish a room in a model home, an interior decorator is to select 2 chairs and 2 tables from a collection of chairs and tables in a warehouse that are all different from each other. If there are 5 chairs in the warehouse and if 150 different combinations are possible, how many tables are in the warehouse?

(A) 6

(B) 8

(C) 10

(D) 15

(E) 30

【答案】 A

【思路】假设有n张tables $C_2^5 \times C_2^n = 150$

$$\frac{5!}{2!(5-2)!} \times \frac{n!}{2!(n-2)!} = 150$$

$$10 \times \frac{n!}{2!(n-2)!} = 150$$

$$\frac{n \times (n-1)}{2} = 30$$

$$(n-6)(n+5) = 0$$

$$N=6(-5 \text{ 不合})$$

2. Of the 20 people who each purchased 2 tickets to a concert, some used both tickets, some used only 1 ticket, and some used neither ticket. What percent of the tickets that were purchased by the 20 people were used by those people?



(1) Of the 20 people, 10 used only 1 ticket.

(2) Of the 20 people, 4 used neither ticket.

【答案】C

【思路】在买 2 张票的 20 人当中，有人用 2 张，有人用 1 张，有人都没用
问：在这 20 人买的票当中，有多少比例是被这 20 人使用？

(1) 并不知道没有用票的人，无法得知比例关系，insufficient

(2) 只知道没有用票的人，不知道用一张票的人数，无法得知比例关系，insufficient

(1)+(2) 由(1)(2)可以得知使用两张票的人：6 人

$$\frac{6 \times 2 + 10 \times 1}{20 \times 2} = \frac{22}{40} = \frac{11}{20} = 0.55, \text{sufficient}$$

3. In a certain senior class, 72 percent of the male students and 80 percent of the female students have applied to college. What fraction of the students in the senior class are male?

(1) There are 840 students in the senior class.

(2) 75 percent of the students in the senior class have applied to college.

【答案】B

【思路】

已知所有男生 in the senior class 为 M 个，所有女生 in the senior class 为 F 个

(1) 840 students in the senior class, $M + F = 840$ 无法求出 fraction of the students in the senior class are male, NOT sufficient

(2) 75 percent of the students in the senior class have applied to college, 求出

$(M + F) \times \frac{75}{100}$, 72 percent of the male students and 80 percent of the female students 可求出

$\frac{72}{100}M + \frac{80}{100}F$, 又 $(M + F) \times \frac{75}{100} = \frac{72}{100}M + \frac{80}{100}F$, $5F = 3M$, 可以求出 fraction, sufficient

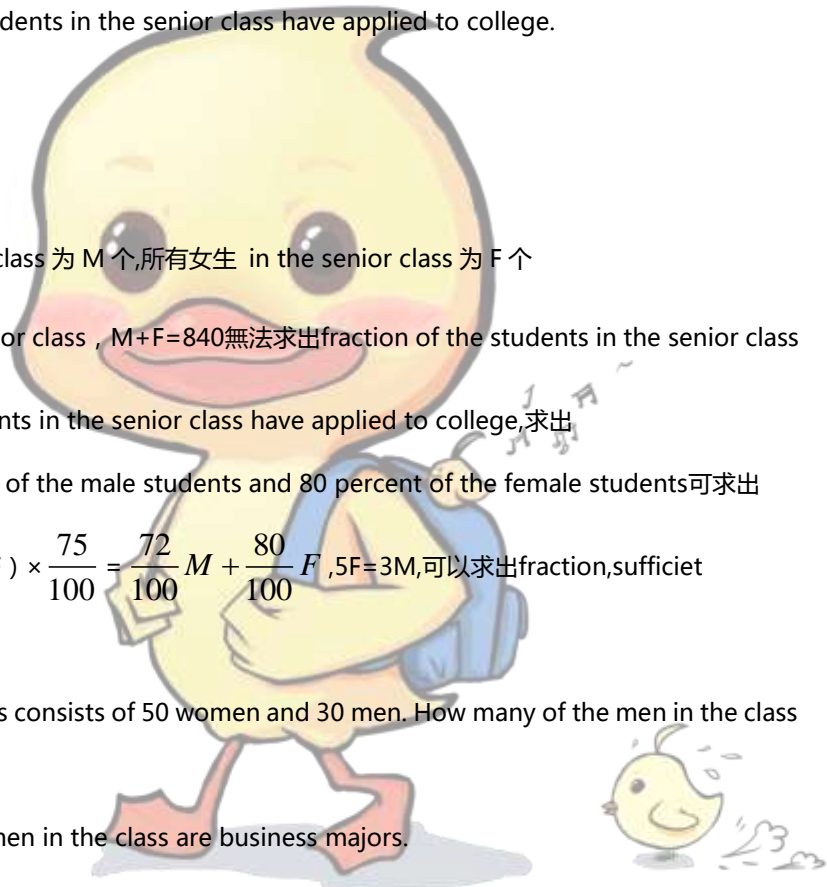
4. A certain economics class consists of 50 women and 30 men. How many of the men in the class are business majors?

(1) 40 percent of the women in the class are business majors.

(2) 50 percent of all the people in the class are business majors.

【答案】C

【思路】已知男生数量求男生有多少人主修 business，需要知道男生修 business 的比例，或全班主修 business 的人数跟女生主修 business 的人数。



- (1) 可得知主修 business 的女生有 $50 \times 40\% = 20$ 人，但题目问的是男生，所以不充分。
- (2) 可得知全班一半的人主修 business，即 $50\%(50 + 30) = 40$ 人，但我们不知道这 40 人中男女生比例，所以不充分。
- (1)+(2) 知道全班有 40 人主修 business，其中有 20 人是女生，即可得知主修 business 的男生人数，所以充分。

5. Linda, Robert, and Pat packed a certain number of boxes with books. What is the ratio of the number of boxes of books that Robert packed to the number of boxes of books that Pat packed?

(1) Linda packed 30 percent of the total number of boxes of books.

(2) Robert packed 10 more boxes of books than Pat did.

【答案】E

【思路】求 R:P，需知道总数，和三人之间的比。

(1) 问 R:P，只给 L 没有帮助，不充分。

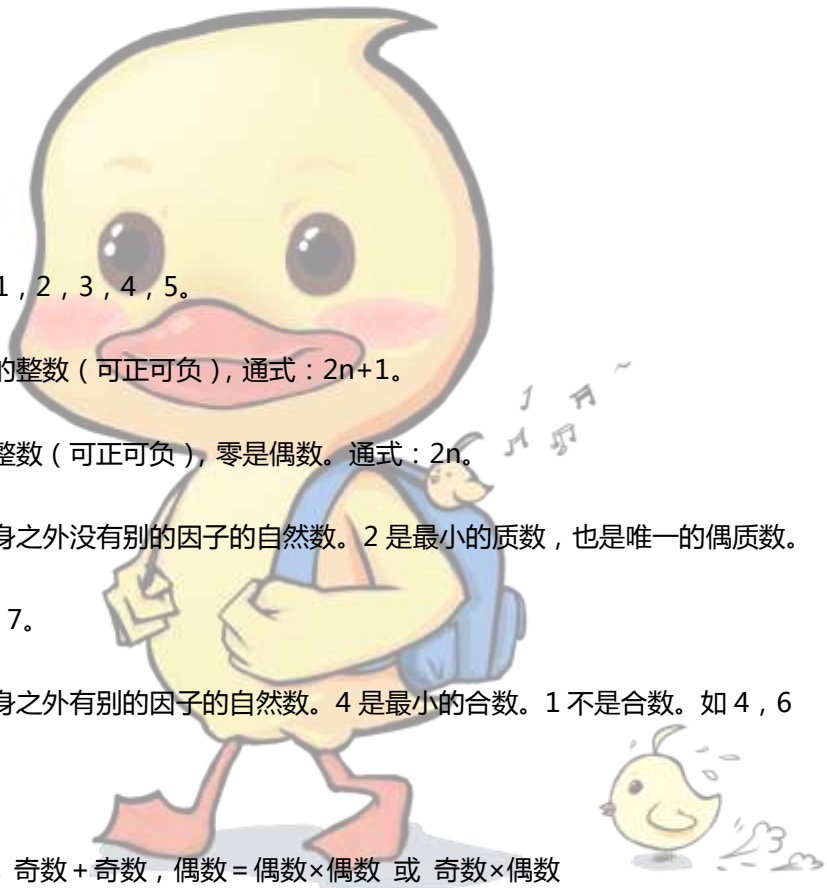
(2) $R - P = 10$ 只知两者差距 10 但无法知道比例，10 和 20 差 10，比例为 $1/2$ ；但 30 和 40 也差 10，可是比例却为 $3/4$ ，所以不充分。

(1)+(2) 从(1)中扣掉 L 的 30%得知 $R + P$ 加起来占了 70%，从(2)知道 $R - P = 10$ ，但因为不知道两人中任一人的数量或是总数，所以仍然无法求出 R:P，不充分

PART 5 自然数理论

知识点总结

1. 自然数：正整数。如 1, 2, 3, 4, 5。
2. 奇数：不能被 2 整除的整数（可正可负），通式： $2n+1$ 。
3. 偶数：能被 2 整除的整数（可正可负），零是偶数。通式： $2n$ 。
4. 质数：除了 1 和它本身之外没有别的因子的自然数。2 是最小的质数，也是唯一的偶质数。
1 不是质数。如 2, 3, 5, 7。
5. 合数：除了 1 和它本身之外有别的因子的自然数。4 是最小的合数。1 不是合数。如 4, 6
6. 奇偶性分析：
 - 1) 偶数 = 偶数 + 偶数 或 奇数 + 奇数，偶数 = 偶数 \times 偶数 或 奇数 \times 偶数
 - 2) 奇数 = 奇数 + 偶数



- 3) 奇数个奇数相加减, 结果为奇数
- 4) 偶数个奇数相加减, 结果为偶数
- 5) 任意个偶数相加减, 结果为偶数
- 6) 若 n 个整数相乘结果为奇数, 则这 n 个整数为奇数
- 7) 若 n 个连续的整数相加等于零, 则 n 为奇数。如: $(-2)+(-1)+0+1+2=0$
- 8) 若 n 个连续的奇数相加等于零, 则 n 为偶数。如: $(-3)+(-1)+1+3=0$
- 9) 若 n 个连续的偶数相加等于零, 则 n 为奇数。如: $(-4)+(-2)+0+2+4=0$
- 10) 两个质数之和为奇数, 其中必有一个是 2。

7. n 个连续自然数的乘积一定能够被 $n!$ 整除。

8. 若 n 能被 a 整除, 且能被 b 整除, 那么 n 一定能够被 $[a, b]$ 整除。

(其中 $[a, b]$ 表示 a 和 b 的最小公倍数, 另外 $\{a, b\}$ 表示 a 和 b 的最大公约数) 特别地, 当 a, b 互质(即无公因子), 则 n 能被 $a \times b$ 整除。(这里用到了公式 $[a, b] = a \times b / \{a, b\}$)

E.G. n 能被 8 和 12 整除, n 也能被 24 整除; n 能被 9 和 11 整除, n 也能被 99 整除。

9. 一些整除性质。

- 1) 已知 $C=A+B$ 且 A 是 m 的倍数, 则 C 是 m 的倍数与 B 是 m 的倍数互为充分必要条件

推论: 一个数是否能够被 5 整除, 只要看它的最后一位是否可被 5 整除。

一个数是否能够被 4 整除, 只要看它的后两位是否可被 4 整除。

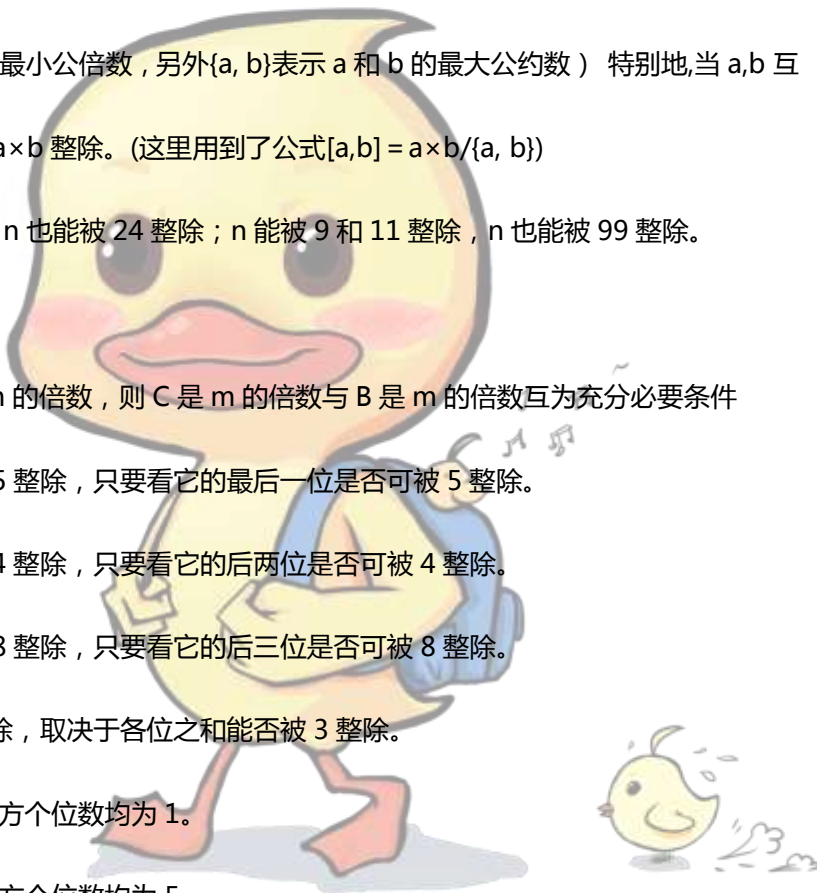
一个数是否能够被 8 整除, 只要看它的后三位是否可被 8 整除。

一个数能否被 3 整除, 取决于各位之和能否被 3 整除。

- 2) 个位数为 1 的数任意次方个位数均为 1。

- 3) 个位数为 5 的数任意次方个位数均为 5。

- 4) 个位数为 6 的数任意次方个位数均为 6。



练习题

1.The product P of two prime numbers is between 9 and 55. If one of the prime numbers is greater than 2 but less than 6 and the other is greater than 13 but less than 25, then P =

- (A) 15
- (B) 33
- (C) 34
- (D) 46
- (E) 51

【答案】E

【思路】前提是p是两个“质数”的乘积

one of the prime numbers is greater than 2 but less than 6 包含3,5

the other is greater than 13 but less than 25 包含 17,19,23

p 可能为51,57,69,85,95,115 (又 $9 < p < 55$),故答案 E

2.What is the sum of the different positive prime factors of 550 ?

- (A) 10
- (B) 11
- (C) 15
- (D) 16
- (E) 18

【答案】E

【思路】 $550 = 5 \times 5 \times 2 \times 11$

the sum of the different positive prime factor= $5+2+11=16$ (5只能加一次因为different)

3.The number 75 can be written as the sum of the squares of 3 different positive integers. What is the sum of these 3 integers?

- (A) 17
- (B) 16
- (C) 15
- (D) 14
- (E) 13

【答案】E

【思路】

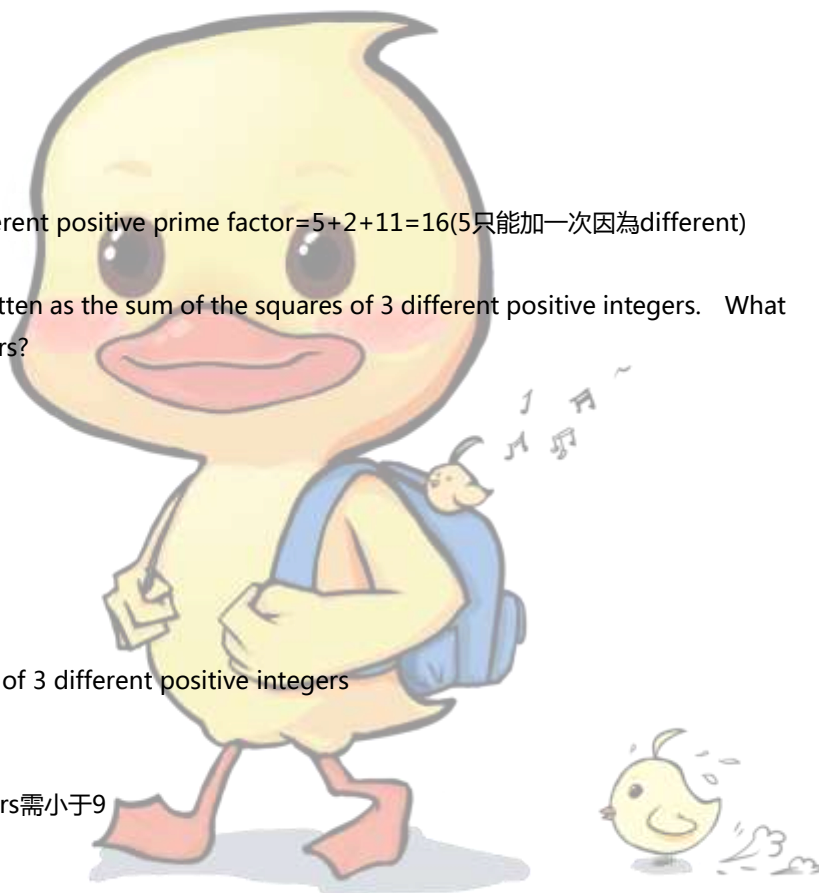
75为the sum of the squares of 3 different positive integers

$$9^2 = 81 > 75$$

故3 different positive integers需小于9

$$7^2 + 5^2 + 1^2 = 75$$

$$7 + 5 + 1 = 13$$



4. If n is a positive integer and the product of all the integers from 1 to n , inclusive, is a multiple of 990, what is the least possible value of n ?

- (A) 10
- (B) 11
- (C) 12
- (D) 13
- (E) 14

【答案】 B

【思路】 $990=2*3*5*3*11$ 题目问the least possible 可知至少要乘到11才能是990的倍数

5. How many different prime numbers are factors of the positive integer n ?

(1) Four different prime numbers are factors of $2n$.

(2) Four different prime numbers are factors of n^2 .

【答案】 B

【思路】 (1) 如果 $2n=2.3.5.7$ 的话 n 的prime factor为3.5.7=>三个

如果 $2n=2^2.3.5.7$ 的话 n 的prime factor为2.3.5.7=>四个,不充分

(2) $n = \sqrt{P_1^a \times P_2^b \times P_3^c \times P_4^d}$, a,b,c,d不管是几次方,都可以确定质数只有四个,充分

6. Is the product of a certain pair of integers even?

(1) The sum of the integers is odd.

(2) One of the integers is even and the other is odd.

【答案】 D

【思路】 设2个整数为x,y 题目问 $xy = 2n$?

(1) $x + y = 2n+1$, 即至少有一个为奇数;一个为偶数, 则xy相乘, 一定为偶数, 充分

(2) 有一个为奇数;一个为偶数, 则xy相乘, 一定为偶数, 充分

7. If n and m are positive integers, what is the remainder when $3^{(4n+2+m)}$ is divided by 10?

(1) $n = 2$

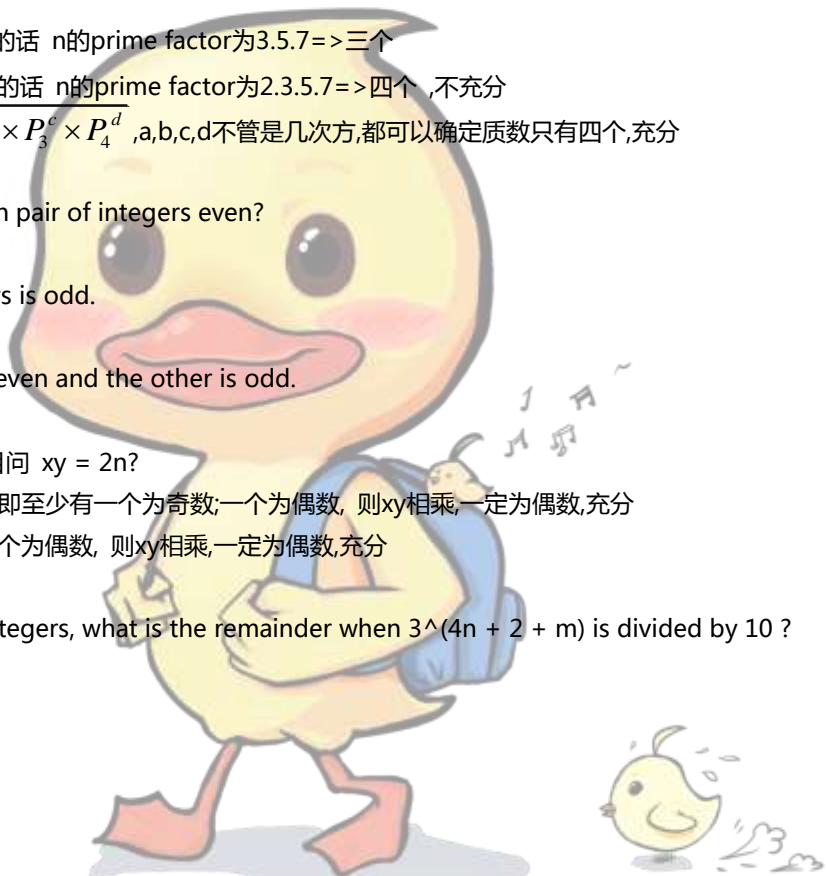
(2) $m = 1$

【答案】 B

【思路】

(1) $n = 2$, 不知 $m = ?$ m 的变化会影响到余数的结果, 余数会不一样, 带几个不同数字进去, 然后除10看看.....不充分

8. If p and n are positive integers and $p > n$, what is the remainder when $p^2 - n^2$ is divided by 15?



(1) The remainder when $p + n$ is divided by 5 is 1.

(2) The remainder when $p - n$ is divided by 3 is 1.

【答案】E

【思路】 $p^2 - n^2 = (p+n)(p-n)$

(1) $p+n=5a+1$ -----insufficient

(2) $p-n=3b+1$ -----insufficient

(1)(2) $(p+n)(p-n)=(5a+1)(3b+1)=15ab+5a+3b+1$ -----insufficient

9. If the integer n is greater than 1, is n equal to 2 ?

(1) n has exactly two positive factors.

(2) The difference of any two distinct positive factors of n is odd.

【答案】B

【思路】若两数相减为奇数，则两数必为一奇数，一偶数。

两奇数或两偶数相减都会是偶数。

(1) n 有两个正因数 \rightarrow 只能得知 n 是个质数。

(2) $n = 2$ 时，两因数相减的结果有 $2 - 1 = 1$, $2 - (-1) = 3$, $(-2) - 1 = -3$, $(-2) - (-1) = -1$ 四种，全是奇数。

■ 若 $n > 2$ ，为偶数： $n - 2$ 会是偶数。

■ 若 $n > 2$ ，为奇数： $n - 1$ 会是偶数。

由以上推理可知2是符合条件要求的唯一数字，故条件充分。

10. An integer greater than 1 that is not prime is called composite. If the two-digit integer n is greater than 20, is n composite?

(1) The tens digit of n is a factor of the units digit of n .

(2) The tens digit of n is 2.

【答案】A

【思路】

Keep in mind that they are not asking for the actual value of n but rather whether n is composite

(1)

■ For every prime numbers, tens digit is not a factor of unit digits. 23,29,31,37,41,47... If it's a factor..it's definitely composite number

■ For every prime no#, tens digit is not a factor of unit digits.

23,29,31,37,41,47... If it's a factor..it's definitely composite number.

■ This means n will be a product of its tens digit and the two-digit number with tens digit 1 and unit digit (n 's unit digit)/(n 's tens digit). For example, 26, 2 is a factor of 6, so 26 is a product of 2 and 13.

■ $n = AB = 10xA + AC = A(10+C)$.. So A is factor, it cannot be prime.

(2) not sufficient, because there are two digit prime numbers that have 2 as their tens digit: 23,

29.

难题冲刺

1. For every positive even integer n , the function $h(n)$ is defined to be the product of all the even integers from 2 to n , inclusive. If p is the smallest prime factor of $h(100) + 1$, then p is

- (A) between 2 and 10
- (B) between 10 and 20
- (C) between 20 and 30
- (D) between 30 and 40
- (E) greater than 40

【答案】E

【思路】假设 $h(100)+1$ 最小因数是 n , $n < 50$

例如47好了, 既然47是因数, 它就能整除 $h(100)+1$

可是发现 $h(100)$ 里面有47的因数, 可以整除 那 $h(100)+1$ 被47除的除数为1, 表示47不是 $h(100)+1$ 的因数

同理, 小于50的其它因式 也是一样的结果

所以 $h(100)+1$ 的因式是大于50外

重点在那50, 假如小于50的质数是 $h(100)$ 的factor, 那就不会是 $h(100)+1$ 的factor, 因为除数会是1

2. If n is a multiple of 5 and $n = (p^2)q$, where p and q are prime numbers, which of the following must be a multiple of 25?

- (A) p^2
- (B) q^2
- (C) pq
- (D) p^2q^2
- (E) p^3q

【答案】D

【思路】题目给的是 n 的质因式分解 因为 n 为5的乘数 那么 n 的质因式中必定包含5这个因子 所以有两种可能 $p=5$ 或 $q=5$

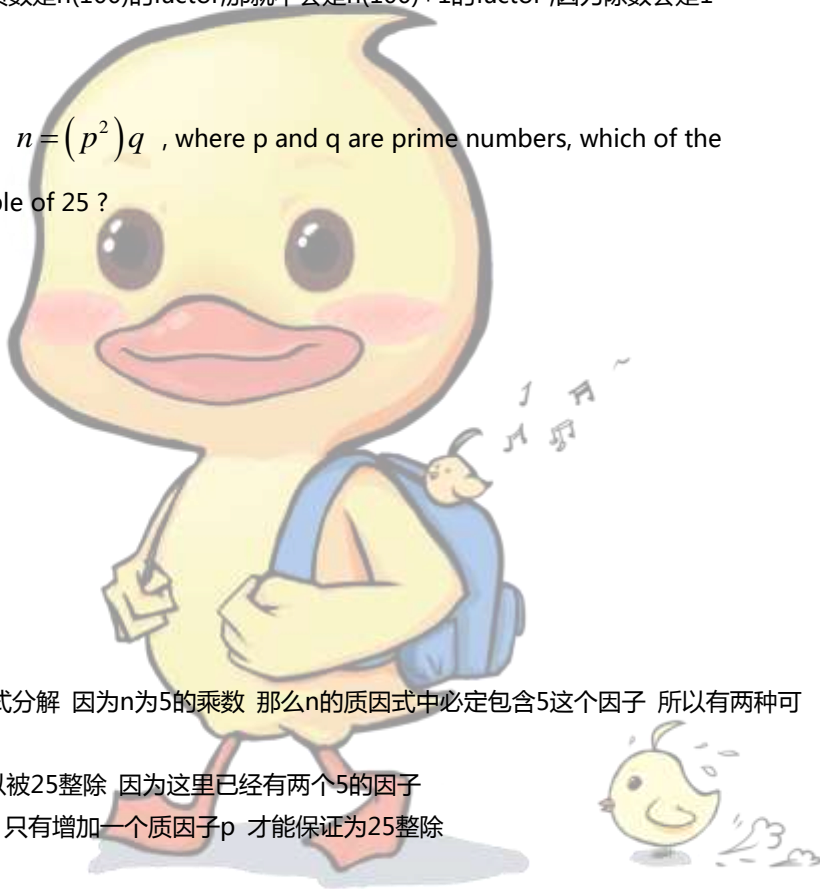
如果 $p=5$ q 不一定 那么 n 可以被25整除 因为这里已经有两个5的因子

但是如果 $q=5$ p 不一定 此时 只有增加一个质因子 p 才能保证为25整除

[另解]

题目说: $p^2q=5k$

这边就要先推论一下, 有两种可能情形:



$$(1)p=5, k=pq$$

$$(2)q=5, k=p^2$$

直接看答案D

$$(p^2)(q^2)=5kq \text{ (由题目推的)}$$

再带入刚刚的两种情形

$$(1)5kq=5(pq)(q)=25q^2$$

$$(2)5kq=5(p^2)(5)=25p^2$$

所以只有 p^2q^2 符合条件

3. A company plans to assign identification numbers to its employees. Each number is to consist of four different digits from 0 to 9, inclusive, except that the first digit cannot be 0. How many different identification numbers are possible?

(A) 3,024

(B) 4,536

(C) 5,040

(D) 9,000

(E) 10,000

【答案】B

【思路】

Each number is to consist of four different digits 为四位数

the first digit cannot be 0 故有1到9 九个数字

the second digit 可以为0但是因为the first digit用到一个数字 所以剩下九个数字

the third digit 剩下八个数字 the fourth digit 剩下七个数字 $9 \times 9 \times 8 \times 7 = 4536$

4.S is a finite set of numbers. Does S contain more negative numbers than positive numbers?

(1) The product of all the numbers in S is -1,200.

(2) There are 6 numbers in S.

【答案】E

【思路】(1) 只告诉有一些正负的数字,insufficient 表示一定要有负数

(2) 六个数字并不能表达什么

(1) + (2)

因为S contain more negative numbers than positive numbers

所以至少要有四个负数以上,但一定要奇数的负数,

The product of all the numbers in S 才会是负数

因此会有5个负数,就可以成立more negative numbers than positive numbers

$(-)(-)(-)(-)(-)(+)=(-)$ 一正五负

但是也可能S contain less negative numbers $(-)(+)(+)(+)(+)(+)=(-)$ 一负五正 所以答案为E

5.The positive integer k has exactly two positive prime factors, 3 and 7. If k has a total of 6

positive factors, including 1 and k , what is the value of k ?

(1) 3^2 is a factor of k .

(2) 7^2 is not a factor of k .

【答案】D

【思路】

对任意正整数 n , 先定义因子个数函数 $\tau(n)$, 可以证明 τ 是一个 multiplicative 函数, 意思是说 $\tau(mn)$

$= \tau(m)\tau(n)$. m, n 互质, 才有 $\tau(mn) = \tau(m)\tau(n)$. 而对于质数 p , 很明显我们有: $\tau(p^r) = (r+1)$.

举例: 用这公式算一下 90 的因子个数, $\tau(p^r) = (r+1)$,

$\tau(90) = \tau[(2)(3^2)(5)] = \tau(2)\tau(3^2)\tau(5) = (1+1)(2+1)(1+1) = (2)(3)(2) = 12$ 个

再举 120 为例: $120 = 2^3 \cdot 3 \cdot 5$ 因子个数为 $(3+1)(1+1)(1+1) = 16$ 个

题目说 $3^x \times 7^y$, 共 6 个因子

因此 $(x+1)(y+1) = 6$, 则共有 $(2,3)(3,2) \rightarrow (x,y)$ 有 either $(1,2)$ or $(2,1)$ 的组合

所以 $3^1, 3^2, 7^2, 7^1$ 的互相乘积皆是可能 factors

(1) 3^2 is a factor of k .

$$(3^0)(7^0) = 1$$

$$(3^0)(7^1) = 7$$

$$(3^1)(7^0) = 3$$

$$(3^1)(7^1) = 21$$

$$(3^2)(7^0) = 9$$

$$(3^2)(7^1) = 63 \text{ (k, itself) } (x \geq 2, \text{ so it must be true that } x=2 \text{ and } y=1)$$

由条件得知 k 's positive factors: 1, 3, 7, 9, 21, k

$k=63$ sufficient

(2) 7^2 is not a factor of k .

k 有 6 个 positive factors, 且只有 3, 7 两个质因子

表示 k 可因式分解为 $3^2 \times 7$ or 3×7^2

此条件排除了第二个可能, $y < 2$, so $x=2$ and $y=1$.

所以 $k=63$ sufficient

类比

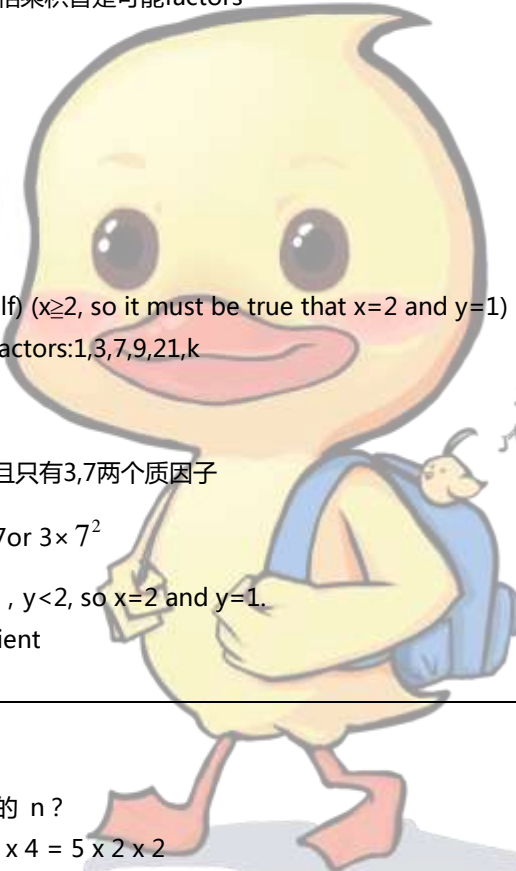
现在要求在 $\tau(n) = 20$ 中最小的 n ?

首先, $\tau(n) = 20 = 10 \times 2 = 5 \times 4 = 5 \times 2 \times 2$

(1) 假如 n 只有一个质因子, 那么要求 n 为最小时此质因子一定是 2, 于是 $n = 2^{19}$, 明显超大的。

(2) 假如 n 有两个质因子, 那么一个是 2, 另一个是 3, 且把大的次方数放在 2 那儿, 就会令 n 最小, 所以 $n = (2)^4(3)^3 = 432$

或是另一个可能 $n = (2)^9(3)$, 这个不用算, 超大的。



(3) 假如 n 有三个质因子，那令 n 最小的一定是 2, 3, 5，又依序把较小次方数的放在较大的质因数，

所以得 $n = (2)4(3)(5) = 240$ 。

很明显答案就是 240 了。

PART 6 几何

知识点总结

1. 平面几何

1. 直角三角形勾股定理： $a^2 + b^2 = c^2$

2. 两直线平行，内错角相等，同位角相等。

3. 圆心角是圆周角的两倍。

4. 面积与周长：

(1) 三角形 (边长为 a, b, c):

面积 = $\frac{1}{2} ab \sin \gamma$ (γ 是 a, b 两边之夹角) or **面积** = $\frac{1}{2}$ 底 * 高

(对于直角三角形, $\gamma = 90^\circ$, $S_{\text{直角三角形}} = \frac{1}{2} ab$)

对于等边三角形, $\gamma = 60^\circ$, $S_{\text{等边三角形}} = \frac{\sqrt{3}}{4} ab$)

周长 = $a + b + c$

(2) 梯形 (上底为 a , 下底为 b , 高为 h)

面积 = $(a + b) \times h / 2$

(3) 平行四边形 (边长为 a, b , 高为 h)

面积 = $a \times h$ **周长** = $2(a + b)$

(4) 矩形 (边长为 a, b)

面积 = $a \times b$ **周长** = $2(a + b)$

(5) 正方形 (边长为 a)



面积= a^2 周长= $4a$

(6) 圆 (半径为 R)

面积= πR^2 周长= $2\pi R$

5. 多边形内角和 : $(n-2) 180^\circ$

2. 立体几何

体积和表面积 :

(1) 长方体 (边长为 a, b, c)

体积= $a \times b \times c$

表面积= $2(a \times b + b \times c + c \times a)$

(2) 正方体 (立方体) (边长为 a)

体积= a^3

表面积= $6a^2$

(3) 圆柱 (底面半径为 R , 高为 h)

体积= $\pi R^2 h$

表面积= $2\pi R^2 + 2\pi R \times h$

(4) 圆锥

体积= $\frac{1}{3}\pi R^2 h$ (底面半径为 R , 高为 h)

3 解析几何

1. 对称

(1) 坐标 (a, b) 关于 $y=x$ 的对称点为 (b, a)

(2) 坐标 (a, b) 关于 $y=-x$ 的对称点为 $(-b, -a)$

2. 直线方程



(1) $y=kx+b$ (斜截式, k 为斜率 slope, b 为截距 intercept)

(2) $x/a + y/b = 1$ (截距式, a 为 x 轴上截距, b 为 y 轴上截距)

(3) $(y-y_2)/(x-x_2) = (y_1-y_2)/(x_1-x_2)$ (两点式, 已知 (x_1, y_1) , (x_2, y_2))

(4) $(y-y_1)/(x-x_1) = k$ (点斜式, 已知 (x_1, y_1) , 斜率 k)

练习题

1. The perimeters of square region S and rectangular region R are equal. If the sides of R are in the ratio 2 : 3, what is the ratio of the area of region R to the area of region S ?

- (A) 25 : 16
- (B) 24 : 25
- (C) 5 : 6
- (D) 4 : 5
- (E) 4 : 9

【答案】B

【思路】

the sides of R are in the ratio 2 : 3 可设 R 的长度宽度分别为 $2x$ 和 $3x$

因为 The perimeters of square region S and rectangular region R are equal

$(2x+3x) \times 2 \div 4 = 2.5x$ the area of region R to the area of region S 为 $6x : 6.25x = 24:25$

2. A circular jogging track forms the edge of a circular lake that has a diameter of 2 miles.

Johanna walked once around the track at the average rate of 3 miles per hour. If t represents the number of hours it took Johanna to walk completely around the lake, which of the following is a correct statement?

- (A) $0.5 < t < 0.75$
- (B) $1.75 < t < 2.0$
- (C) $2.0 < t < 2.5$
- (D) $2.5 < t < 3.0$
- (E) $3 < t < 3.5$

【答案】c

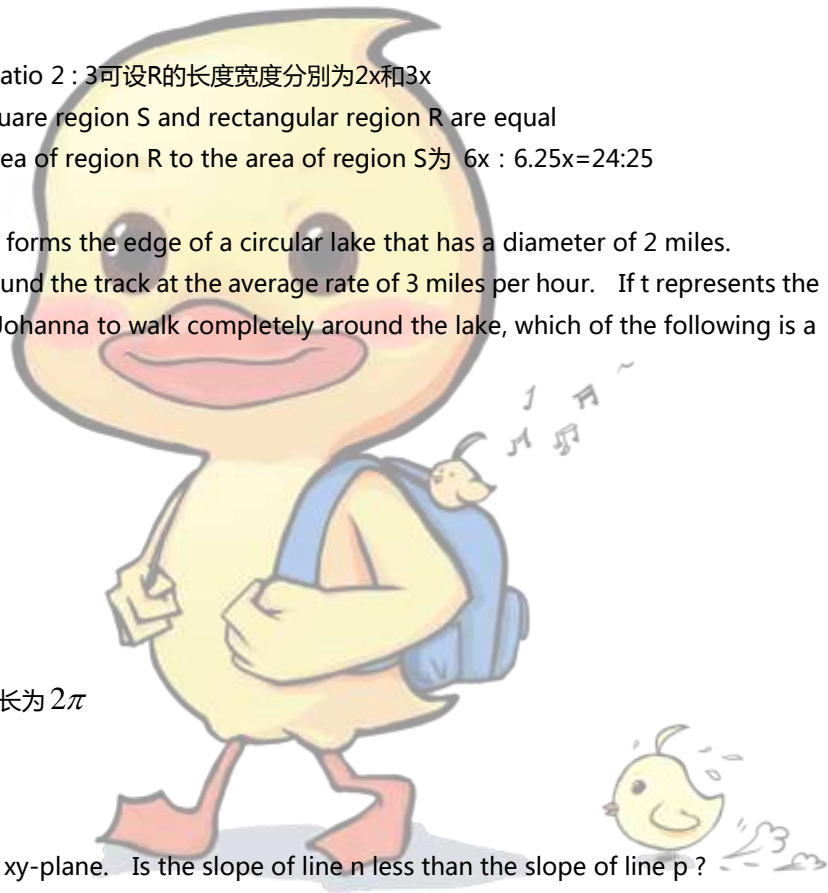
【思路】

直径 2 miles 可得圆湖的周长为 2π

$2 \times 3.14 \div 3 = 2.09$

3. Lines n and p lie in the xy -plane. Is the slope of line n less than the slope of line p ?

- (1) Lines n and p intersect at the point $(5, 1)$.



(2) The y-intercept of line n is greater than the y-intercept of line p.

【答案】C

【思路】

(1) 两线同过一个点, 根本比较不出来, 所以不充分...

(2) 两线的与Y所截取的值N线比P线大, 也比较不出来, 因为不知两线的各另外一点, 所以不充分

(1)+(2)合并即能算出, 因为条件一已经给了两个线段的一点, 再加上条件二给了两个线段分别在Y坐标上点的相对位置, 画个圆即能算出来, 线段较陡的斜率一定比线段较平的斜率大, 所以选(C)

4. In isosceles $\triangle RST$ what is the measure of $\angle R$

(1) The measure of $\angle T$ is 100° .

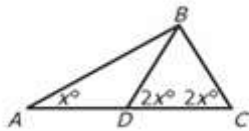
(2) The measure of $\angle S$ is 40° .

【答案】A

【思路】(1) 等腰三角形中有一角为 100° , 表示另外两角必为 40° 、 40° 。(因总和为 180° , 100° 不可能为等边的角度) \leftarrow 选项(1)成立

(2) 等腰三角形中有一角为 40° , 此角可能为对等的角度(40° 、 40° 、 100°), 也有可能是顶角(40° 、 70° 、 70°) \leftarrow 选项(2) 不成立

5.



In triangle ABC above, what is the length of side BC ?

(1) Line segment AD has length 6.

(2) $x = 36$

【答案】B

【思路】用三角形角度合为 180° 来推算 $\angle ABD$ 是否等于 $\angle DAB$ 。发现: 1. $\angle CBD = 180^\circ - 4x$ 2. \angle

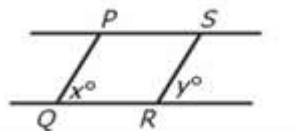
$ABD = 180^\circ - x^\circ - 2x^\circ - (180^\circ - 4x)^\circ = x^\circ = \angle DAB$, 因而得知 $\triangle ADB$ 为等腰三角形。

(1) 因已得知 $\triangle ADB$ 和 $\triangle BDC$ 皆为等腰三角形, 所以边 $AD =$ 边 $BD =$ 边 BC 。即知边 $AD = 6$ 即可推边 BC 。

\leftarrow 选项(1)成立

(2) 不知道任何边长资讯, 所以无法得知边 BC 。 \leftarrow 选项(2)不成立





6. In the figure above, if x and y are each less than 90 and $PS \parallel QR$ is the length of segment PQ less than the length of segment SR ?

(1) $x > y$

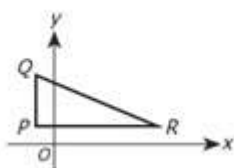
(2) $x + y > 90$

【答案】A

【思路】已知 PS 和 QR 平行 \Rightarrow 角度越大的在平行的两条线之间会越短

(1) 得知 $x > y$, 所以 $SR > PQ$ \leftarrow 选项(1)成立

(2) 得知 $x + y > 90$, 但无法得知哪一个角度较大 \leftarrow 选项(2)不成立



7. In the figure above, segments PQ and PR are each parallel to one of the rectangular coordinate axes. What is the sum of the coordinates of point P ?

(1) The x -coordinate of point Q is -1 .

(2) The y -coordinate of point R is 1 .

【答案】C

【思路】已知 PO 和 PR 分别和 y 轴、 x 轴平行, 所以可得知 P 点的 x 坐标和 Q 点一样、 y 坐标和 R 点一致。

(1) 知道 Q 点的 x 坐标 \Rightarrow 即知道 P 的 x 坐标, 但无法得知 y 坐标 \leftarrow 选项(1)不成立

(2) 知道 R 点的 y 坐标 \Rightarrow 即知道 P 的 y 坐标, 但无法得知 x 坐标 \leftarrow 选项(2)不成立

(1)+(2) 知道 P 点的 x 、 y 坐标, 即可得知 $x+y$ 的合 \leftarrow 选项(1)加(2)成立

8. If m , r , x , and y are positive, is the ratio of m to r equal to the ratio of x to y ?

(1) The ratio of m to y is equal to the ratio of x to r .

(2) The ratio of $m + x$ to $r + y$ is equal to the ratio of x to y .

【答案】B

【思路】问 $m:r = x:y \rightarrow r*x = m*y$?

(1) $m/y = x/r \rightarrow x*y = m*r$ insufficient

(2) $(m+x)/(r+y) = x/y \rightarrow r*x + x*y = m*y + x*y$ (x, y 皆为正数, $x*y$ 对消)

故 $rx = my$ sufficient



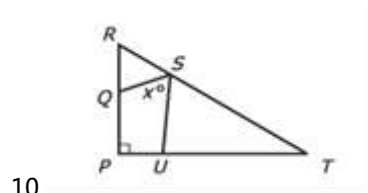
9. In the xy -plane, what is the y -intercept of line l

- (1) The slope of line l is 3 times its y -intercept.
 (2) The x -intercept of line l is $-\frac{1}{3}$.

【答案】E

【思路】(1) 直线方程式为 $y=mx+b$, m 为斜率, b 为截距, 通过 $(0, Y_1)$, $Y_1=3b(x)+b$, $Y_1=b$, 不充份

(2) $y=mx+b$, $(-1/3, 0)$ 帶入, $b = -\frac{1}{3}m$, 不充份 (1) + (2) 还是不充份 故选E



10.

In the figure shown, what is the value of x ?

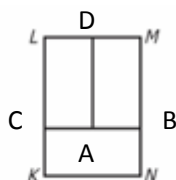
- (1) The length of line segment QR is equal to the length of line segment RS .
 (2) The length of line segment ST is equal to the length of line segment TU .

【答案】C

【思路】

- (1) $QR=RS$ 不能判断 x
 (2) $ST=TU$ 不能判断 x
 (1)+(2) $\angle RSQ = \angle RQS$ & $\angle TSU = \angle SUT$
 $\angle SUP = x + \angle TSU$
 $\angle SQP = x + \angle RSQ$
 $\angle SUP + \angle SQP = 180 + x$
 $\angle QPU + 180 + 2x = 360$ (四边形内角和)
 $2x = 90$, $x = 45$

难题冲刺



1. In the figure above, what is the ratio $\frac{KN}{MN}$



- (1) The perimeter of rectangle KLMN is 30 meters.
- (2) The three small rectangles have the same dimensions.

【答案】B

【思路】设其他点来说明

- (1) 已知长方形KLMN的周长为30，但无法得知其他资讯←选项(1)不成立
- (2) 已知个小的面积一样,且有共用边(ex:AD为LDAC和MDAB的共用边)可推出AC=AB. 三个长方形的面积相等,表示LDAC=CKNB (设共用边BC为2k)=> 长*k=2K*宽 =>可得知长为2k, 宽为k. KN=2k, MN=3k
- 即可求出 $\frac{KN}{MN}$. ←选项(2)成立



2. In the figure shown, the measure of angle PRS is how many degrees greater than the measure of angle PQR ?

- (1) The measure of angle QPR is 30°
- (2) The sum of the measures of angles PQR and PRQ is 150°

【答案】D

【思路】

$$PRS - PQR = ?$$

$$PQR + QPR + PRQ = 180 \text{ (三角形PQR)}$$

$$PRS + RPS + 90 = 180 \text{ (三角形RPS)} \Rightarrow PRS + RPS = 90$$

$$180 = PRQ + PRS$$

$$(1) \text{ } PQR = 30$$

$$1. \text{ } PQR + QPR + PRQ = 180 \text{ (三角形PQR)}$$

$$\Rightarrow PQR + PRQ = 150 \text{ (=180-30)}$$

$$2, \text{ } 180 = PRQ + PRS$$

1,2 互相結合

$$PQR + QPR + PRQ = PRQ + PRS$$

$$\Rightarrow PQR + QPR = PRS$$

$$PRS - PQR = QPR = 30 \dots \text{sufficient}$$

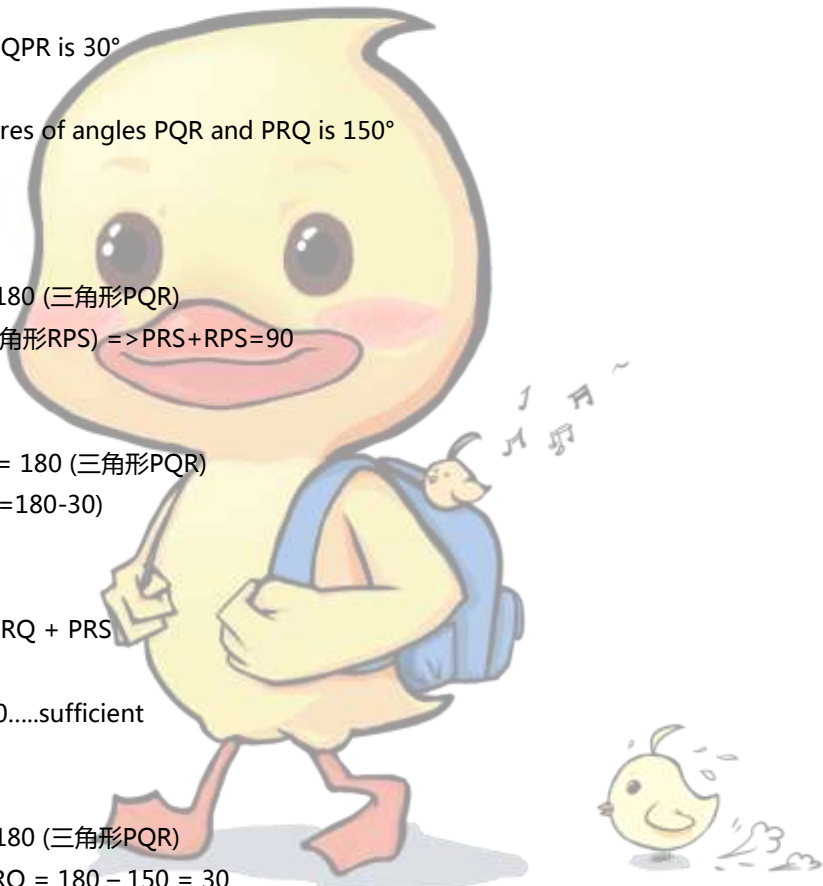
$$(2) \text{ } PQR + PRQ = 150$$

$$PQR + QPR + PRQ = 180 \text{ (三角形PQR)}$$

$$QPR = 180 - PQR - PRQ = 180 - 150 = 30$$

其他解題同(1)

$$\text{故 } PRS - PQR = QPR = 30 \dots \text{sufficient}$$



法2 另一种想法

Extend SQ out and that external angle is $180 - \text{PQR}$, which must be equal to the two internal angles, one of which is 30 degrees given in one and two and the other is $180 - \text{PRS}$. Using algebra the difference is 30 degrees.

$$\begin{aligned}\text{PRQ} &= (180 - \text{PQR}) - \text{QPR} \\ &= 180 - \text{PQR} - 30 \\ \Rightarrow \text{PQR} + \text{PRQ} &= 150\end{aligned}$$

3. In the xy -plane, does the line with equation $y = 3x + 2$ contain the point (r, s) ?

(1) $(3r + 2 - s)(4r + 9 - s) = 0$

(2) $(4r - 6 - s)(3r + 2 - s) = 0$

【答案】C

【思路】 r is the x coordinate, s is the y , so we want to know whether: $s = 3r + 2$

(1) Essentially, one equation or the other must be equal to zero (if $xy = 0$, the $x = 0$ or $y = 0$).

either $3r + 2 - s = 0$ or $4r + 9 - s = 0$, so: $s = 3r + 2$ or $s = 4r + 9$

If it is true that $3r + 2 - s = 0$, then we see that (r, s) must be included in $3r + 2 = s$, since we have that exact equation. However, it is possible that $4r + 9 = s$, in which case our equation doesn't have to be $3r + 2 = s$

In the first case, (r, s) falls in the line. In the second case, (r, s) does not fall in the line.

无法确定 (r, s) 会落在哪一条线.....insufficient because it could be the second case

(2) either $4r - 6 - s = 0$ or $3r + 2 - s = 0$, so: $s = 4r - 6$ or $s = 3r + 2$

In the first case, (r, s) falls in the line. In the second case, (r, s) does not fall in the line.

无法确定 (r, s) 会落在哪一条线.....insufficient because it could be the second case

(1)+(2)

$y = 3x + 2$ will contain (r, s) only if $s = 3r + 2$.

(1) $s = 3r + 2$ OR $4r + 9$. Insufficient.

(2) $s = 4r - 6$ OR $3r + 2$. Insufficient.

Combining: $s = 3r + 2$. Suff. C

当(1)、(2)同时存在时，会出现交集，此交集上的任何一点皆可以满足题目 $s = 3r + 2$ ，故选 C。

4. On the number line, what is the distance between the point $2x$ and the point $3x$?

(1) On the number line, the distance between the point $-x$ and the point x is 16.

(2) On the number line, the distance between the point x and the point $3x$ is 16.

【答案】D

【思路】 $|3x - 2x| = x = ?$

(1) $|x - (-x)| = 16$ $x - (-x) = 16$ $2x = 16$ $x = 8$. Suff.

(2) $|x - 3x| = 16$ $3x - x = 16$ $2x = 16$ $x = 8$. Suff.

5. In the xy -plane, line k passes through the point $(1, 1)$ and line m passes through the point $(1, -1)$. Are lines k and m perpendicular to each other?

- (1) Lines k and m intersect at the point $(1, -1)$.
 (2) Line k intersects the x -axis at the point $(1, 0)$.

【答案】E

【思路】

(1) line k passes through the point $(1, 1)$ 和 $(1, -1)$ 可求出 k 斜率 line m passes through the point $(1, -1)$, 条件不足无法求出 m 的斜率, 故不知道是否 lines k and m perpendicular to each other, NOT sufficient

(2) line k passes through the point $(1, 1)$ 和 $(1, 0)$ 可求出 k 斜率 line m passes through the point $(1, -1)$, 条件不足无法求出 m 的斜率, 故不知道是否 lines k and m perpendicular to each other, NOT sufficient

(1)+(2) line k passes through the point $(1, 1)$ 和 $(1, 0)$ $(1, -1)$ 可求出 k 斜率 line m passes through the point $(1, -1)$, 条件不足无法求出 m 的斜率, 故不知道是否 lines k and m perpendicular to each other, NOT sufficient

PART 7 数据充分性

知识点总结

1. 在 GMAT 考试中 DS 题均为：

A 为(1)充分，(2)不充分。

B 为(1)不充分，(2)充分。

C 为(1)和(2)在一起充分，但分别不充分。

D 为(1)和(2)自己分别充分。

E 为(1)和(2)在一起也不充分。

在做此类题的过程中：

先看条件(1)，只要(1)充分，答案不是 A 就是 D；再看条件(2)，只要(2)充分，答案不是 B 就是

D；如果(1)(2)都充分，则答案一定是 D；如果一个充分一个不充分，答案就是 A 或者 B

(只要(1)不充分，答案肯定不是 A 或者 D)；C 是好的，E 是坏的



做此类题的技巧：

- (1) 不要求出具体值，只需要知道求出即可。
- (2) 选 C 时应该注意是否可选 A 或 B
- (3) 唯一性。($x=2$ 和 $x^2=4$)

练习题

1.If \$1,000 is deposited in a certain bank account and remains in the account along with any accumulated interest, the dollar amount of interest, I , earned by the deposit in the first n years is given by the formula $I=1000 \left[\left(1 + \frac{r}{100}\right)^n - 1 \right]$, where r percent is the annual interest rate paid by the bank. Is the annual interest rate paid by the bank greater than 8 percent?

(1) The deposit earns a total of \$210 in interest in the first two years.

(2) $\left(1 + \frac{r}{100}\right)^2 > 1.15$

【答案】A

【思路】(1) $210 = 1000 \left[\left(1 + \frac{r}{100}\right)^2 - 1 \right]$ $0.21 = 1 + \frac{r}{50} + \frac{r^2}{10000} - 1$ $0.21 = \frac{r}{50} + \frac{r^2}{10000}$

求得166.4..... The deposit earns a total of \$210 in interest in the first two years.
是的.....sufficient

(2) $\left(1 + \frac{r}{100}\right)^2 > 1.15 = 1 + 2\frac{r}{100} + \left(\frac{r}{100}\right)^2 > 1.15 = 10000 + 200r + r^2 > 11500 = (r + 100)^2 > 11500$
 $= r + 100 > 105$ (大约值) 所以 r 可能大于也可能小于 8.....insufficient

2.If a , b , k , and m are positive integers, is a^k a factor of b^m ?

(1) a is a factor of b .

(2) $k \leq m$

【答案】C

【思路】

问题是要问 b^m / a^k 是否可以没有余数remainder

(1) b/a 没有余数是不够的

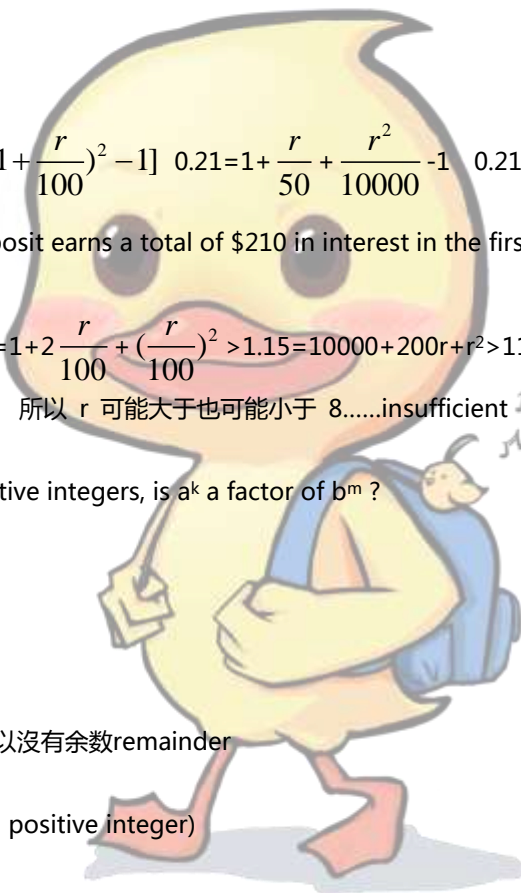
例如 $b=ax$ (x being an positive integer)

$b^m = (ax)^m = a^m x^m$

$b^m / a^k = (a^m x^m) / a^k$,

$a = 2$ and $b = 10$

2 has one factor 2 and say $k = 2$ and $m = 2$



In this case $2^2 = 4$ is a factor of 100, However, if $m = 1$ then 4 is not a factor of 10.
we don't know the value of k & m .

(2) $m-k \geq 0$

No information about a & b .

(1)+(2)

$$b^m / a^k = (a^m x^m) / a^k = x^m a^{(m-k)}$$

$$= (\text{integer}) \times (\text{integer} \geq 1)$$

So ans = C

3. If Company M ordered a total of 50 computers and printers and Company N ordered a total of 60 computers and printers, how many computers did Company M order?

(1) Company M and Company N ordered the same number of computers.

(2) Company N ordered 10 more printers than Company M.

【答案】E

【思路】(1) company M 和 company N 订购同样数量的电脑,但题目并未告知companyM,N 订购 computers和printers,不充份 (2)假设订购printers= x , x , $x-10$ 为M,N订购printers, 总数量为110,但并不是printers总数,所以不充份

(1) +(2)还是无法得知电脑或是printers的总数,故选E

4. Linda put an amount of money into each of two new investments, A and B, that pay simple annual interest. If the annual interest rate of investment B is 1.5 times that of investment A, what amount did Linda put into investment A?

(1) The interest for 1 year is \$50 for investment A and \$150 for investment B.

(2) The amount that Linda put into investment B is twice the amount that she put into investment A.

【答案】E

【思路】假设 investmentsA= X , investmentsB= y , interest rateA= $X1$, interest rateB= $Y1$, $1.5X1=Y1$

(1) $X(X1)=50$, $Y(Y1)=150$, $Y(1.5X1)=150$, 无法得investment,不充份

(2) $Y=2X$ 无法得investment,不充份

(1)+(2) $2X(1.5X1)=150$, $Y(1.5X1)=150$ 无法得 investment 故选E

5. If x , y , and z are integers and $xy + z$ is an odd integer, is x an even integer?

(1) $xy + xz$ is an even integer.

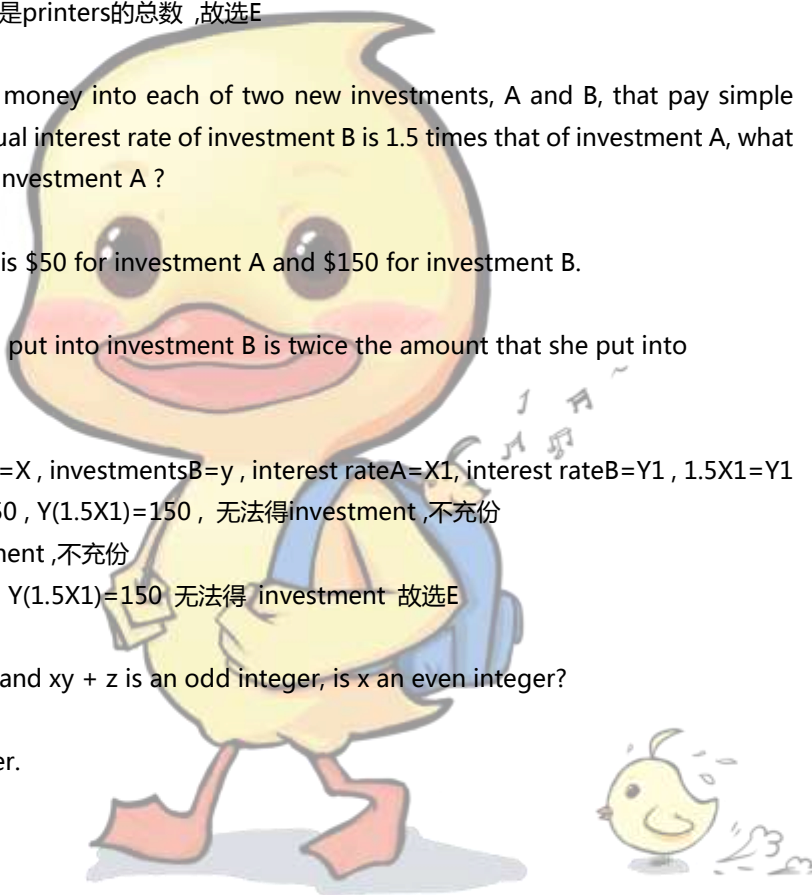
(2) $y + xz$ is an odd integer.

【答案】A

【思路】 $xy + z = \text{odd integer}$, $xy(\text{odd})+z(\text{even})= \text{odd integer}$, $xy(\text{even})+z(\text{odd})=\text{odd integer}$

(1) $xy + xz = \text{even}$, $xy(\text{even})+xz(\text{even})=\text{even}$, so z 应该为odd, x 为even, xz 才为even,

$xy(\text{odd})+xz(\text{odd})=\text{even}$ z 为even, xz 不等于odd, 充份



(2) $y + xz = \text{odd integer}$, $y(\text{odd}) + xz(\text{even}) = \text{odd}$, $z = \text{even}$, $x = \text{even}$,
 $y(\text{even}) + xz(\text{odd}) = \text{odd}$, $z = \text{odd}$, $x = \text{odd}$, 所以不充份

6. On the number line, the distance between x and y is greater than the distance between x and z . Does z lie between x and y on the number line?

(1) $xyz < 0$

(2) $xy < 0$

【答案】E

【思路】(1) $xyz < 0$, 可能为 $++-, -++$, $---$, 所以不充份

(2) $xy < 0$, $-+, +-$, 不充份 (1)+(2) = $-++$, $+-+$, 不充份 选E

7. Is $|x - y| > |x| - |y|$?

(1) $y < x$

(2) $xy < 0$

【答案】B

【思路】(1) $y < x$, $(y = -, x = +)$, $(y = +, x = +)$, $(y = -, x = -)$, 当 $x = +, y = -$, $|x - y| > |x| - |y|$, 当
 $(y = +, x = +)$ 时, $|x - y| = |x| - |y|$, 所以不充份

(2) $xy < 0$, $(y = -, x = +)$ or $(y = +, x = -)$, $|x - y| > |x| - |y|$, 充份

8. If the operation Δ is one of the four arithmetic operations addition, subtraction, multiplication, and division, is $(6 \Delta 2) \Delta 4 = 6 \Delta (2 \Delta 4)$?

(1) $3 \Delta 2 > 3$

(2) $3 \Delta 1 = 3$

【答案】A

【思路】

$(6 \Delta 2) \Delta 4 = 6 \Delta (2 \Delta 4)$ 乘或加, 两式相等. 除或减, 两式不相等

(1) Δ 可以是乘或加

(2) Δ 可以是乘或除

9. If the product of the three digits of the positive integer k is 14, what is the value of k ?

(1) k is an odd integer.

(2) $k < 700$

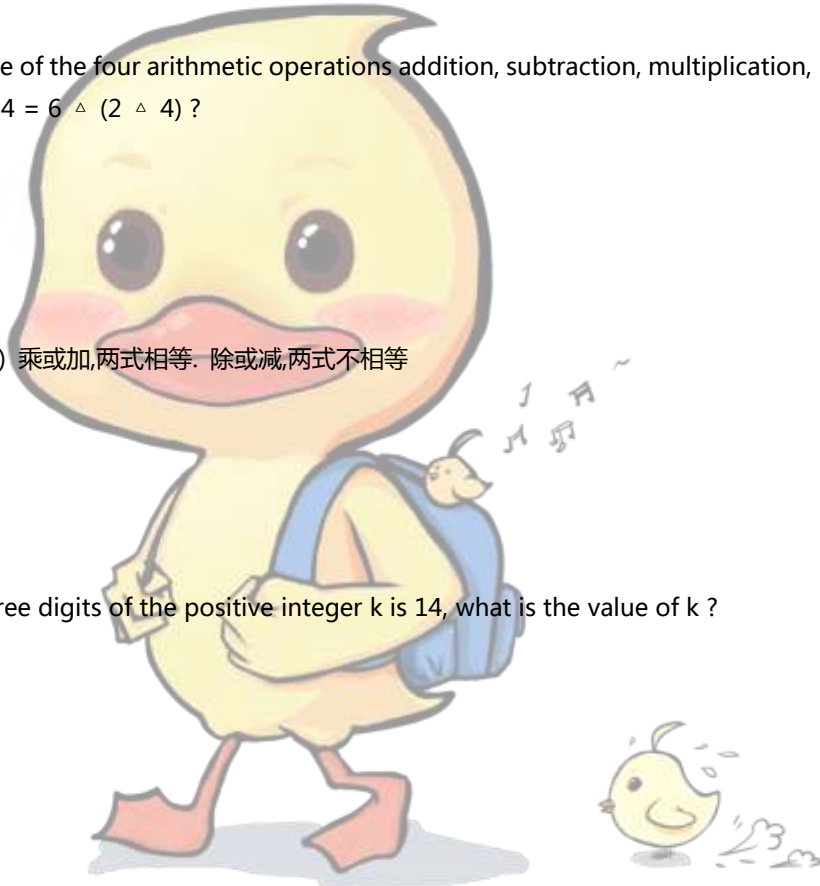
【答案】E

【思路】 $14 = 1 \times 2 \times 7$

(1) $\{271, 721, 127, 217\}$

(2) $\{172, 127, 271, 217\}$

(1)+(2) $\{271, 127, 127\}$



10. Sets A, B, and C have some elements in common. If 16 elements are in both A and B, 17 elements are in both A and C, and 18 elements are in both B and C, how many elements do all three of the sets A, B, and C have in common?

(1) Of the 16 elements that are in both A and B, 9 elements are also in C.

(2) A has 25 elements, B has 30 elements, and C has 35 elements.

【答案】A

【思路】

(1) $A \cap B = 16, C = 9$

$A \cap B = [A \cap B - C] + [A \cap B \cap C]$, 可得 $A \cap B \cap C = 9$, 充分

(2) $A = 25, B = 30, C = 35$

$A \cup B \cup C = A + B + C - A \cap B - B \cap C - C \cap A + A \cap B \cap C$

$A \cup B \cup C = 25 + 30 + 35 - 16 - 18 - 17 + A \cap B \cap C$

我们不知道 $A \cup B \cup C$ 是多少, 所以无法算出 $A \cap B \cap C$, 不充分

难题冲刺

1. The cost of a square slab is proportional to its thickness and also proportional to the square of its length. What is the cost of a square slab that is 3 meters long and 0.1 meter thick?

(1) The cost of a square slab that is 2 meters long and 0.2 meter thick is \$160 more than the cost of a square slab that is 2 meters long and 0.1 meter thick. $4y2x \ 32xy$

(2) The cost of a square slab that is 3 meters long and 0.1 meter thick is \$200 more than the cost of a square slab that is 2 meters long and 0.1 meter thick.

【答案】D

【思路】

根据这题意：

因为是square, 所以long and width是一样的.

1. a square slab is proportional to its thickness

假设thickness = $T \Rightarrow T * k_1$

2. also proportional to the square of its length

假设length = $L \Rightarrow L * k_2$

成本 $C = T * k_1 * L^2 * k_2^2$

$\Rightarrow C = T * L^2 * (k_1 * k_2^2)$ 由于不管length or thickness怎样变动, 常数(constant)都不会变, 为他是成比例的, 因此简化为 $(k_1 * k_2^2) = K \Rightarrow T * L^2 * K$

$C = \text{cost}, k = \text{constant}, T = \text{thickness}, L = \text{length}$

$C = k * T * L^2$

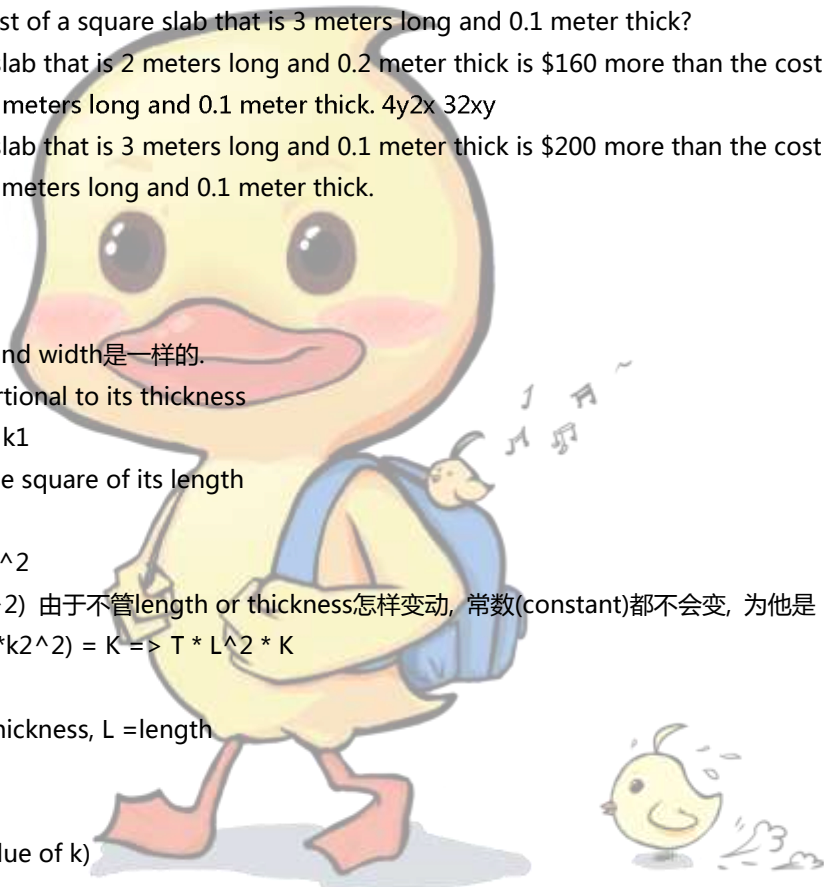
so when $T = 0.1, L = 3, C = ?$

(we need to know the value of k)

(1) The cost of a square slab that is 2 meters long and 0.2 meter thick

$\rightarrow C = k * 0.2 * 2^2$

the cost of a square slab that is 2 meters long and 0.1 meter thick



$$\rightarrow C = k * 0.1 * 2^2$$

difference of two costs=160 (given)

$$\rightarrow (k * 0.2 * 2^2) - (k * 0.1 * 2^2) = 160$$

so you can get the value of $k=400$ ---> **sufficient**

(2) The cost of a square slab that is 3 meters long and 0.1 meter thick

$$\rightarrow C = k * 0.1 * 3^2$$

the cost of a square slab that is 2 meters long and 0.1 meter thick

$$\rightarrow C = k * 0.1 * 2^2$$

difference of two costs=200 (given)

$$\rightarrow (k * 0.1 * 3^2) - (k * 0.1 * 2^2) = 200$$

so you can get the value of $k=400$ ---> **sufficient**

2. Some of the students enrolled at College T are part-time students and the rest are full-time students. By what percent did the number of full-time students enrolled at College T increase from the fall of 1999 to the fall of 2000 ?

(1) There were 50 more full-time students enrolled at College T in the fall of 2000 than in the fall of 1999.

(2) The total number of students enrolled at College T increased by 5 percent from the fall of 1999 to the fall of 2000.

【答案】E

【思路】2000 part-A, 2000 full-B, 1999 part-C, 1999 full-D, 问 $\frac{B-D}{D} = ?$

(1) $B=D+50$ ----- insufficient

(2) $\frac{(A+B)-(C+D)}{C+D} = \frac{5}{100}$ ----- insufficient

(1)(2) $\frac{(A+D+50)-(C+D)}{C+D} = \frac{5}{100}$ ----- insufficient

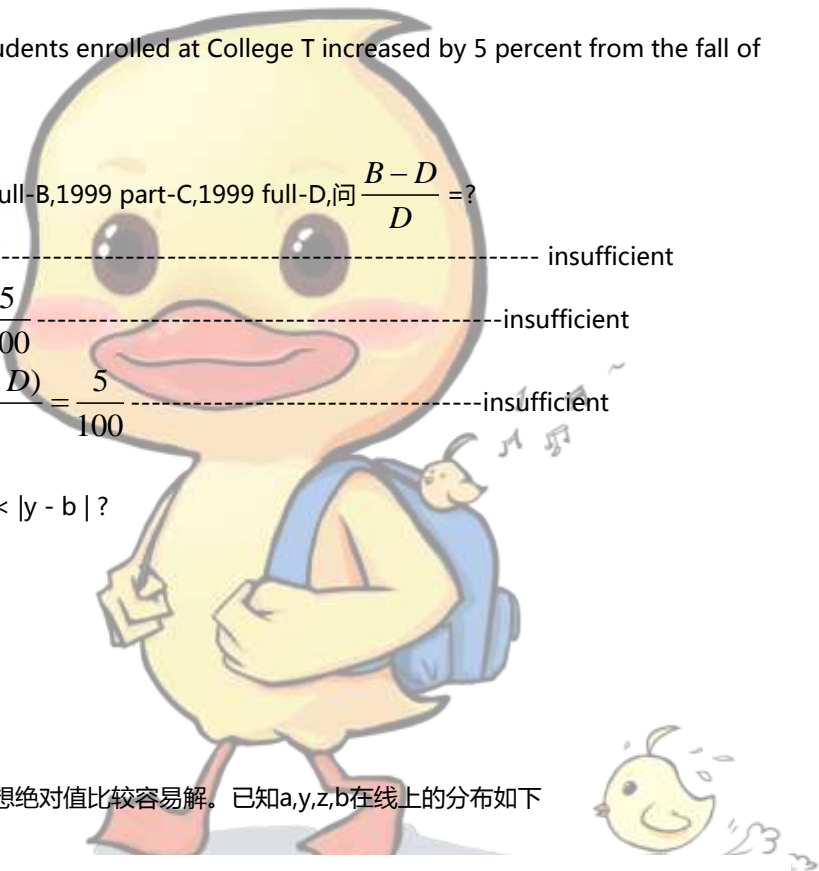
3. If $a < y < z < b$, is $|y - a| < |y - b|$?

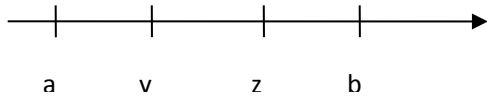
(1) $|z - a| < |z - b|$

(2) $|y - a| < |z - b|$

【答案】D

【思路】这题用距离的概念来想绝对值比较容易解。已知a,y,z,b在线上的分布如下





(1) 在此条件下知道 $\overline{az} < \overline{zb}$ ($\overline{yb} = \overline{yz} + \overline{zb}$ 且 $\overline{az} = \overline{ay} + \overline{yz}$) $\Rightarrow \overline{ay} < \overline{az} < \overline{zb} < \overline{yb} \Rightarrow |y - a| < |y - b| \leftarrow$ 选项(1)成立

(2) 在此条件下知道 $\overline{ay} < \overline{zb} \Rightarrow \overline{ay} < \overline{yb}$ (因 $\overline{yb} > \overline{zb}$) $\Rightarrow |y - a| < |y - b| \leftarrow$ 选项(2)成立

4. If r and t are positive integers, is rt even?

(1) $r + t$ is odd.

(2) r^t is odd.

【答案】A

【思路】

(1) $r+t$ is odd, 当 r is odd, s is even; 另一个情形 r is even, s is odd (表示 r 和 s 不可能同时为偶数或是奇数) 所以 rt is even (odd \times even = even; even \times odd = even), sufficient

(2) 偶数的奇次方或是偶数的偶次方都是偶数; 但奇数的偶次方 = 偶数, 如: $3^2 = 9$ 或是奇数的奇次方, 如:

$5^3 = 125$ 无法知道 r^t 情形下, t 是 odd or even, $r \times s =$ even or odd, so insufficient

5. Is $y < \frac{x+z}{2}$?

(1) $y - x < z - y$

(2) $z - y < \frac{z-x}{2}$

【答案】D

$$y - x < z - y$$

【思路】(1) $2y < x + z$

$$y < \frac{x+z}{2}$$



$$z - y < \frac{z - x}{2}$$

$$-y < \frac{z - x}{2} - z$$

$$(2) \quad -y < \frac{z - x - 2z}{2}, -y < \frac{-z - x}{2} \text{ (同乘 } -1 \text{)}$$

$$y > \frac{z + x}{2}$$

所以可知 y is not $< \frac{x+z}{2}$, sufficient



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