新东方在线
GRE 数学教程

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目录

Chapter 1 Arithmetic

Objectives:

A. 掌握基本算数运算及概念
B. 了解常见运算涉及的数学术语及题型
C. 掌握基本的做题套路（百分数增减，比例表达）和速度提升方法

1.1 Integers 整数

1.1.1 Key terms' definition

基本运算：
add/ plus 加→sum 和
subtract/ minus 减 → difference 差
multiply/ time 乘 →product 积
divide 除 → quotient 商 divided by
divisible 可被整除的 divisible by
remainder 余数

integer 整数
positive number 正数
negative number 负数
factor=divisor 因数
multiple 倍数
least common multiple 最小公倍数
greatest common divisor/ greatest common factor 最大公因数
odd integer 奇数
even integer 偶数
prime number 质数
prime factor / divisor 质因数
composite number 合数
digit 数位
consecutive number 连续数
round to the nearest 四舍五入

1.1.2 Strategies

余数需要注意：80 divided by 100 is 0 remainder 80.

除法的表达：The result of 19 divided by 7 is the quotient 2 with remainder 5, or simply "2 remainder 5.

乘法的表达：2/3 as many A as B → 表示 A=2/3*B

twice as many A as B表示 A=2*B

The least common multiple: 两个非零整数a和b的最小公倍数是a和b共同倍数中最小的正整数。例如30和75的最小公倍数是150.

The greatest common divisor (or **greatest common factor):** 两个非零整数a和b的最大公因数是a和b共同因数中最大的正整数，例如30和75的最大公因数是15.

prime number: 质数是所有大于 1 的整数中，只有 1 和它自己两个因数的整数。前十个质数是 2，3，5，7，11，13，17，19，23，和 29.

prime factor/prime divisor：质因数是一个数的因数中属于质数的数。下图为分解质因数的例子。

$$12 = (2)(2)(3) = (2^2)(3)$$
$$14 = (2)(7)$$
$$81 = (3)(3)(3)(3) = 3^4$$
$$338 = (2)(13)(13) = (2)(13^2)$$
$$800 = (2)(2)(2)(2)(2)(5)(5) = (2^5)(5^2)$$
$$1{,}155 = (3)(5)(7)(11)$$

1.1.3 Exercise

1） If 7.36 is rounded to the nearest tenth and the rounded number is multiplied by 3, what is the result?
A. 21.0
B. 21.9
C. 22.0
D. 22.1
E. 22.2

answer: 7.36 被四舍五入到十分位是 7.4，随后用得到的结果 7.4 乘以三，答案是 22.2，选择 E。

2) What number exceeds 50 percent of itself by 10?
A. 5
B. 10
C. 15
D. 20
E. 60

answer: D，注意 exceed by 表达超过的幅度。

1.2 Fractions& Decimals 分数&小数

1.2.1 Key terms' definition

fraction 分数

numerator 分子
denominator 分母
common denominator 公分母
reciprocal 倒数
mixed number 带分数
improper fraction 假分数

decimal 小数
units digit / ones digit 个位
tens digit 十位
hundreds digit 百位
thousands digit 千位
tenths digit 十分位
hundredths digit 百分位
thousandths digit 千分位

rational numbers 有理数
irrational number 无理数

1.2.2 Strategies

rational numbers: 有理数指所有可以表示为分数式的数字。

terminate or repeat decimal: 所有会终止或者循环的小数，都可以是有理数；如果一个小数既不终止（**terminate**）也不循环（**repeat**），那么它是一个无理数（**irrational number**）.

1.3 Exponents and Roots 指数及根

1.3.1 Key terms' definition

base 底数
exponent 指数
power 次方
square 平方
square root 平方根
cube root 立方根
fourth root 四次方根

1.3.2 Strategies

power 次方：2^5=the fifth power of 2.

root: 不同次根的表达是需要掌握的重点之一。比如 **square root** 平方根，**cube root** 立方根，以及上升到 4 次以上的表达为序数词+root，例如 **fourth root**.

对于奇数次的根（odd-order roots），每一个数字 n 来说都有且仅有一个根，不论 n 是正数还是负数；

对于偶数次的根（even-order roots），每一个正数有且仅有两个根，每一个负数没有根。

例如：8 有且仅有一个立方根 2，但是有两个四次方根；-8 有且仅有一个立方根-2，但是因为是负数，它没有四次方根。

1.4 Real Numbers 实数

1.4.1 Key terms' definition

real number 实数
real number line 实数轴
less than 小于
greater than or equal to 大于等于
absolute value 绝对值
interval 区间
inclusive 包含的

1.4.2 Strategies

triangle inequality: $|a+b| \le |a|+|b|$

interval:不等式中包含的所有取值组成的集合。

常见的表达中，需要掌握**大于等于(more than or equal to),小于等于（less than or equal to）.**

1.5 Ratio& Percent 比例/百分比

1.5.1 Key terms' definition

ratio 比例
proportion 比例式
percent 百分比
percent change 百分比变化
percent increase/decrease 百分比增/减

1.5.2 Strategies

percent change: 运算中最重要的是记得百分比变化计算式的分母一定是**原始数值(original value).** 例如，100 下降到 80 的 percent change 计算方法是（100-80）/100=20%。

increase by x percent / decrease by x percent: 增长/降低了百分之 x.

1.5.3 Exercise

The population of Country *X* for 1980 was *p*. The population of Country *X* increased by 3.8 percent in each of the next two years.

Quantity A: The population of Country *X* for 1982.
Quantity B: 1.076*p*

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.

answer: A

1.6 New Definition Problem 新定义题

1.6.1 Strategies

- 1. 分析定义-长难句
- 2. 看懂例子
- 3. 套用条件

1.6.2 Exercise

1) An integer k is a "half square" if 2k is the square of a positive integer. For example, 18 is a half square because $2X18=36=6^2$. What is the smallest half square that is greater than 100?

answer： 本题新定义为"half square"，注意学生不应纠结翻译字面，而是理解实际的定义含义，即一个数如果两倍是一个平方数，那么该数就是一个"半平方数"，要求大于100的最小的半平方数，应寻找大于200的最小的平方数，但225除以二无法得到整数，所以应该是第二小的平方数256的一半，答案为128.

2）A number is a palindrome if it can be written the same backwards and forwards (6336 is an example of a palindrome). What number divides into every 4 digit palindrome?
A. 2
B. 3
C. 7
D. 11

answer：本题的新定义是palindrome回文数字，即从左往右和从右往左一样的数字，问句问哪个数字一定可以整除回文数字，可以用xyyx来表示回文数字，则其实际大小为1000x+100y+10y+x=1001x+110y,明显可被11整除。答案为D

Mixed Practice 1

1. Which of the integers 2, 9, 19, 29, 30, 37, 45, 49, 51, 83, 90, and 91 are prime numbers?

2. A particular stock is valued at $40 per share. If the value increases by 20 percent and then decreases by 25 percent, what will be the value of the stock per share after the decrease?

3. When the positive integer n is divided by 3, the remainder is 2 and when n is divided by 5, the remainder is 1. What is the least possible value of n ?

Answers:

1. 2, 19, 29, 37 and 83

2. $36 per share

3. 11

Mixed Practice 2

1. n is an integer.

Quantity A: $(-1)^n(-1)^{n+2}$

Quantity B:1

A. Quantity Ais greater.
B. Quantity B is greater.
C. The two quantities are equal
D. The relationship cannot be determined from the information given.

2. x is an integer greater than 3.

Quantity A: The number of even factors of $2x$
Quantity B: The number of odd factors of $3x$

A. Quantity Ais greater.
B. Quantity B is greater.
C. The two quantities are equal
D. The relationship cannot be determined from the information given.

3. What is the remainder when 3^{283} is divided by 5?
A. 0
B. 1
C. 2
D. 3
E. 4

4. If $x < y$, which of the following must be true?

A. $2x < y$
B. $2x > y$
C. $x^2 < y^2$
D. $2x-y < y$
E. $2x-y < 2xy$

5. If n and m are positive integers and m is a factor of 2^6, what is the greatest possible number of integers that can be equal to both 3n and $2^6/m$?
A. Zero
B. One
C. Three
D. Four
E. Six

6. Amy and Jed are among the 35 people, who are standing in a line, one behind the other, waiting to buy movie tickets. The number of people in front of Amy plus the number of people behind Jed is 24. If there are 15 people behind Amy, including Jed, how many people are in front of Jed?
A. 23
B. 25
C. 27
D. 29
E. 31

7. The discounted price of a certain suit is 20 percent less than the original price of the suit. If the discounted price of the suit plus a sales tax of 5 percent of the discounted price equals $67.20, what was the original price of the suit?
A. $ 70.50
B. $73.90
C. $76.00
D. $79.80
E. $80.00

8. Greg's weekly salary is $187, which is 15 percent less than Karla's weekly salary. If Karla's weekly salary increases by 10 percent, by what percent must Greg's weekly salary increase in order to equal Karla's new weekly salary?
Give your answer to the nearest tenth of a percent.

9. xy>o
Quantity A: x^4y^3
Quantity B: 0
A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

10. Which of the following pairs of integers have reciprocals whose sum is either less than 1/3 or greater than 1/2?
Indicate all such pairs.
□ 1 and 14
□ 3 and 12
□ 5 and 10
□ 7 and 8

11. In 1988 Mr. Smith's annual income was greater than Mrs. Smith's annual income. In 1989 Mr. Smith's annual income decreased by *p* percent. Whereas Mrs. Smith's annual income increased by *p* percent. *p*>0
Quantity A: Mr. and Mrs. Smith's combined annual income in 1988
Quantity B: Mr. and Mrs. Smith's combined annual income in 1989

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

12. The original value of machine X is V dollars, while the original value of machine Y is $2V$ dollars. Both machines depreciate in value at a constant rate of 10 percent of their original value per year.
Quantity A: The value of machine X after 3 years
Quantity B: The value of machine Y after 6 years

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

13. A number is considered "odd-mult" if it is the product of exactly two consecutive odd numbers. How many positive numbers less than 400 are "odd-mult"?

14. The 20 people at a party are divided into n mutually exclusive groups in such a way that the number of people in any group does not exceed the number in any other group by more than 1.
Quantity A: The value of n if at least one of the groups consists of 3 people
Quantity B: 6

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

15. If n is a positive integer, then n^+ denotes a number such that $n < n^+ < n+1$.
Quantity A: $20^+/4^+$
Quantity B: 5^+

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

16. Both P and Q are positive numbers, and S is a negative number. Which of the following fractions could be undefined?
A. P/Q+S
B. Q/P+S
C. S/P+Q
D. Q/S-P
E. S/P-Q

17. For a certain event, 148 people attended. If all 148 had paid full admission price, the total revenue would be three times the cost of sponsoring the event. (Admission price was the only source of revenue.) As it happens, only 50 paid the full admission price, and the others paid nothing.
Quantity A: the total revenue
Quantity B: the cost of sponsoring the event

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

18.
$$\begin{array}{r} A\ A\ B \\ \times \qquad B \\ \hline C\ B\ 5\ B \end{array}$$

If A,B and C represent different digits in the multiplication, then A+B+C=

A. 9
B. 12
C. 14
D. 15
E. 17

19.

Quantity A: The number of distinct prime factors of 20^6
Quantity B: The number of distinct prime factors of 32^{10}

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

20. n is a positive integer, and k is the product of all integers from 1 to n inclusive. If k is a multiple of 1440, then the smallest possible value of n is__

A. 8
B. 12
C. 16
D. 18
E. 24

21. M is a positive two-digit number. When the digits are reversed, the number is N. If K = M + N, which of the following is true?

A. K must be even
B. K cannot be square
C. K cannot be divisible by 13
D. K must be divisible by 11
E. If M is even then K must be even

22. If n = 2×3×5×7×11×13×17, then which of the following statements must be true?

I. n^2 is divisible by 600
II. n + 19 is divisible by 19
III. $\frac{n+4}{2}$ is even

A. I only
B. II only
C. III only
D. I and III
E. None of the above

23. How many integers from 1 to 900 inclusive have exactly 3 positive divisors?

A. 10
B. 14
C. 15
D. 29
E. 30

Answers and Explanations

1. C
2. D
3. C
4. D
5. A
6. 注意站队时的前后人数是不包括May和Jed的。
7. E 本题虽然有plus 但是消费税率应该是乘以打折后的价格才是消费税
8. 29.4，本题先计算Karla的薪金，再用其1.1倍除以187得到倍数。需要注意percent less, increase by的数学含义,by表达增减幅度
9. xy 大于 0 有可能都为正数，也有可能都为负数，前一种情况 A 大于 B，后一种则反之。
10. ACD
11. A
12. B
13. 最接近 400 的是 19X21=399，因为是连续数字，从 1X3 开始总共 10 对
14. 本题需要理解题干的分组要求，每一组的人数之间差值不得大于 1。Quantity A 中若有一组人数为 3，其他组人数要么都为 3 但不可能，要么有 2 人组；要么有 4 人组。分情况讨论 2 人+3 人组，则最多有 9 组，最少 7 组，都大于 Quantity B；讨论 2 人+4 人组，则可以有 6 组-4 人组 4 个，2 人组 2 个，等于 Quantity B,所以答案无法判断。
15. 20^{+} 和 4^{+}各自的大小不定，其比值也会是不定的，Quantity B 也是不定值,其关系无法判断。
16. ABE. 数学术语 undefined 意为不可定义，分数不可定义的情况是分母为 0，正负数相加可能为 0,正数相减也可能为 0.
17. A
18. E. 题中的 B 可以是 1，5，6。 B=1 可以直接排除，将 5 代入发现无论 A 取何值，积的十位数都不为 5，排除；将 6 代入后，发现要使积的十位数为 5，则 A 可以取 2 or 7；将 2 代入后发现积的百位不为 6，排除；将 7 代入后发现都满足，且得出 C 为 4；所以 6+7+4=17.
19. A. 注意 distinct prime factor 是不同的质因数，也就是重复的都不算。则 20 只有 2 和 5，6 次方后有 12 个质因数；32 只有一个 2 是质因数，10 次方则有 10 个质因数。
20. A
21. D
22. E
23. A

Chapter 2 Algebra

2.1 Operations with Algebraic Expression 代数表达式的运算

2.1.1 objectives

A.代数表达式的定义

B.代数表达式的加减乘除运算

2.1.2 Key terms' definition

equation 等式，方程
linear equation 线性方程
quadratic equation 二次方程
algebraic term 代数项
like terms / similar terms 同类项
variable 变量
numerical coefficient 数字系数
literal coefficient 字母系数
constant 常数
identity 恒等式

2.1.3 Strategies

identity: 恒等式，e.g.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^2 - b^2 = (a+b)(a-b)$$

代数式运算的三大基本形式：

$3x + 5 = -2$	**A linear equation in one variable,** x
$x - 3y = 10$	**A linear equation in two variables,** x and y
$20y^2 + 6y - 17 = 0$	**A quadratic equation in one variable,** y

2.2 Rules of Exponents 指数运算法则

2.2.1 objectives

A.记忆指数运算的七大法则

B.熟练运用七大法则

2.2.2 Key terms' definition

base 底

exponent 指数

2.2.3 Strategies

1. $x^{-a} = \frac{1}{x^a}$
2. $(x^a)(x^b) = x^{a+b}$
3. $\frac{x^a}{x^b} = x^{a-b} = \frac{1}{x^{b-a}}$
4. $x^0 = 1$
5. $(x^a)(y^a) = (xy)^a$
6. $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
7. $(x^a)^b = x^{ab}$

2.2.4 Exercises 练习题

1. n is an integer.

Quantity A: $(-1)^n(-1)^{n+2}$
Quantity B: 1

E. Quantity A is greater
F. Quantity B is greater
G. The two quantities are equal.
H. The relationship cannot be determined from the information given

answer：根据公式第二条 Quantity A：负 1 的指数为 2（n+1），为偶数，那么负数的偶数次幂为正数，

所以 Quantity A 与 Quantity B 相等都为一 ，选 C。

2.3 Solving Linear Equation 线性方程

2.3.1 objectives

A.线性方程及方程组的求解

2.3.2 Key terms' definition

equivalent equations 等价方程式
a system of equations 方程组
simultaneous equations 联立方程

2.3.3 Strategies

equivalent equations: 拥有相同解的方程
substitution: 连立法
elimination: 消元法

2.4 Solving Quadratic Equations 二次方程求解

2.4.1 objectives

A. 熟练使用求根公式

2.4.2 Key terms' definition

Quadratic Equations 二次方程

2.4.3 Strategies

一元二次方程形如:

$ax^2 + bx + c = 0$

quadratic formula 求根公式:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

除了求根公式之外，还有一种方法，十字交叉/因式分解法（factoring）：

$$5x^2 + 3x - 2 = 0$$
$$(5x - 2)(x + 1) = 0$$

2.5 Solving Linear Inequalities 解一次不等式

2.5.1 objectives

A.掌握一次不等式求解

2.5.2 Key terms' definition

Inequalities 不等式

solution set 解集

2.5.3 Strategies 主要知识点

<	less than
>	greater than
≤	less than or equal to
≥	greater than or equal to

不等式两边同时乘/除以负数时需要反号.

2.6 Functions 函数

2.6.1 objectives

A.理解函数定义

2.6.2 Key terms' definition

function 函数

range 值域
domain 定义域

2.6.3 Strategies 主要知识点

函数一般形如：f(x) = 3x+5

2.7 Applications 函数应用

2.7.1 Key terms' definition

interest 利息
rate 率
interest rate 利率
principal 资本
profit 利润
margin 利润
single interest 单利
compound interest 复利
discount 折扣
original price 原价
list price 标价
sale price 卖价
purchasing price 买价
retail value 零售价
tax 税
mark up 涨价
mark down 降价

2.7.2 Strategies 主要知识点

Simple interest 单利模式：Simple interest is based only on the initial deposit, which serves as the amount on which interest is computed, called the principal, for the entire time period. If the amount P is invested at a simple annual interest rate of r percent, then the value V of the investment at the end of t years is given by the formula:

$$V = P\left(1 + \frac{rt}{100}\right)$$

compound interest 复利模式：In the case of compound interest, interest is added to the principal at regular time intervals, such as annually, quarterly, and monthly. Each time interest is added to the principal, the interest is said to be compounded. After each compounding, interest is earned on the new principal, which is the sum of the preceding principal and the interest just added. If the amount P is invested at an annual interest rate of r percent, compounded annually, then the value V of the investment at the end of t years is given by the formula:

$$V = P\left(1 + \frac{r}{100}\right)^t$$

If the amount P is invested at an annual interest rate of r percent, compounded n times per year, then the value V of the investment at the end of t years is given by the formula:

$$V = P\left(1 + \frac{r}{100n}\right)^{nt}$$

2.7.3 Exercise 练习

1. A mixture of 12 ounces of vinegar and oil is 40 percent vinegar, where all of the measurements are by weight. How many ounces of oil must be added to the mixture to produce a new mixture that is only 25 percent vinegar?

answer: 7.2

2. If an amount P is to be invested at an annual interest rate of 3.5 percent, compounded annually, what should be the value of P so that the value of the investment is $1,000 at the end of 3 years?

answer: 901.94

2.8 Coordinate Geometry 坐标几何

2.8.1 objectives

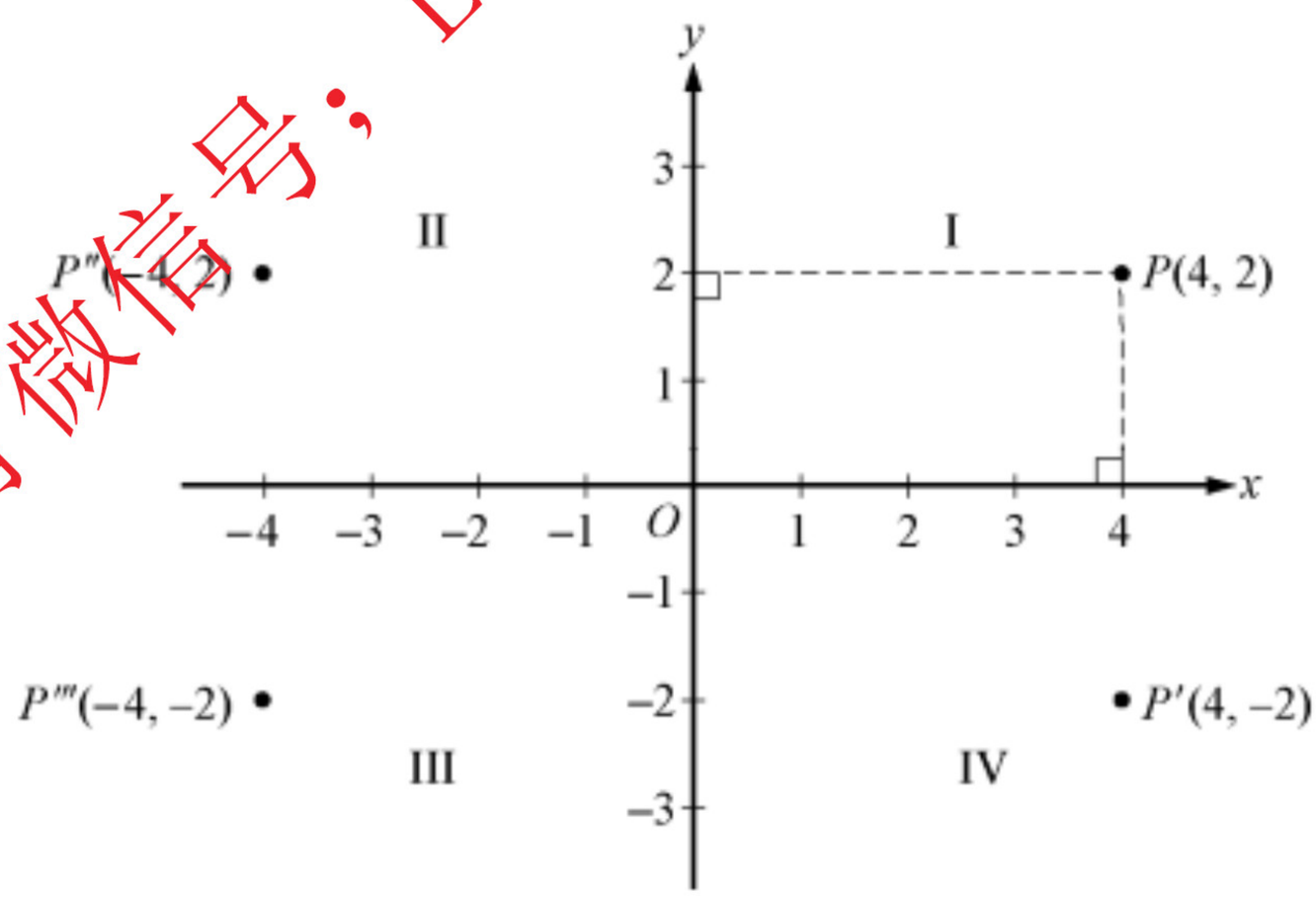
A. 理解定义

B. 熟悉利用图像解线性不等式方程组（线性规划）

2.8.2 Key terms' definition

coordinate system 坐标系
rectangular coordinate system 直角坐标系
xy-coordinate system / xy-plane 平面直角坐标系
origin 原点
x-axis x 轴
y-axis y 轴
x-coordinate / abscissa 横坐标
y-coordinate / ordinate 纵坐标
number line 数轴
quadrant 象限
slope 斜率
intercept 截距
symmetric 对称

2.8.3 Strategies 主要知识点



- P' is the reflection of P about the x-axis, or P' and P are symmetric about the x-axis.
- P'' is the reflection of P about the y-axis, or P'' and P are symmetric about the y-axis.
- P''' is the reflection of P about the origin, or P''' and P are symmetric about the origin

两点之间距离计算：

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

线性方程斜率计算：

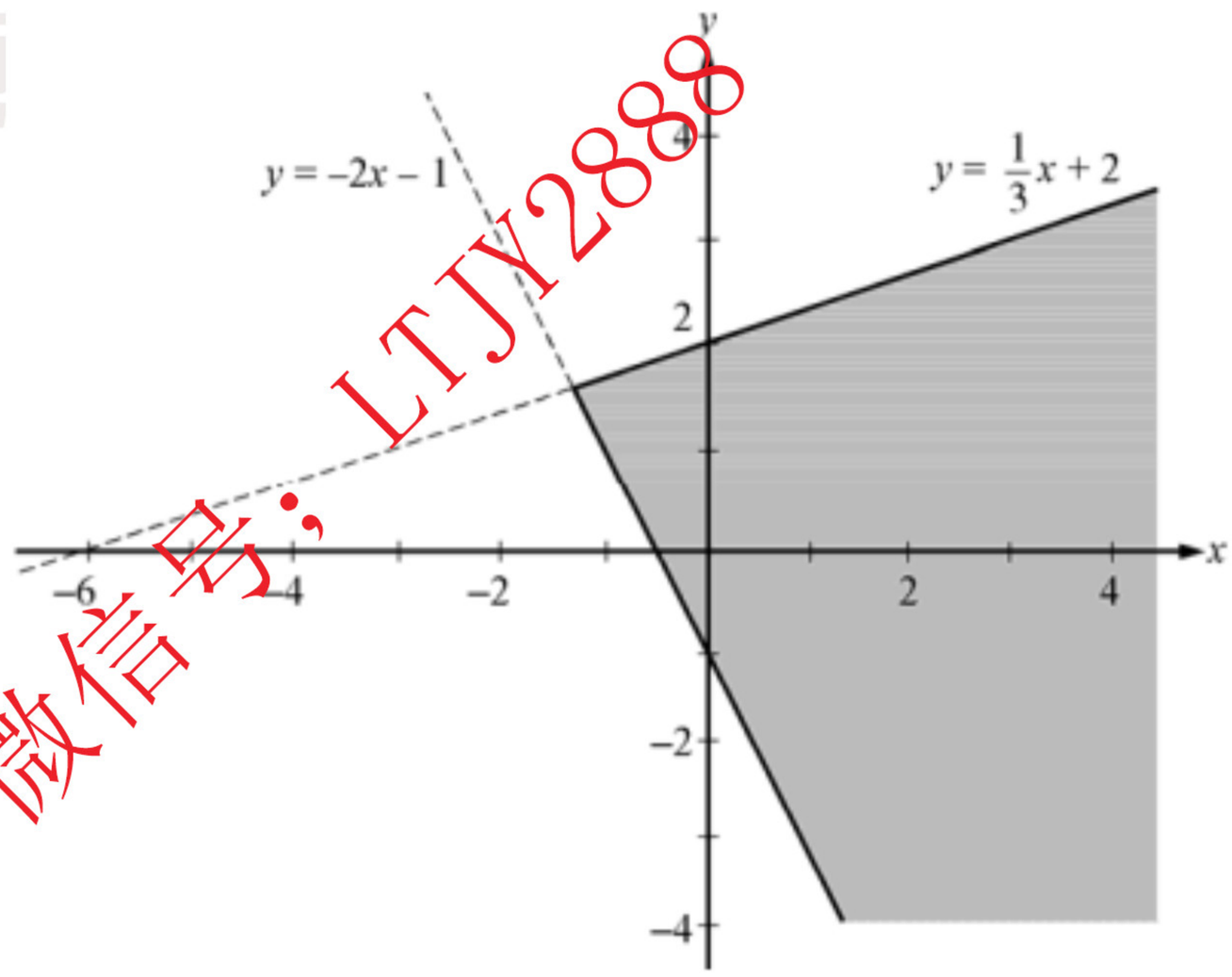
$$\frac{y_2-y_1}{x_2-x_1}$$

若两条直线平行则斜率相等，若两条直线垂直则斜率互为负倒数.

2.8.4Exercises 练习题

Example 2.8.3: Consider the following system of linear inequalities.

$$x-3y\geq -6$$
$$2x+y\geq -1$$



2.9 Graphs of Functions 函数图像

2.9.1 objectives

A. 熟悉各类函数图像特征

B. 熟悉图像的变化和位移与各系数变化的对应关系

2.9.2 Key terms' definition

parabola 抛物线
vertex 顶点
vertices 顶点（复数）
line of symmetry 对称轴
piecewise-defined function 分段函数

2.9.3 Strategies 主要知识点

1. 二次函数 y = ax2 + bx + c

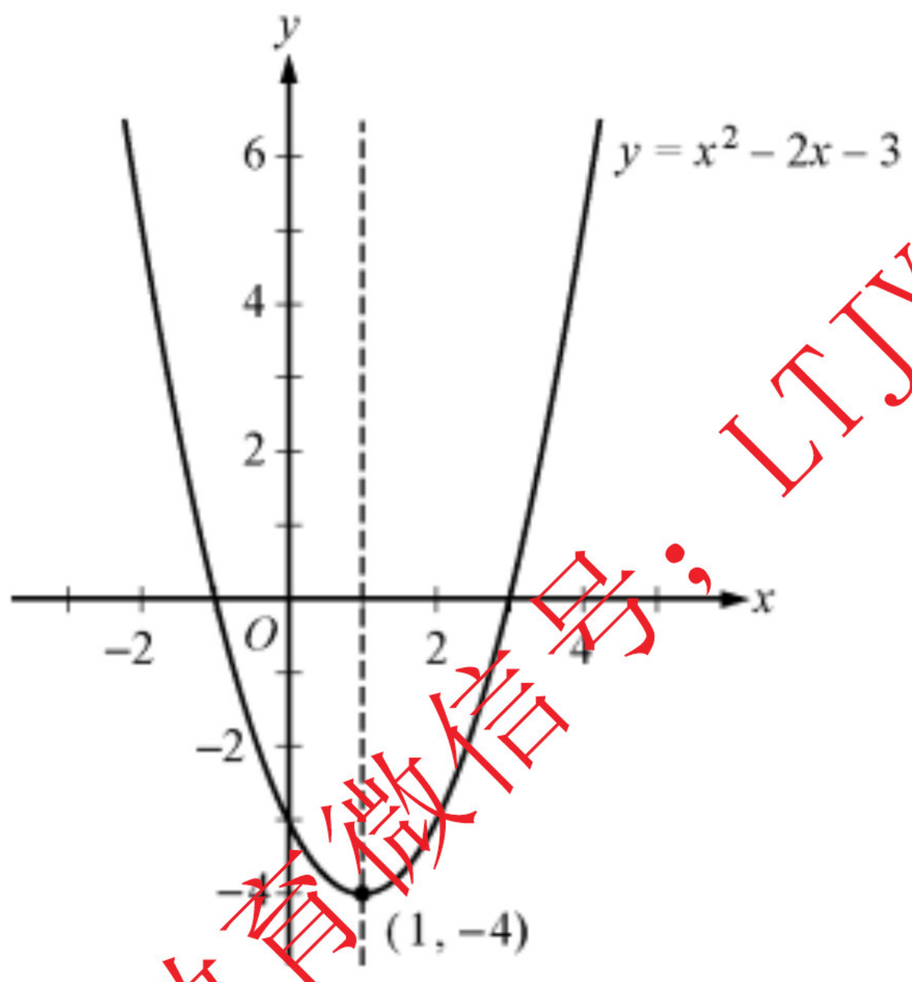
a 的正负决定开口方向（a>0 开口向上，a<0 开口向下）. c 为 y 轴截距.

line of symmetry 对称轴：

$$x = -\frac{b}{2a}$$

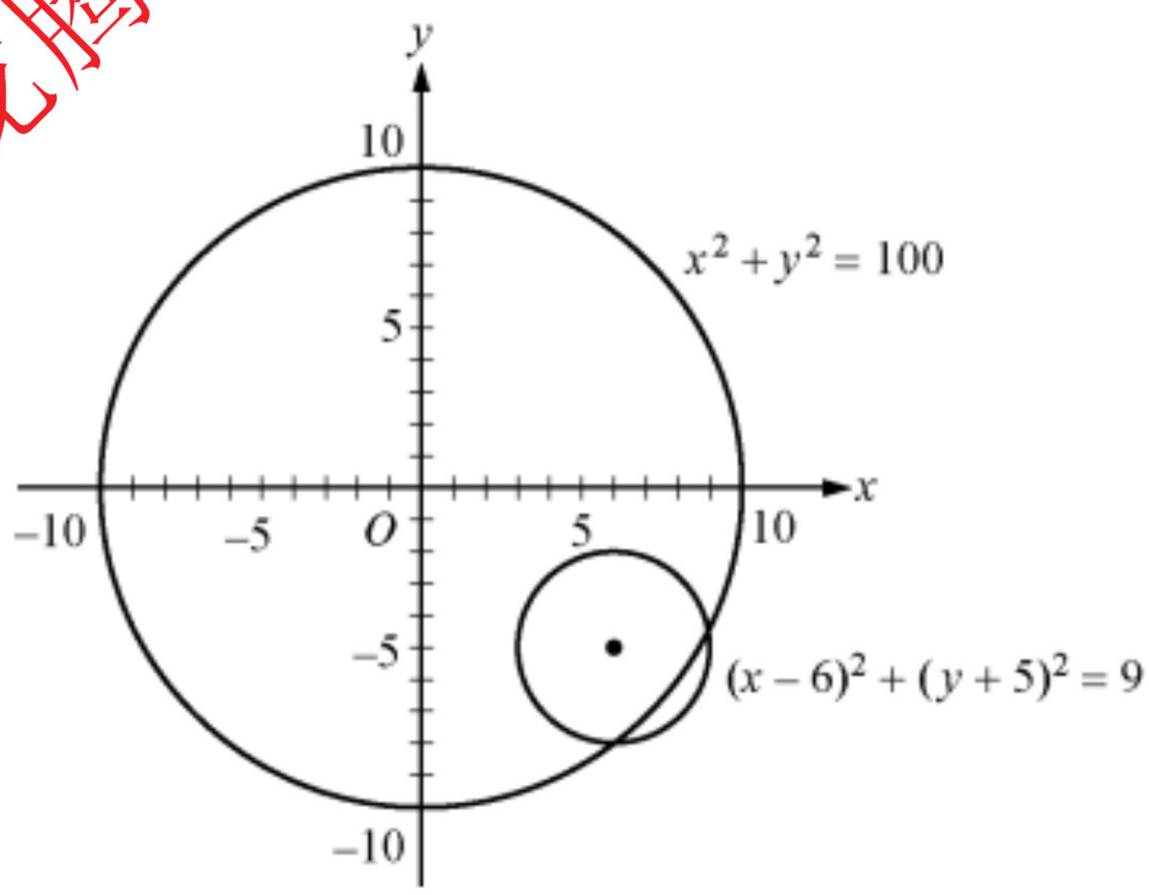
vertex 顶点：

$$\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$



2. 圆的方程：(x – a)2 + (y – b)2 = r2

r 为圆的半径，圆形坐标为(a , b)

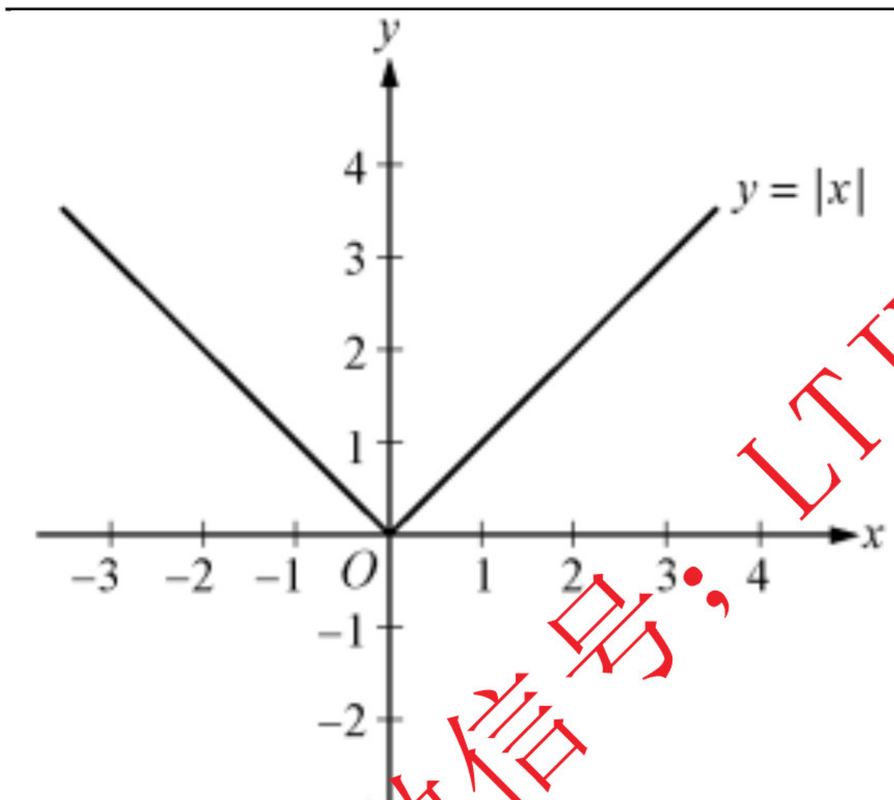


3. piecewise-defined function 分段函数：不同定义域区间上有不同函数方程.

例如绝对值函数：

h(x) = |x|

$\rightarrow\ h(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$



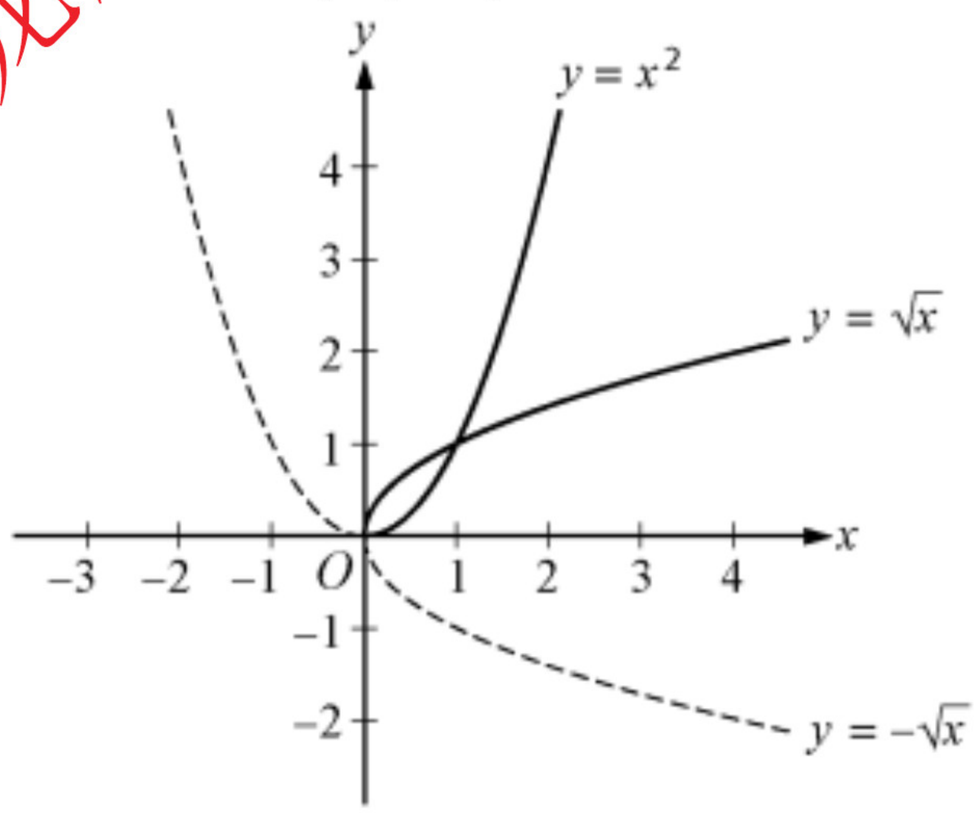
4. 幂函数 f(x)=xa

a>0 时，图像经过点(1,1)和(0,0).

函数的图像在区间[0,+∞)上是增函数.

当 α<0 时，图像都通过点(1,1).

6、图像在区间（0，+∞）上是减函数.



5. 函数图像的位移

In general, for any function h(x) and any positive number c, the following are true.

- The graph of h(x) + c is the graph of h(x) shifted upward by c units.
- The graph of h(x) - c is the graph of h(x) shifted downward by c units.
- The graph of h(x + c) is the graph of h(x) shifted to the left by c units.
- The graph of h(x - c) is the graph of h(x) shifted to the right by c units.
- The graph of ch(x) is the graph of h(x) stretched vertically by a factor of c if c > 1.
- The graph of ch(x) is the graph of h(x) shrunk vertically by a factor of c if 0 < c < 1.

Mixed Practice

1.

For each of the following functions, give the domain and a description of the graph $y = f(x)$ in the xy-plane, including its shape, and the x- and y-intercepts.

(a) $f(x) = -4$

(b) $f(x) = 100 - 900x$

(c) $f(x) = 5 - (x + 20)^2$

(d) $f(x) = \sqrt{x + 2}$

(e) $f(x) = x + |x|$

2.

In the coordinate system below, find the following.

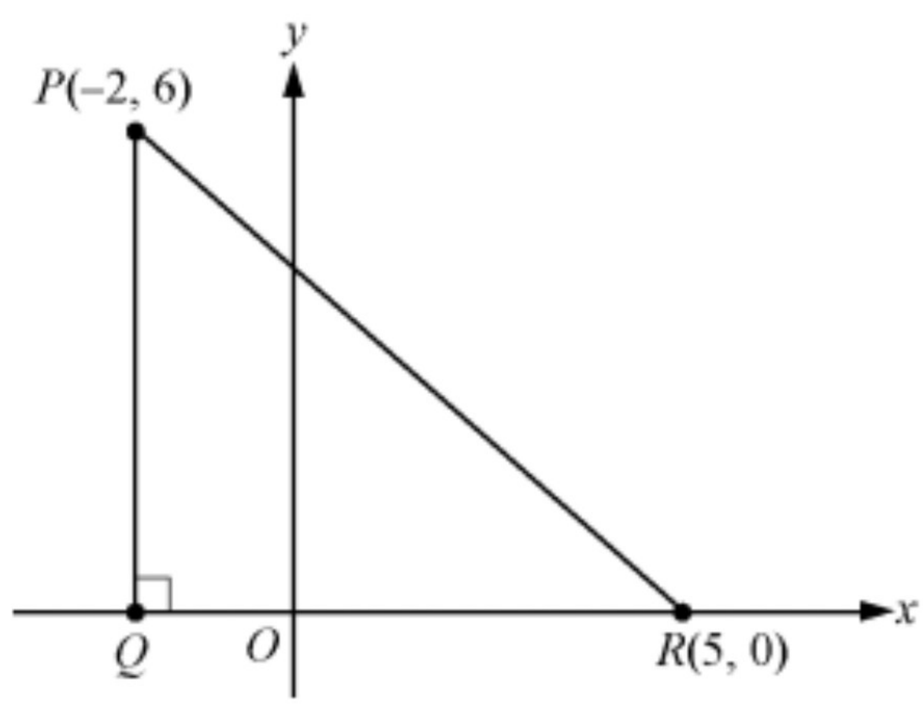
(a) Coordinates of point Q

(b) Lengths of PQ, QR, and PR

(c) Perimeter of $\triangle PQR$

(d) Area of $\triangle PQR$

(e) Slope, y-intercept, and equation of the line passing through points P and R



3.

Pat invested a total of \$3,000. Part of the money was invested in a money market account that paid 10 percent simple annual interest, and the remainder of the money was invested in a fund that paid 8 percent simple annual interest. If the interest earned at the end of the first year from these investments was \$256, how much did Pat invest at 10 percent and how much at 8 percent?

4.

Two cars started from the same point and traveled on a straight course in opposite directions for exactly 2 hours, at which time they were 208 miles apart. If one car traveled, on average, 8 miles per hour faster than the other car, what was the average speed of each car for the 2-hour trip?

Answers:

1.

(a) Domain: the set of all real numbers. The graph is a horizontal line with y-intercept -4 and no x-intercept.

(b) Domain: the set of all real numbers. The graph is a line with slope -900, y-intercept 100, and x-intercept $\frac{1}{9}$.

(c) Domain: the set of all real numbers. The graph is a parabola opening downward with vertex at $(-20, 5)$, line of symmetry $x = -20$, y-intercept -395, and x-intercepts $-20 \pm \sqrt{5}$.

(d) Domain: the set of numbers greater than or equal to -2. The graph is half a parabola opening to the right with vertex at $(-2, 0)$, x-intercept -2, and y-intercept $\sqrt{2}$.

(e) Domain: the set of all real numbers. The graph is two half-lines joined at the origin: one half-line is the negative x-axis and the other is a line starting at the origin with slope 2. Every nonpositive number is an x-intercept, and the y-intercept is 0. The function is equal to the following piecewise-defined function

$$f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

2.

(a) $(-2, 0)$

(b) $PQ = 6$, $QR = 7$, $PR = \sqrt{85}$

(c) $13 + \sqrt{85}$

(d) 21

(e) Slope: $-\frac{6}{7}$; y-intercept: $\frac{30}{7}$; equation of line: $y = -\frac{6}{7}x + \frac{30}{7}$, or $7y + 6x = 30$

3. \$800 at 10% and \$2,200 at 8%

4. 48 mph and 56 mph

Chapter 3 Geometry

Objectives:

A. 了解平面几何中线、角、三角形、四边形和多边形的性质，能够转化边角关系， 熟练计算各类特殊三角形中的边、角问题等。理解各类混合图形中的测算问题，并通过平面几何的计算能够解决实际生活中遇到的问题。
B. 熟悉圆的各项性质：半径、直径、弧长、圆心角和圆周角等内容， 熟练计算圆的周长面积。
C. 培养立体几何的空间概念， 能够借助辅助线将三维图形转化成平面几何中的三角形、四边形或者圆来计算， 能够运用简单的立体几何中表面积和体积等概念去解决实际生活中的现实问题。

3.1 Lines and Angles

3.1.1. Objective:

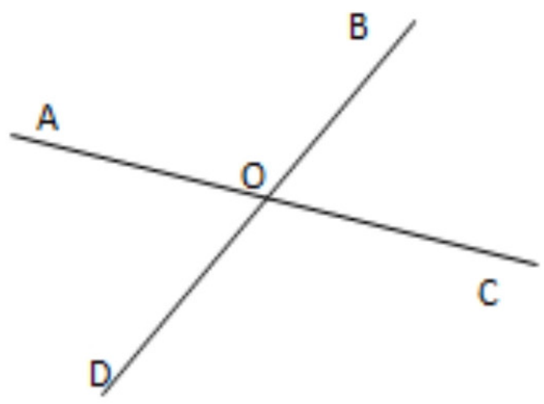
1. 理解线、线段、中点， 端点等基础概念
2. 了解各种类型的角，特别在两线平行时各类角的相互关系。
3. 能够结合特殊三角形、正方形、长方形和圆等图形，熟悉特定角度和边长的转化关系。

3.1.2. Key terms' definition

lines 直线
line segment 线段
ray 射线
endpoint 端点
congruent line segments 全等线段: Line segments that have equal lengths
midpoint 中点
bisect 平分

perpendicular 垂直
parallel 平行
length 长度

angle 角
degree 角度
intersect 相交
opposite angles / vertical angles 对顶角（∠AOD and ∠BOC)
congruent angles 等角： angles that have equal degrees



acute angle 锐角
right angle 直角
obtuse angle 钝角
interior angle 内角
exterior angle 外角
alternate interior angle 内错角
Corresponding angle 同位角
interior angles on the same side 同旁内角
adjacent angle 邻角
complementary 互余
complementary angle 余角
supplementary 互补
supplementary angle 补角

Figure not drawn to scale 图形不按比例绘制

3.1.3. Exercises

1. In the diagram, AE = 20, AD = 14, CD = 6 and EB = 17. What is the length of line segment BC?
Note: Figure not drawn to scale.

A.2
B.3
C.4
D.5
E.6

Answer: D

3.2 Polygons

3.2.1. Objective:

A. 掌握多边形的内角和计算公式，了解多边形的内角、对角线、边长和面积的计算方法。
B. 了解正多边形的各类特殊性质和特定角度计算。

3.2.2. Key terms' definition

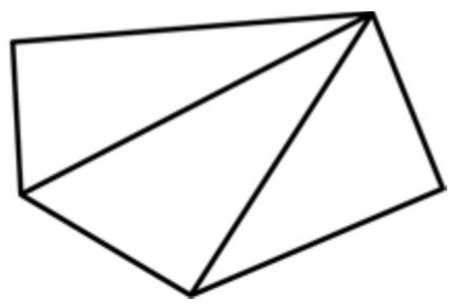
polygon 多边形
side 边
vertex 交点
vertices 交点（复数）
triangle 三角形
quadrilateral 四边形
pentagon 五边形
hexagon 六边形
octagon 八边形
decagon 十边形
regular polygon 正多边形
perimeter 周长
area 面积

3.2.3. Strategies

n 边形的内角和公式 =（n-2）×180°,

例如：六边形的内角和=（6-2）×180° =720°

n 边形内联结任意不相邻的三个顶点可以形成图内三角形，图内三角形的总个数 = n-2



多边形考点比较简单集中：内角和公式一定要熟练应用，只有正多边形能计算出单个内角和的值。其余情况可以参考连接对角线，转化成三角形计算。

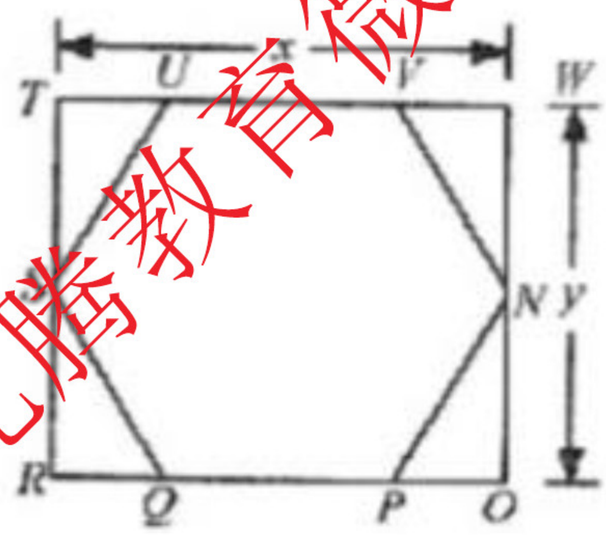
多边形的面积计算基本转化成多个三角形的叠加计算。

3.2.4. Exercises

1. The figure shows a regular 9-sided polygon. What is the value of x ?

Answer： 140

2. Polygon SUVNPQ is equilateral and equiangular and TWOR is a rectangle.



Quantity A	Quantity B
$\frac{x}{y}$	1

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

Answer: A

3.3 Triangles

3.3.1. Objective:

A. 了解三角形的边长和角的关系，能够在三角形内做出辅助线化解一般三角形或者多边形中的复杂问题，掌握周长和面积的计算方法。
B. 重点掌握直角三角形、等腰三角形和正三角形的计算问题，包括：边长、角度、周长和面积等。
C. 对于特殊直角三星的边长比例和角度关系要能够熟练进行转换，能够将复杂图形内的问题化简回归到同一三角形内解决。

3.3.2. Key terms' definition

right triangles 直角三角形
hypotenuse 斜边
leg 直角边

Pythagorean theorem 勾股定理
equilateral triangle 等边三角形
isosceles triangle 等腰三角形

base 底
height 高

congruent triangle 全等三角形
similar triangles 相似三角形

3.3.3 Strategies

1. 请熟记常见直角三角形的边长关系， 最常考到的边长比为 3:4:5 或 6:8:10 或 5:12:13.

2. 熟记特殊三角形的角度与边长关系:

30°：60°：90° - 1:√3：2

45°：45°：90° - 1:1:√2

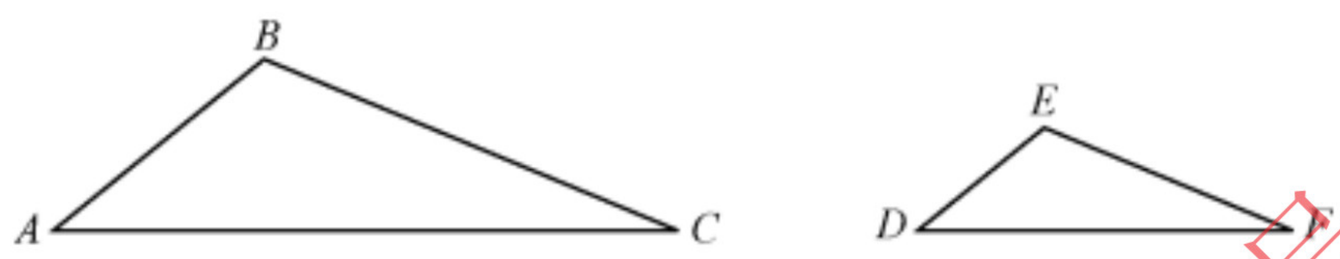
60°：60°：60° - 1:1:1

3. 三角形的面积公式 S=底边×高÷2，三角形的任一边都可以成为底边，高则是垂直于底边的线段（起于对应顶点，终于底边）。

4. 全等三角形 （congruent triangle）：两个三角形的三条边和三个角完全一致。只要满足下列任一条件，即可判断两个三角形为全等三角形。

- 边边边（Side-Side-Side）：三边对应相等的三角形是全等三角形。
- 边角边（Side-Angle-Side）：两边及其夹角对应相等的三角形是全等三角形。
- 角边角（Angle-Side-Angle）：两角及其夹边对应相等的三角形全等。

5. 相似三角形（similar triangles）：如果一个三角形的两条边和另一个三角形的两条边对应成比例,并且对应的夹角相等,那么这两个三角形相似。对应相似边的比例相同，面积比为边长比的平方。



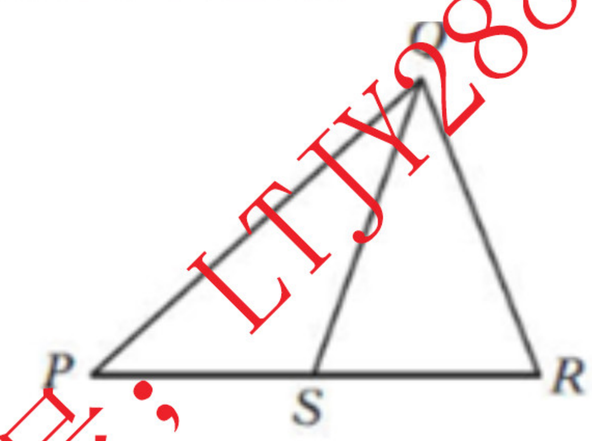
Since triangles ABC and DEF are similar, we have $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$. By cross multiplication, we can obtain other proportions, such as $\frac{AB}{BC}=\frac{DE}{EF}$.

6. 一旦涉及到三角形的比较大小题目，除非标明 drawn to scale，大多数题目的图形不按比例绘制， 不可根据图形解题 。 实际考试中推荐自己绘图 ，适当考虑添加辅助线。

7. 三角形两边之和大于第三边，两边之差小于第三边.

3.3.4 Exercises

1.



$PQ = PR$

Quantity A	Quantity B
PS	SR

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

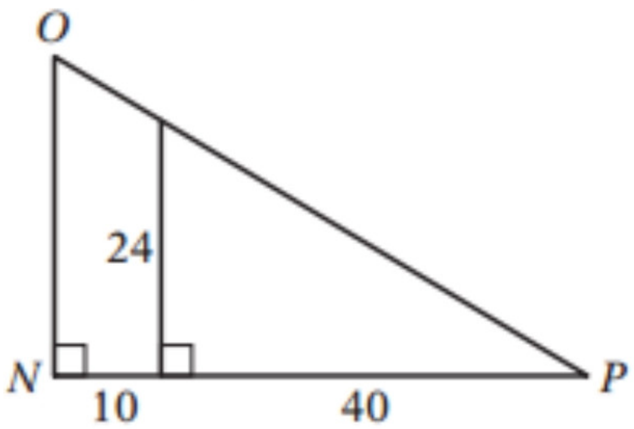
Answer: D

2. If $x > 0$, and two sides of a certain triangle have lengths $2x+1$ and $3x+4$ respectively, which of the following could be the length of the third side of the triangle?

A. $4x + 5$
B. $x + 2$
C. $6x + 1$
D. $5x+6$
E. $2x + 17$

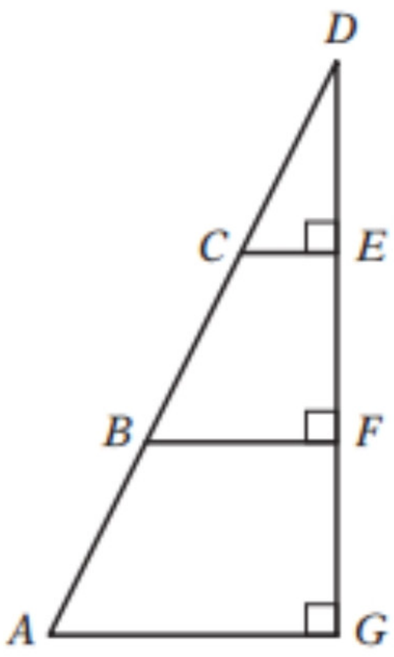
Answer: A C E

3. What are the lengths of sides NO and OP in triangle NOP below?



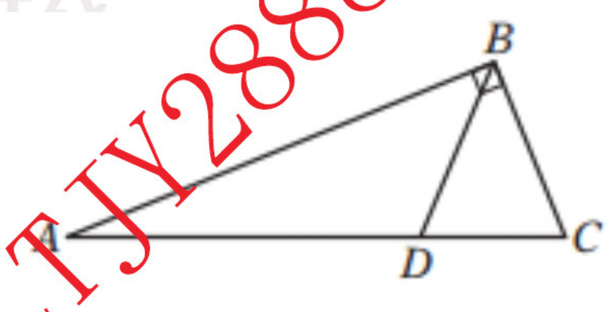
Answer: NO= 30 and OP= $10\sqrt{34}$

4. In the figure below, If the area of triangle AB = BC = CD. CDE is 42, what is the area of triangle ADG?



Answer: 378

5.



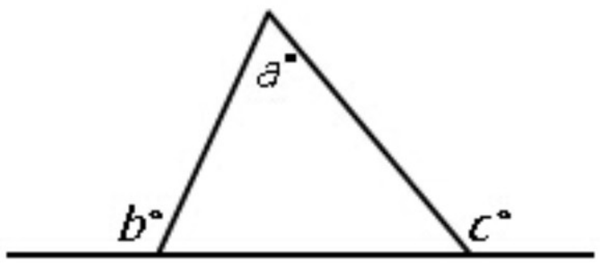
The length of AB is $10\sqrt{3}$.

Which of the following statements individually provide(s) sufficient additional information to determine the area of triangle ABC above?
Indicate all such statements.

A . DBC is an equilateral triangle.
B . ABD is an isosceles triangle.
C . The length of BC is equal to the length of AD.
D . The length of BC is 10.
E . The length of AD is 10.

Answer: A , D

6.



Quantity A	Quantity A
180+a	b+c

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

Answer: C

3.4 Quadrilaterals

3.4.1. Objective

A. 了解四边形的概念 ，能够转化各类四边形成为以单个三角形为单位的运算。
B. 熟练掌握正方形、长方形、梯形的周长面积运算。
C. 理解平行四边形（包括正方形、长方形、菱形）的角、对角线、边长的特定性质。

3.4.2. Definition of key terms

rectangle 长方形
length 长
width 宽
square 正方形
side 边，边长
parallelogram 平行四边形
trapezoid 梯形
rhomb 菱形
bisector 平分线
diagonal 对角线

3.4.3. Strategies

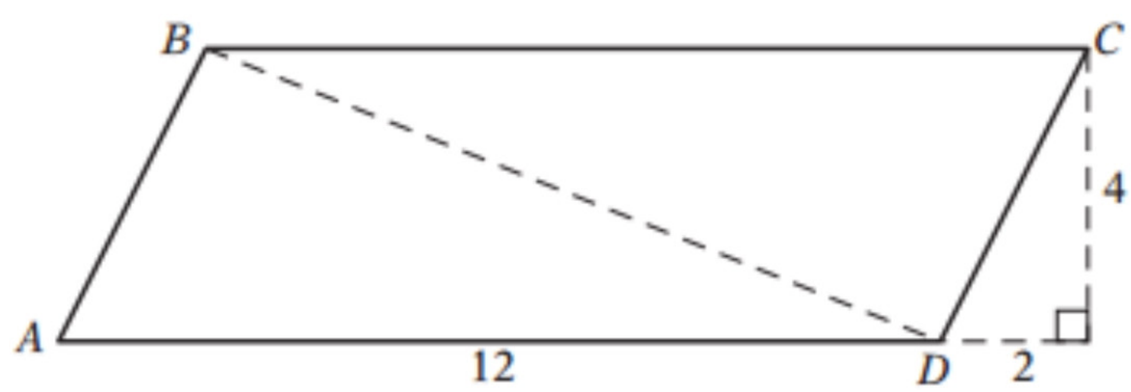
1．平行四边形的重点复习正方形和长方形的对角线计算，例如：正方形的两条对角线相互垂直。

2．平行四边形计算角度的问题一般先转化成单个三角形， 然后利用三角形的内角和来进行计算。

3．平行四边形面积 S=base×height，梯形面积 $S=\frac{1}{2}(base_1+base_2)\times height$

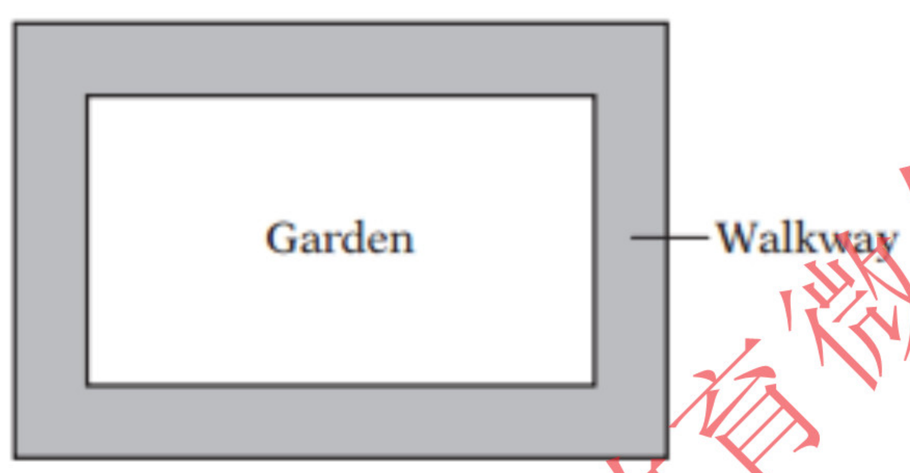
3.4.4. Exercises

1. In parallelogram ABCD below, find the length of diagonal BD



Answer: $\sqrt{29}$

2.



The figure above represents a rectangular garden with a walkway around it. The garden is 18 feet long and 12 feet wide. The walkway is uniformly 3 feet wide, and its edges meet at right angles. What is the area of the walkway?

Answer: 216 square feet.

3. A, B, and C are three rectangles. The length and width of rectangle A are 10 percent greater and 10 percent less, respectively, than the length and width of rectangle C. The length and width of rectangle B are 20 percent greater and 20 percent less, respectively, than the length and width of rectangle C.

Quantity A :The area of rectangle A
Quantity B:The area of rectangle B

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

Answer: A

3.5 Circles

3.5.1. Objectives

A. 了解圆的基本性质：半径、直径、周长和面积的计算。这也包括各类圆与其他图形的结合题 。
B. 掌握圆心角、圆周角的运算方法，能够计算对应弧长。
C. 灵活应对各类圆形的复杂图形：同心圆，扇形等的周长面积等运算。
D. 了解相切的定义，能够运算内切和外切的圆类题目。
E. 熟练掌握圆内接三角形的性质，能够有效转换边长和半径、直径的长度关系，充分利用三角形的知识解决复杂的图形问题 。

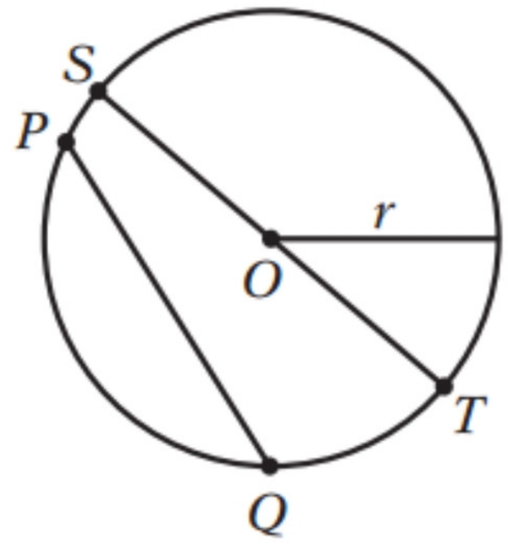
3.5.2. Definition of key terms

circle 圆
center 圆心
radius 半径
radii 半径（复数）
diameter 直径
chord 弦
circumference 圆的周长
arc 弧
radian 弧度
central angle 圆心角
semicircle 半圆
sector 扇形
tangent 相切
tangent line 切线
point of tangency 切点
intersect 相交
inscribe 内接 / 内切
circumscribe 外接 / 外切
internally tangent 内切
externally tangent 外切

concentric circles 同心圆

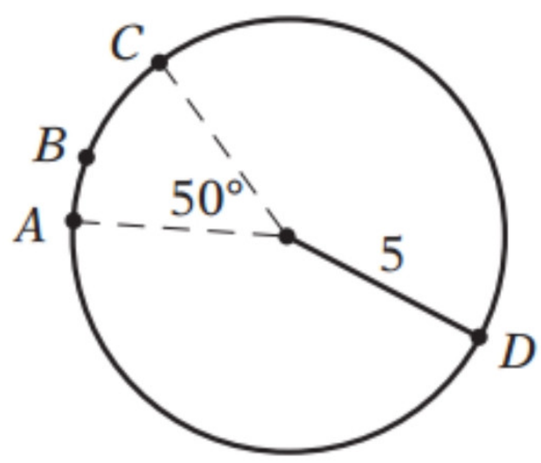
3.5.3. Strategies

1. chord 弦： 连接圆上任意两点的线段叫做弦,经过圆心的弦叫做直径,直径是一个圆里最长的弦。例如：图中的 PQ，ST 都是弦。

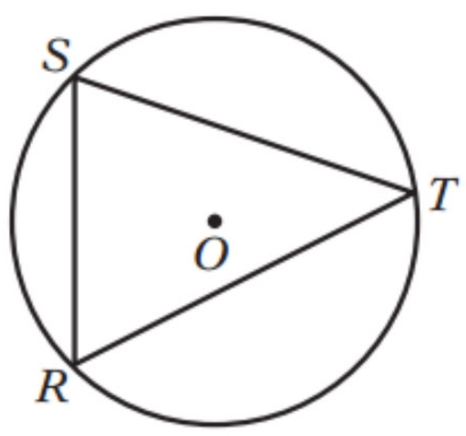


2. 圆的周长（circumference）公式 $C=2\pi r=\pi d$，面积公式 $S=\pi r^2$，$\pi\cong 3.14$。

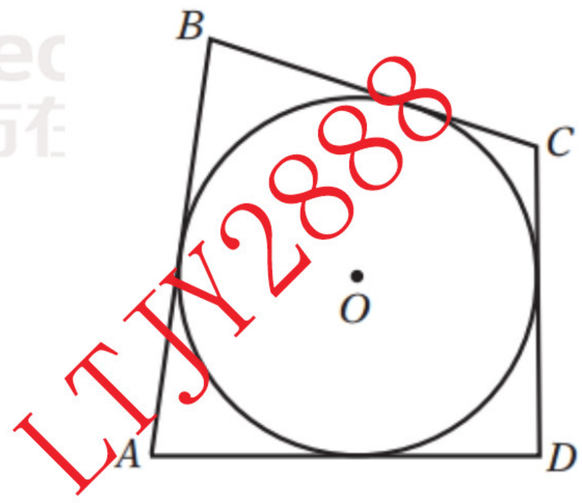
3. 弧（arc）：圆上任意两点间的部分叫做圆弧，简称弧。圆弧的度数可以直接用圆心角表达， 例如：弧 ABC 的度数为 50°， 圆弧 ADC 的度数为 310°。圆弧的弧长通过弧度与圆周角（2π）的比例来计算，例如 弧 ABC 的长度= 50°/360° ×2π×5≈4.4。



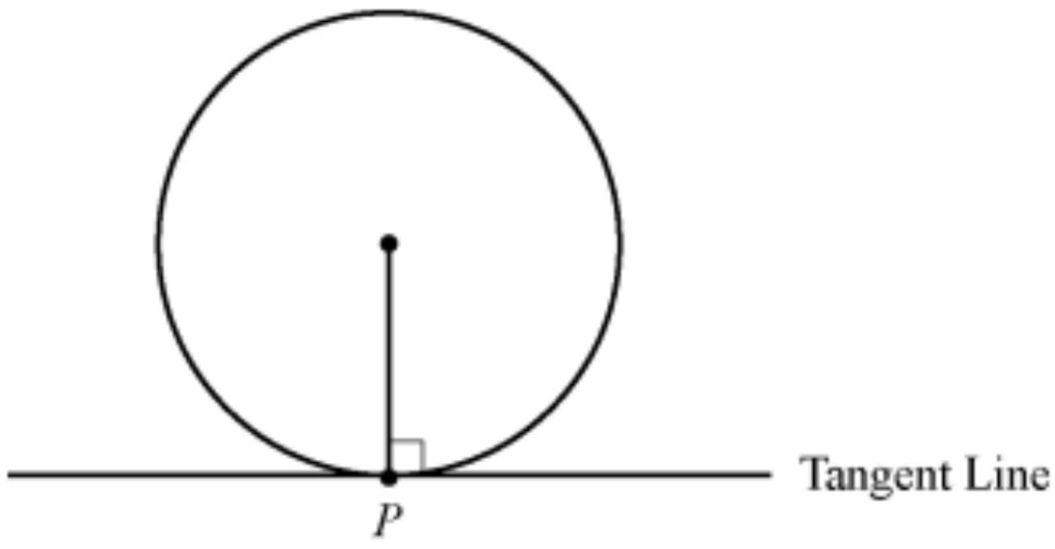
4. A polygon is inscribed in a circle if all its vertices lie on the circle, or equivalently, the circle is circumscribed about the polygon.



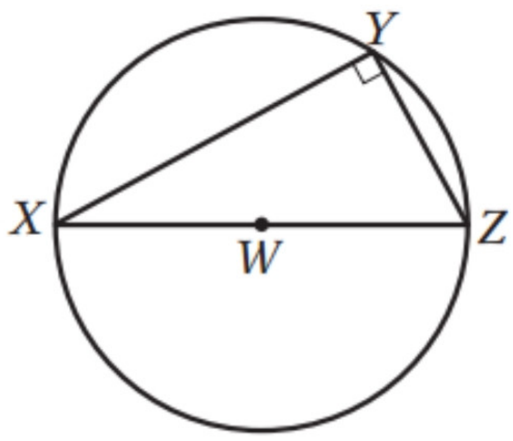
A polygon is circumscribed about a circle if each side of the polygon is tangent to the circle, or equivalently, the circle is inscribed in the polygon. In the figure below, quadrilateral ABCD is circumscribed about the circle with center O.



5. 相切（tangent）： 一条直线和一个圆有且仅有一个交点的时候，该圆与直线相交。这条直线为切线（tangent line），该交点为切点（point of tangency，例如：P 点）。**连接圆心和切点的线段比为半径，且垂直于切线**；反之亦然。



6. 当圆内接三角形有一边是直径的时候，该边对应的角为直角；反之，当圆内接三角形有一个角为直角时，该三角形必有一边为直径。



3.5.4. Exercises

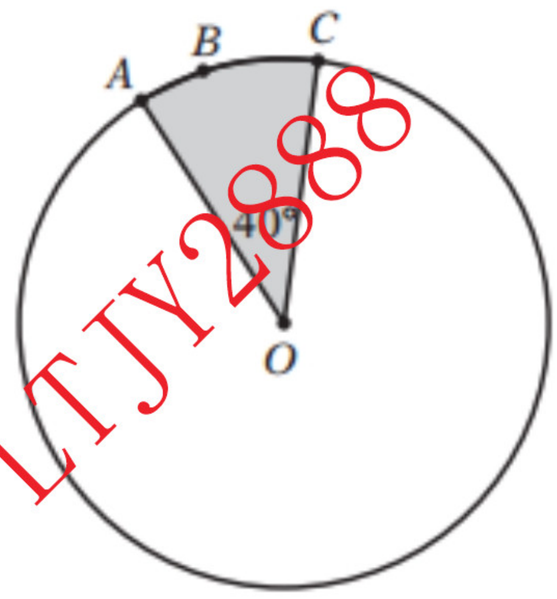
1 . The area of a circle is equal to the area of a square.

Quantity A	Quantity B
The circumference of the circle.	The perimeter of the square.

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

Answer: B

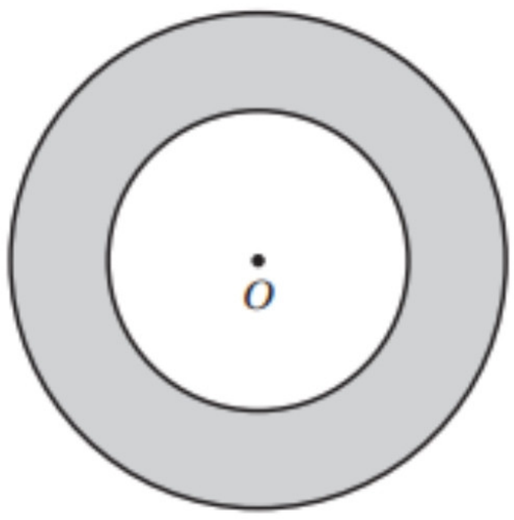
2 .



The circle with center O above has radius = 4. Find the following.
(a) Circumference of the circle
(b) Length of arc ABC
(c) Area of the shaded region

Answer: (a) 8π (b) $\frac{8\pi}{9}$ (c) $\frac{16\pi}{9}$

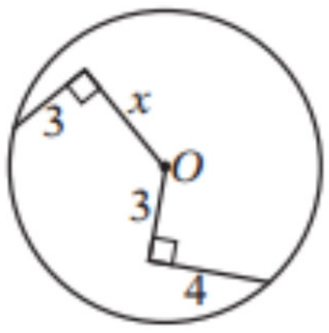
3.
The figure below shows two concentric circles, each with center O. Given that the larger circle has radius 12 and the smaller circle has radius 7, find the following.



(a) Circumference of the larger circle
(b) Area of the smaller circle
(c) Area of the shaded region

Answer: (a) 24π (b) 49π (c) 95π

4.



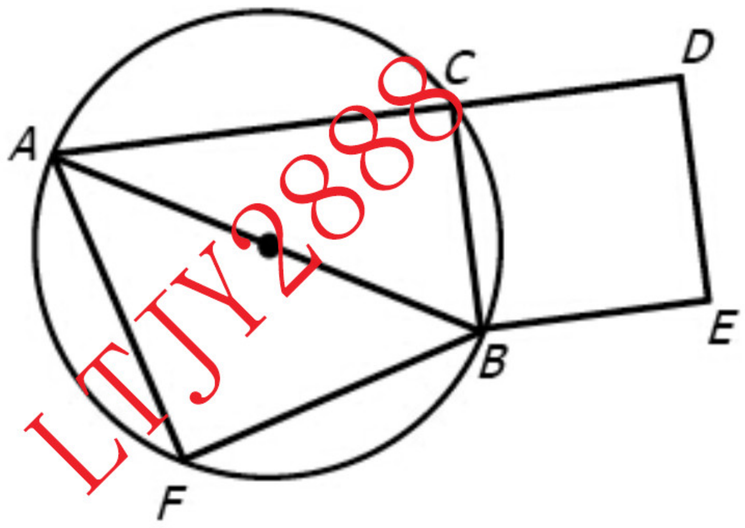
O is the center of the circle above.

Quantity A: x
Quantity B: 5

A. Quantity A is greater
B. Quantity B is greater
C. The two quantities are equal.
D. The relationship cannot be determined from the information given

Answer: B

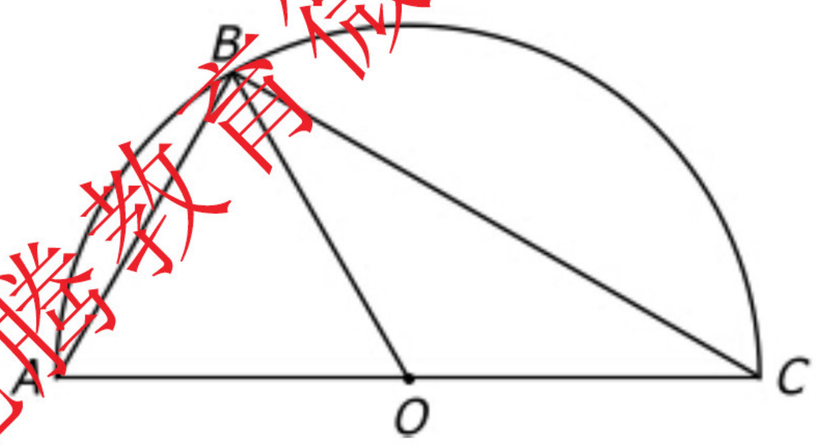
5.



AB is the diameter of the circle. If AF=BF=$3\sqrt{2}$ and AC=5, what is the area of square BCDE?

Answer: 11

6.



O is the center of the semicircle. If $\angle BCO = 30$ and BC =6 $\sqrt{3}$, what is the area of ΔABO?

A. $4\sqrt{3}$
B. $6\sqrt{3}$
C. $9\sqrt{3}$
D. $12\sqrt{3}$
E. $14\sqrt{3}$

Answer: C

3.6 Three-dimensional

3.6.1. Objective:

A. 了解立方体的基本概念，周长、表面积和体积的计算公式。
B. 重点掌握立方体、正方体、圆柱体的特性。
C. 能够独立将三维立体空间问题转化为二维平面问题进行简化计算。

3.6.2. Key terms' definition

face 面
vertex 顶点
vertices 顶点（复数）
edge 棱
rectangular solid 长方体
length 长

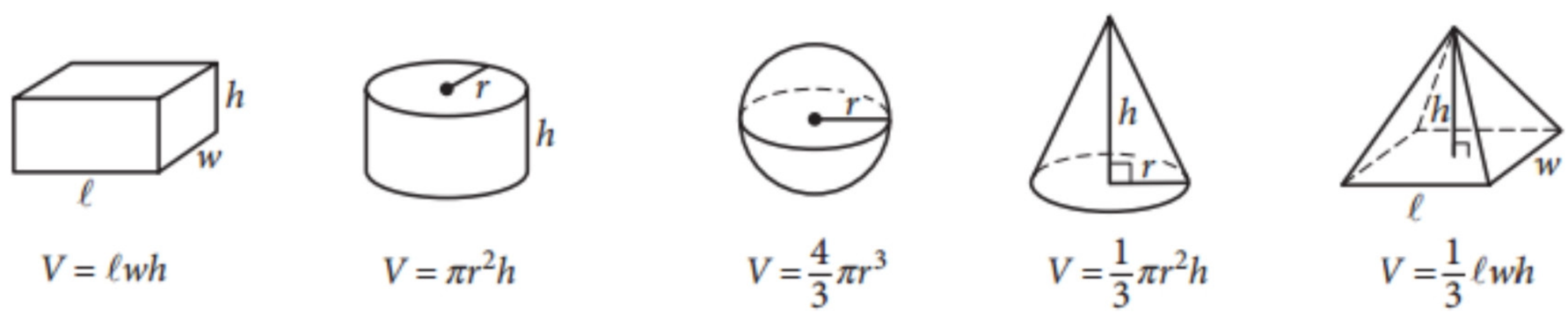
width 宽
height 高
cube 正方体
volume 体积
surface area 表面积
circular cylinder 圆柱体
lateral surface 侧面（圆柱侧曲面）
axis 轴（圆柱中心线）
right circular cylinder 直圆柱
sphere / global 球体
cylinder 圆柱
cone 圆锥
prism 棱柱
pyramid 棱锥
dimension 维度

3.6.3 Strategies

圆柱体的体积公式 $V=\pi r^2 h$, r 是圆柱体底面的半径, h 是圆柱体的高。

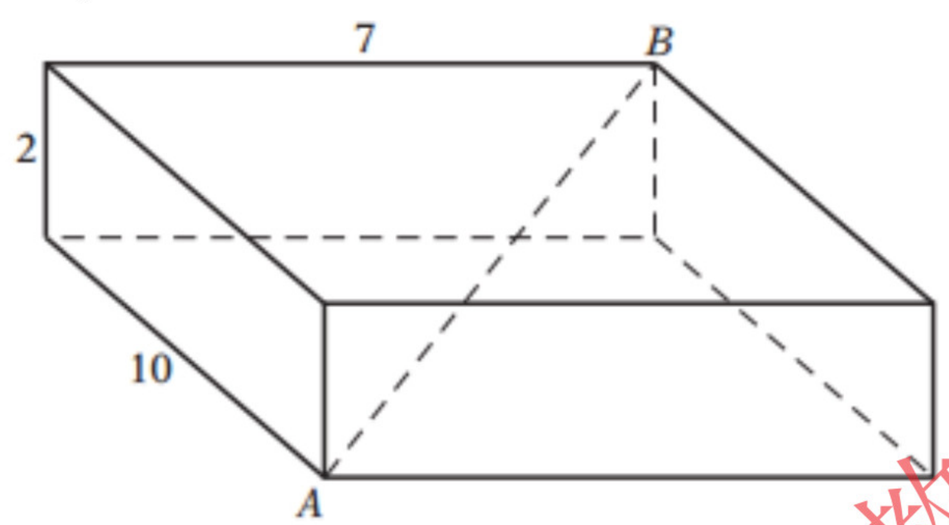
圆柱体的表面积公式 $S=2(\pi r^2)+2\pi rh$

其余的立方体表面积公式及体积公式如下图所示：



3.6.4 Exercises

1. For the rectangular solid below, find the following.
(a) Surface area of the solid
(b) Length of diagonal AB



Answer: (a) 208 (b) $3\sqrt{17}$

Chapter 4 Data Analysis

4.1 Graphical Methods for Describing Data

4.1.1 Objectives

A. 了解基本图形描述变量时所要用到的名词
B. 了解各种图形表达法，并且可以熟练应用
C. 了解各种图形的读图方法

4.1.2 Key terms' definition

individual 个体
object 个体
population 群体，总体
variable 变量

- In data analysis, a **variable** is any characteristic that can vary for the population of individuals or objects being analyzed .

Distribution of Data 数据分布

- **Distribution of data** indicates the values of the variable and how frequently the values are observed in the data.

frequency / count 频数:

- The **frequency**, or **count**, of a particular category or numerical value is the number of times that the category or value appears in the data.

frequency distribution 频数分布:

- A **frequency distribution** is a table or graph that presents the categories or numerical values along with their associated frequencies.

relative frequency 频率，相对频数:

- The **relative frequency** of a category or a numerical value is the associated frequency divided by the total number of data. Relative frequencies may be expressed in terms of percent, fractions, or decimals.

relative frequency distribution 频率分布:

- **Relative Frequency Distribution** （相对频率分布）**:**A **relative frequency distribution** is a table or graph that presents the relative frequencies of the categories or numerical values.

univariate 单变量的: data observed for one variable.
bivariate 双变量的:

- sometimes data are collected to study two different variables in the same population of individuals or objects. Such data are called **bivariate data**.

bar graph / bar chart 柱状图

segmented bar graph 分段柱状图
circle graph / pie chart 饼状图
sector 扇区
histogram 直方图
interval 区间
scatter plot 散点图
trend 趋势
time plot / time series 时间图表

4.1.3 Strategies

- **Frequency and Relative Frequency**:

Example. Here is a list of numbers: 1 3 4 4 4 5 6 7 7 7 7

Here are the resulting frequency and relative frequency distributions of the data.

Frequency Distribution Table（频数分布表）

Number	Frequency (频数)
1	1
3	1
4	3
5	1
6	1
7	4
TOTAL	11

在这个例子当中，各个数字发生的次数叫做 **Frequency**（频数）；而这样在一组数据中的频数汇总分布，就叫做 **Frequency Distribution**（频数分布）。简略来说，频数分布就是对各个变量频数的特征进行分析。

Relative Frequency Distribution Table（相对频数分布表）

Number	Relative Frequency (相对频数)
1	1/11
3	1/11
4	3/11
5	1/11
6	1/11

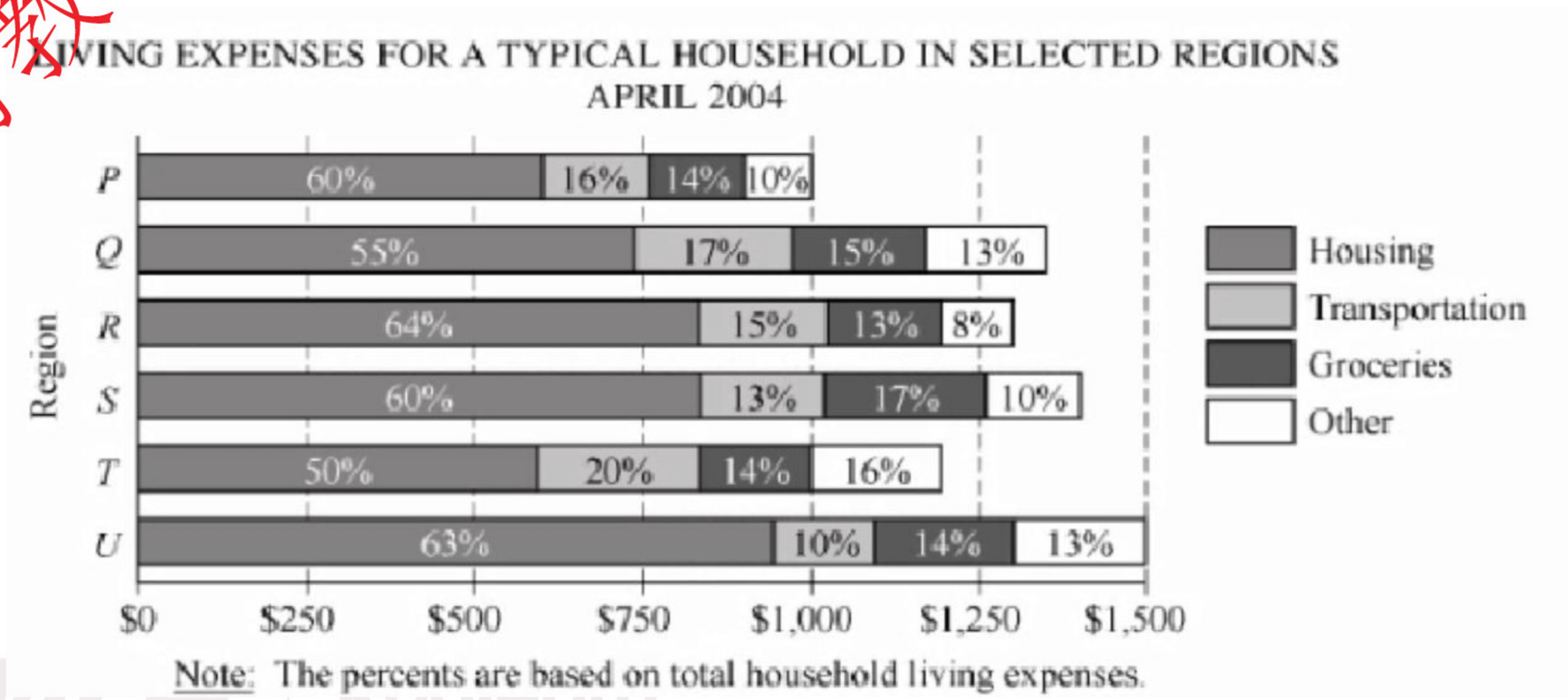
7		4/11
TOTAL		1

相对频数=频数/总数，一组数据中相对频数的总和为 **1**。

- **Bar Graphs**（柱状图）：

柱状图适用于离散数据或者分类数据的描述，长方形的 **bars** 被用来表示数据的种类，所有长方形的宽度相等，而长度表示的是所代表种类的 **frequency** 或 **relative frequency**，每一个长方形间会有固定的举例做间隔。柱状图可以水平陈列也可以竖直陈列。

柱状图的主要考察点：不同数据的 **frequency** 或 **relative frequency** 的比较。

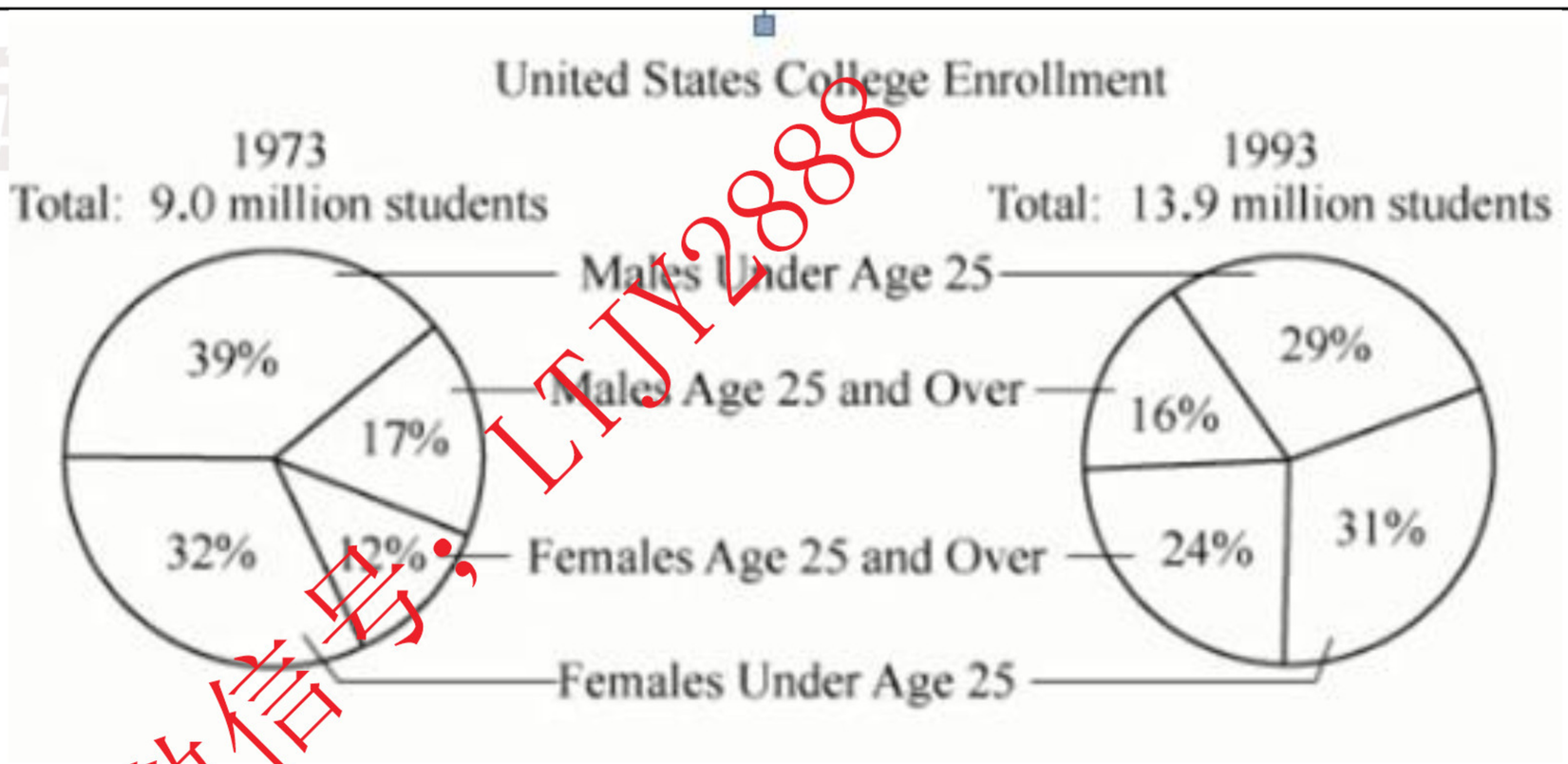


如上图，有的时候柱状图中的长方形还会被分为不同的部分，来表示一个大数据种类下的各个小类别的数据频数或相对频率的分布 ，这种图形是柱状图的一种，叫做 **segmented bar graph** 。

- **Circle Graphs / pie charts** （饼状图）：

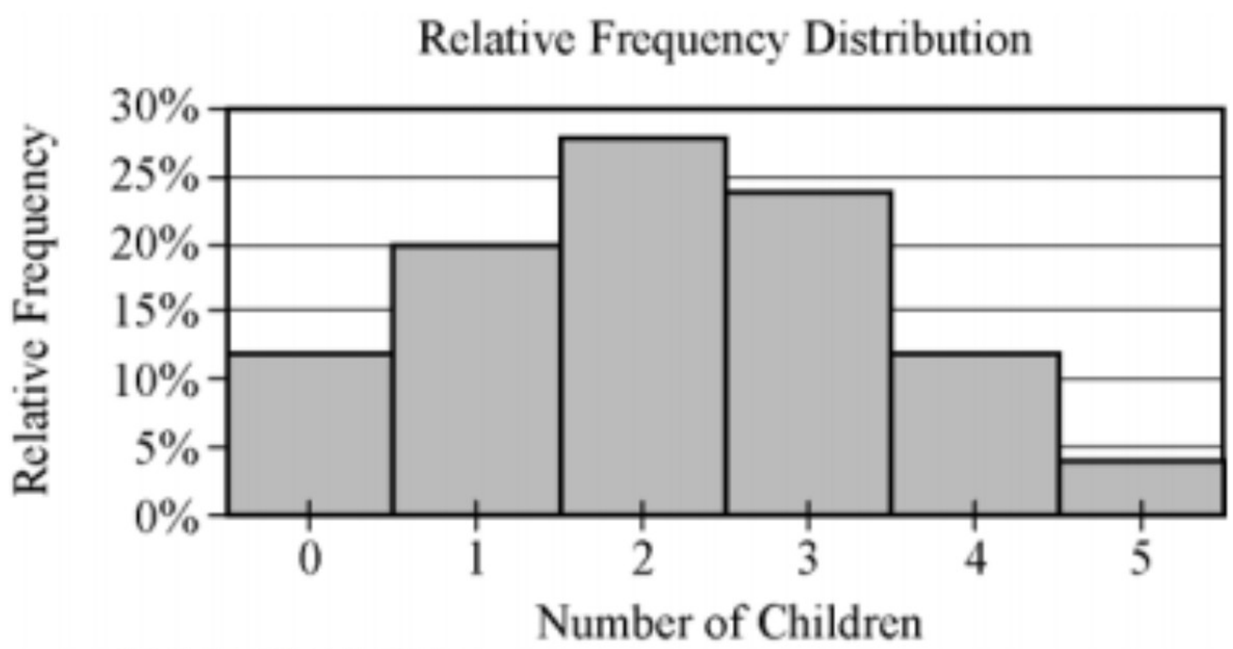
在饼状图中，一组数据被描述成一个圆形的图形，这组数据中的每一个类别，根据其相对频数占据一定比列的面积，每一个种类所占的圆的一部分被叫做 **sector**。

饼图的主要考察点：相对频数计算法。



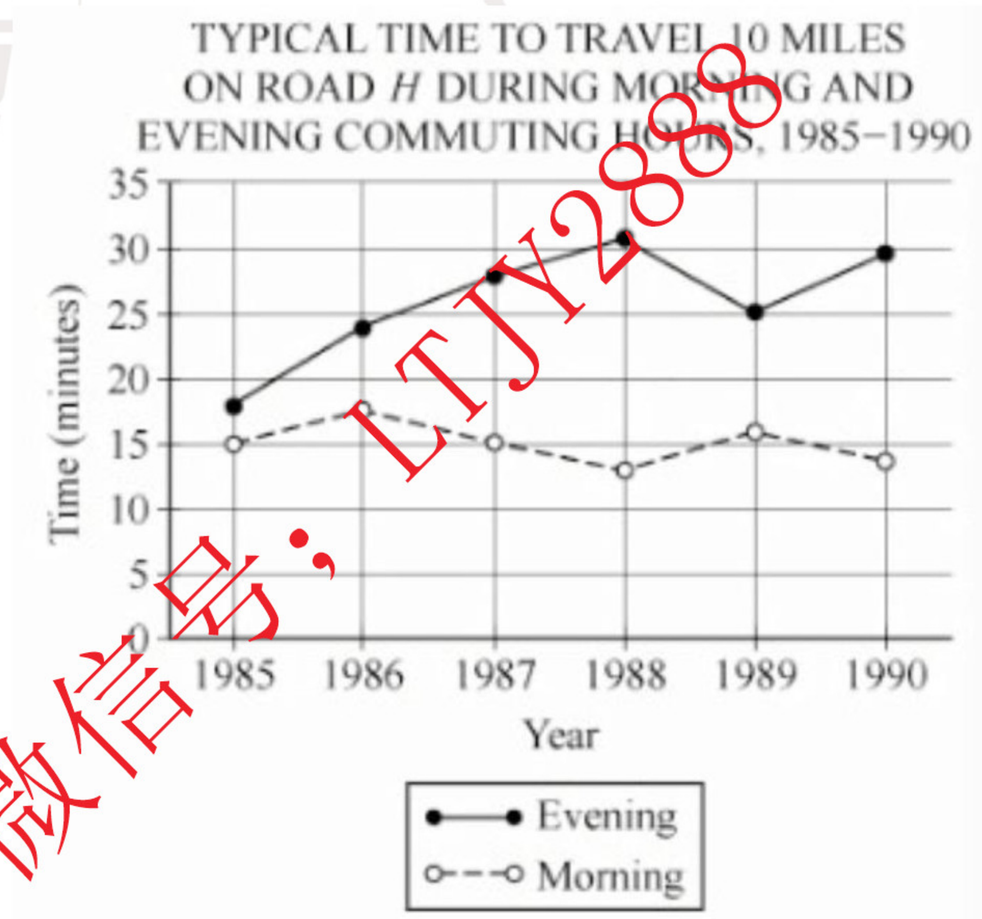
- **Histograms**（直方图）：

直方图和柱状图很像，但是有两点区别。一，直方图的长方形间是没有固定空隙的；二，直方图的长度和宽度都是有意义的，其宽度代表了该组数据的 **interval**（区间）。

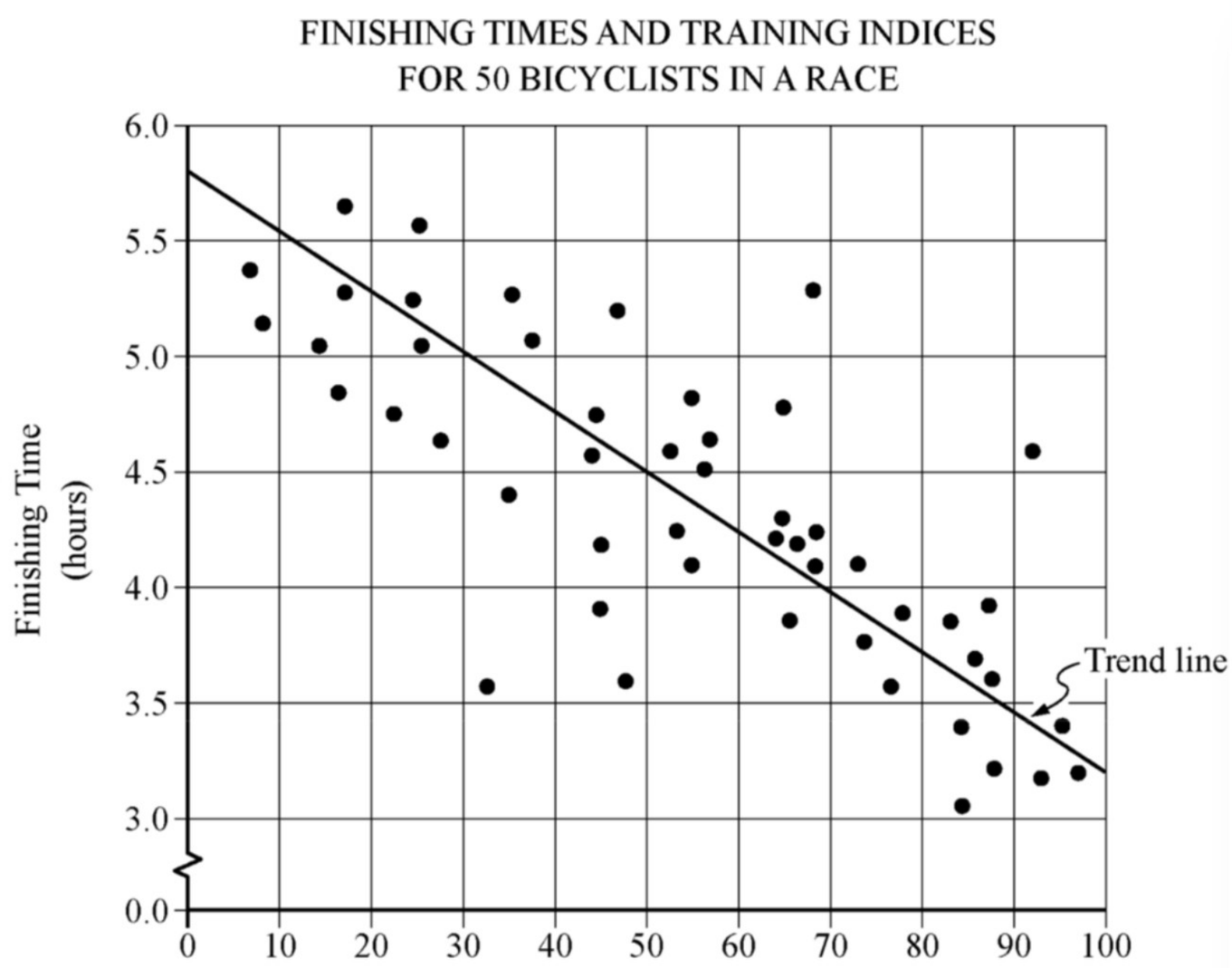


直方图的主要考察点：相对频数的计算。

- **Scatterplots** （散点图）：



当描述两个变量关系时，我们可以把一个变量放在横向坐标，而另一个放在纵向坐标，而图像中的点来表述两个变量的关系。而有时，当两个变量的关系很清晰时，我们可以用一定的趋势线来描述这种 **trend**（趋势）。

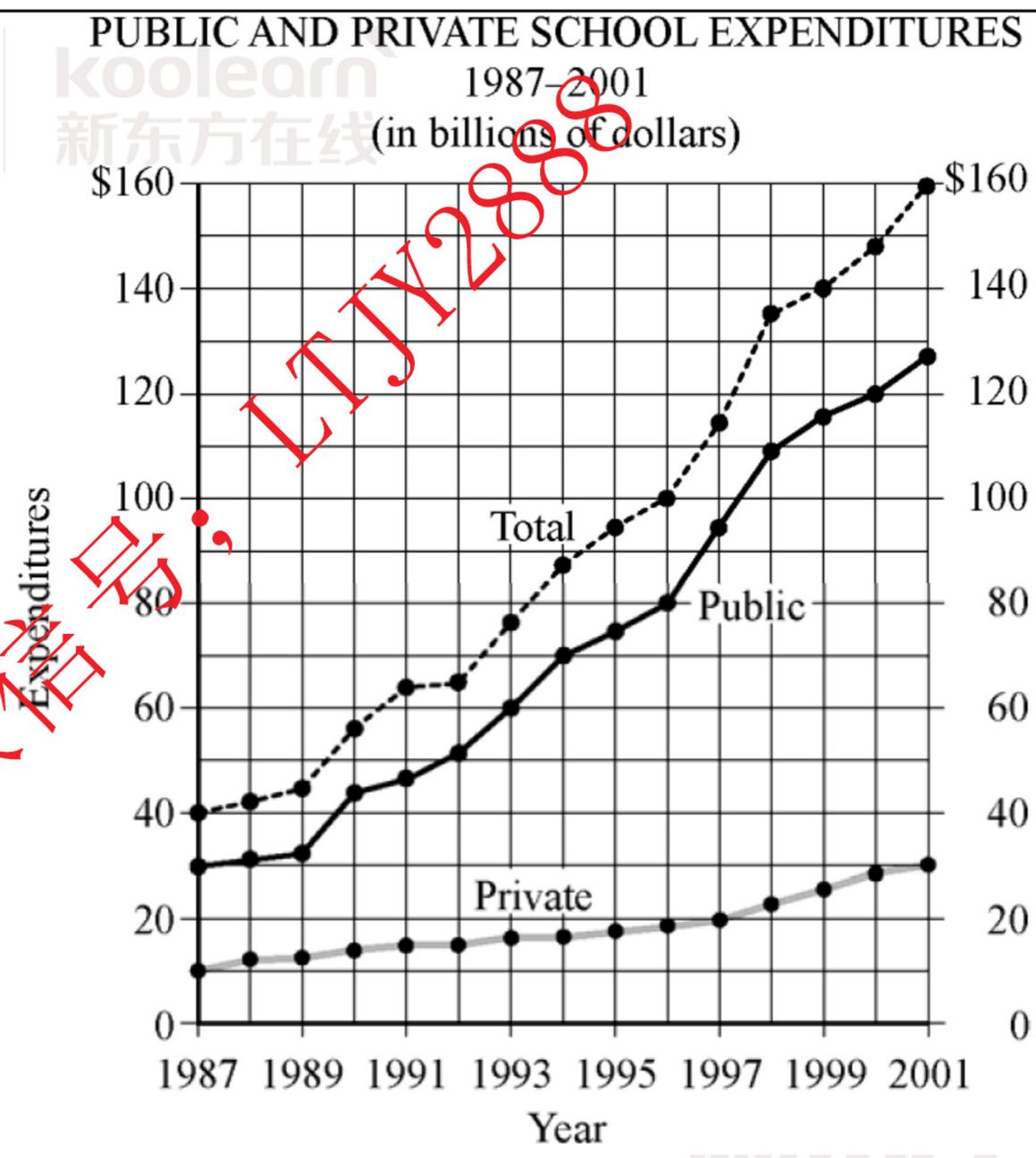


- **Time Plots / time series**（时间图表）

在时间图表中，横坐标为时间，纵坐标为该时间点上变量的取值。

4.1.4 Exercises

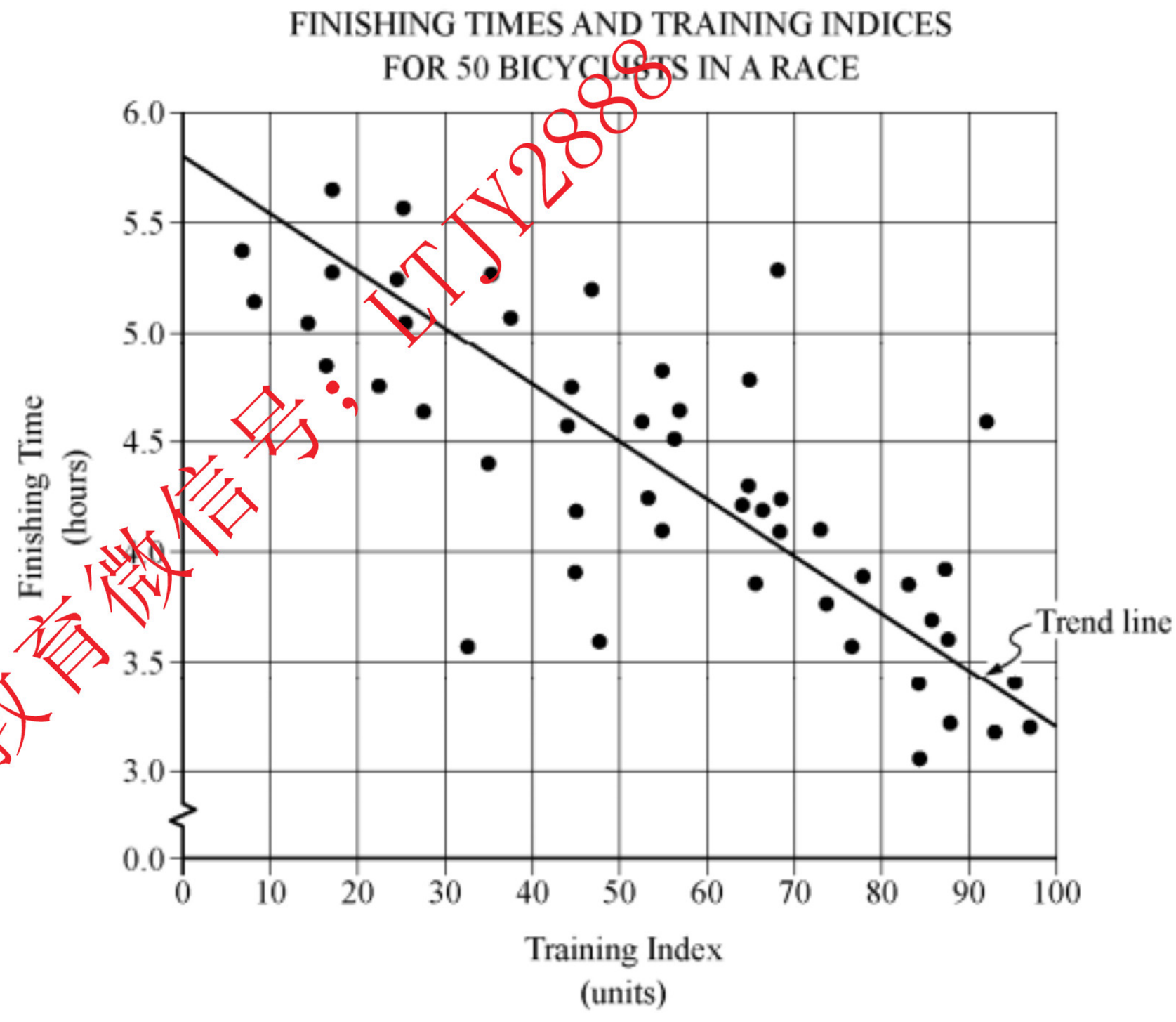
1.



(a) For which year did total expenditures increase the most from the year before?
(b) For 2001, private school expenditures were approximately what percent of total expenditures?

Answer: (a) 1998 (b) 19%

2. A bicycle trainer studied 50 bicyclists to examine how the finishing time for a certain bicycle race was related to the amount of physical training in the three months before the race. To measure the amount of training, the trainer developed a training index, measured in "units" and based on the intensity of each bicyclist's training. The data and the trend of the data, represented by a line, are displayed in the scatterplot above.



(a) How many of the 50 bicyclists had both a training index less than 50 units and a finishing time less than 4.5 hours?

(b) What percent of the 10 fastest bicyclists in the race had a training index less than 90 units?

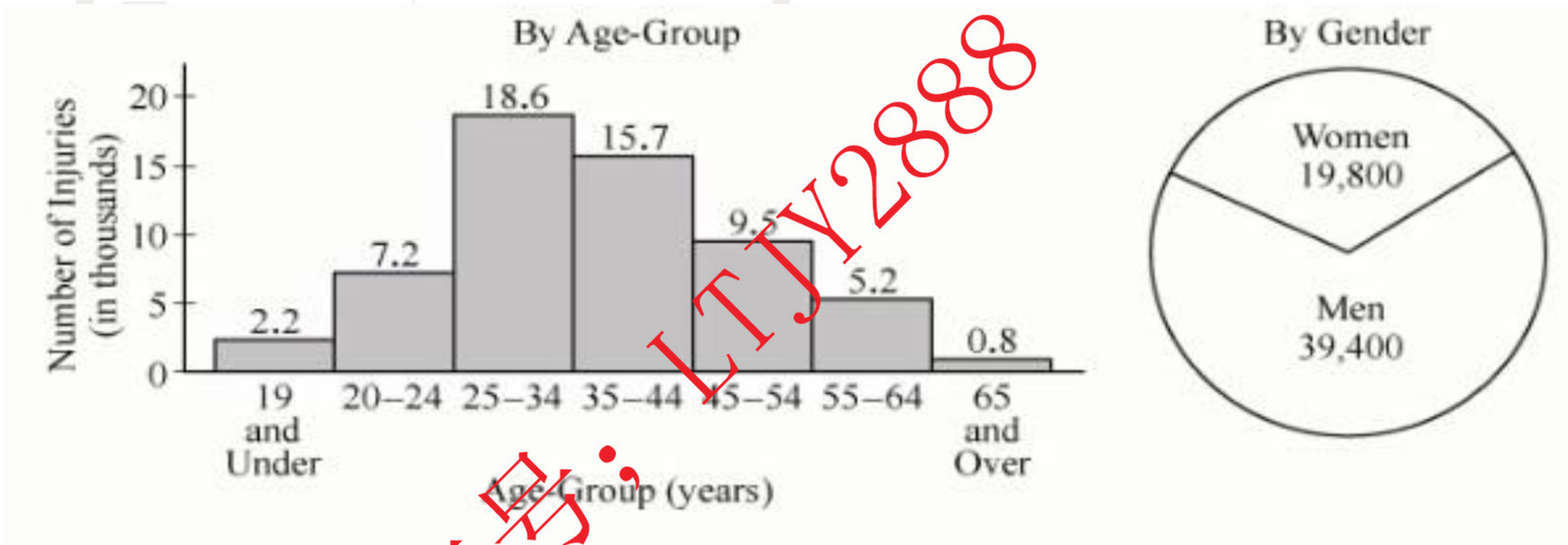
Answer:

(a) 5

(b) 70%

Questions 3 and 5 are based on the following data.

NUMBER OF OCCUPATIONAL INJURIES IN STATE X, 1998



3. How many of the age-groups each accounted for more than 15 percent of the total number of occupational injuries in State X in 1998?

A. One
B. Two
C. Three
D. Four
E. Five

Answer： C

4. In 1998, if one-half of the occupational injuries in the combine 34-and-under age- groups were incurred by men, what was the number of occupational injuries incurred by men in the combined 35-and-over age-groups?

A. 33,500
B. 31,900
C. 30,500
D. 25,400
E. 21,700

Answer： D

5. For the 55-64 age-group in 1998, the average (arithmetic mean) number of work- hours lost per occupational injury was 48.5. If a workweek is 40 work-hours, which of the following is closest to the total number of workweeks lost due to occupational injuries in the 55-64 age-group in 1998?

A. 4,500
B. 5,200
C. 5,500
D. 5,900
E. 6,300

Answer： E

6.

POPULATION OF THE UNITED STATES
IN 1800 AND 1900

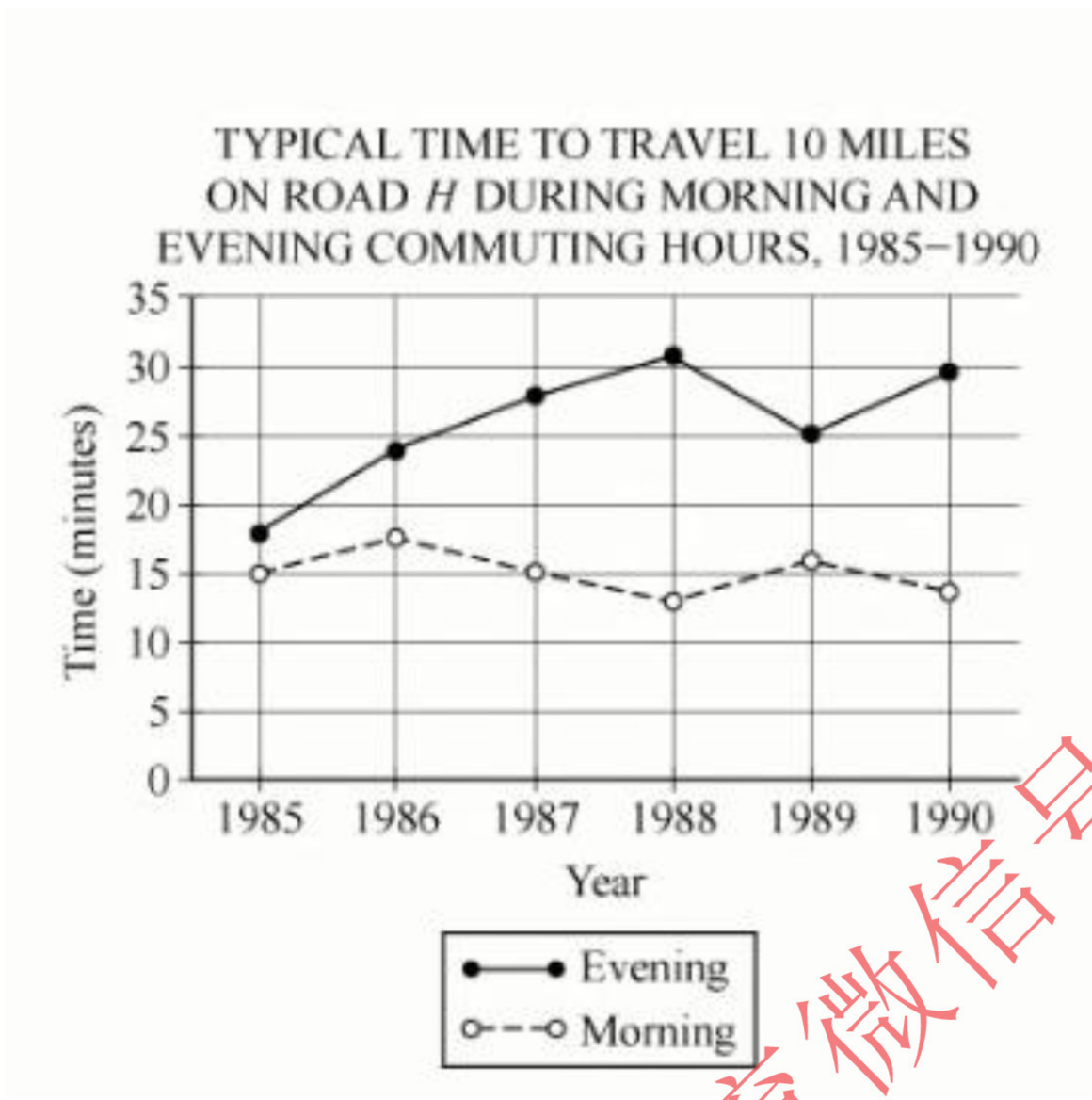
Year	Population	Population per Square Mile
1800	5.3 million	6.1
1900	76.0 million	25.6

By approximately how many square miles did the area of the United States increase from 1800 to 1900?

- ○ 360,000
- ○ 2,000,000
- ○ 3,625,000
- ○ 20,000,000
- ○ 36,250,000

Answer： B

Questions 7 and 9 are based on the following data.



7.

For 1988, the typical travel time during the morning commuting hours was approximately what fraction of the typical travel time during the evening commuting hours?

- $\frac{1}{3}$
- $\frac{2}{5}$
- $\frac{5}{9}$
- $\frac{8}{13}$
- $\frac{2}{3}$

Answer：B

8.

The typical travel time during the morning commuting hours decreased by approximately what percent from 1986 to 1988?

- 5%
- 10%
- 25%
- 40%
- 45%

Answer：C

9.

During the morning commuting hours in 1987, what was the average speed, in miles per hour, of a car that traveled the 10 miles on Road H if the car took the typical amount of time to travel the 10 miles?

- 15
- 20
- 25
- 33
- 40

Answer：E

4.2 Numerical Methods for Describing Data

4.2.1 Objectives

A.了解描述数据中心趋势的各项计算方法
B.了解描述数据扩散趋势的各项计算方法
C.了解箱线图

4.2.2 Key terms' definition

- **Central Tendency 中心趋势：** Measures of **central tendency** indicate the "center" of the data along the number line and are usually reported as values that represent the data. There are three common measures of central tendency: (i) the **arithmetic mean**—usually called the **average or simply the mean**, (ii) the **median**, and (iii) the **mode**.

- **Arithmetic Mean 算术平均数：** To calculate the **mean** of n numbers, take the sum of the n numbers and divide it by n.

- **Median 中位数**

- **Mode 众数**

- **Weight 权重：** The number of times a value appears in the list, or the frequency, is called the **weight** of that value.

- **Weighted Mean 加权平均数：** When several values are repeated in a list, it is helpful to think of the mean of the numbers as a weighted mean of only those values in the list that are different. Weighted average=sum（weight*value）

- **Positions 位置：** The three most basic **positions**, or locations, in a list of data ordered from least to greatest are the beginning, the end, and the middle. It is useful here to label these as L for the least, G for the greatest, and M for the median. Aside from these, the most common measures of position are **quartiles** and **percentiles**.

- **Quartiles 四分位数**

- **Percentiles 百分位数**

- **Dispersion 扩散**：Measures of **dispersion** indicate the degree of "spread" of the data. The most common statistics used as measures of dispersion are the range, the interquartile range, and the standard deviation. These statistics measure the spread of the data in different ways.

- **Range 极差**：The **range** of the numbers in a group of data is the difference between the greatest number G in the data and the least number L in the data; that is, $G - L$.

- **Interquartile Range 四分位差**：A measure of dispersion that is not affected by outliers is the **interquartile range**. It is defined as the difference between the third quartile and the first quartile, that is, $Q - Q$.

- **standard deviation 标准差**：Unlike the range and the interquartile range, the standard deviation is a measure of spread that depends on each number in the list. Using the mean as the center of the data, the standard deviation takes into account how much each value differs from the mean and then takes a type of average of these differences.

- **Outlier 异常值**

- **standardization 标准化**

- **population standard deviation 总体标准差**

- **sample standard deviation 样本标准差**

4.2.3 Strategies

arithmetic mean/average	$\frac{\sum_{i=1}^{n} X_i}{N}$
weighted average/mean	$\sum_{i=1}^{n} X_i \cdot p_i$
population standard deviation	$\sqrt{\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{N}}$
sample standard deviation	$\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$

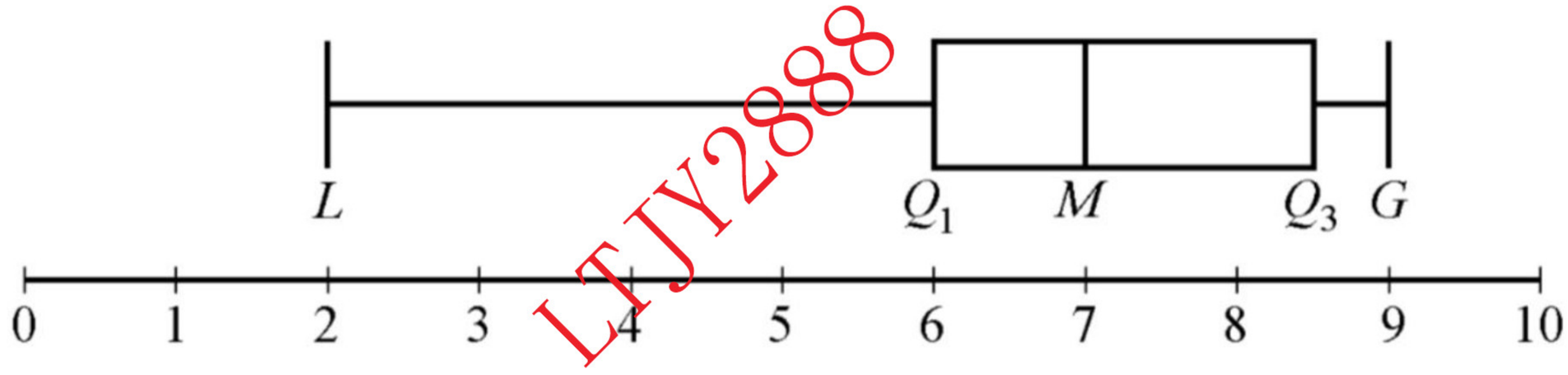
- 基本公示表:

- **Quartile and Boxplot**（四分位数和箱线图）

第一分位数 Q_1，第二分位数 Q_2 (也就是中位数), 和第三分位数 Q_3 将一组数据等分为四份。当一组数据按照递增的方式进行排列的时候，第一组数据包含了从最小值到Q_1 的所有数据，第二组包括了从Q_1 到中位数的所有数据，第三组数据包括了从中位数到 Q_3 的所有数据,而第四组数据包括了从 Q_3 到最大值的所有数据。因为数组中所包含的数据不一定可以被4整出，在决定Q_1 和 Q_3时，不同的统计学家给出了不同的方法，但是在决定第二分位数时，大家达成了一致： $Q_2 = M$ 。一般说来，我们采用的方法为：首先通过$Q_2 = M$ 的方法，将一组数据一分为二，得到一组较小的数据和一组较大的数据，而将第一分位数Q_1 取为较小数据组的中位数，第三分位数Q_3 取为较大数据组的中位数。

Interquartile Range（四分位差）是一个描述数组离散程度的数据，它主要描述一组数据中间半段的离散程度。它的计算方法为IQR= Q_3- Q_1,也就是说，四分位差不会受outliers（异常值）的影响。

一个描述一组数据中间趋势和离散趋势的方式是描述五个数字L（最小值）,Q_1第一分位数， Q_2 第二分位数, Q_3 第三分位数和G最大值。当我们把这五个数字在数轴上标示出来，并在数轴上方画出如下图的图形时，我们就可以得到boxplots or box-and-whisker plots（箱线图）。在箱线图中，“箱子描述的是四分位数，而两端的延伸描述的是最小值和最大值。



- **Standarization（标准化）**

每一个数值都被它所处的数组中的均值和标准差所限制和影响。为了可以比较位于不同均值和标准差的数组的数据，我们可以引入一个概念叫做 **standardization（标准化）。在标准化的过程中，述职要减去它所处数组的均值，而这个差值也将继续除以该数据所处数组的标准差。标准化是一个非常有效的工具，因为它为完全不可以直接相比的数据提供了一个比较的途径。**

$$Z\text{-}score=\frac{x_i-mean}{standard\ \ deviation}$$

4.2.4 Exercises

1. The numbers of passengers on 9 airline flights were 22, 33, 21, 28, 22, 31, 44, 50, and 19. The standard deviation of these 9 numbers is approximately equal to 10.2.

(a) Find the mean, median, mode, range, and interquartile range of the 9 numbers.

(b) If each flight had had 3 times as many passengers, what would have been the mean, median, mode, range, interquartile range, and standard deviation of the nine numbers?

(c) If each flight had had 2 fewer passengers, what would have been the interquartile range and standard deviation of the nine numbers?

Answer:

(a) mean = 30, median = 28, mode = 22, range = 31, interquartile range = 17

(b) mean = 90, median = 84, mode = 66, range = 93, interquartile range = 51, standard deviation ≈30.7

(c) interquartile range = 17, standard deviation ≈ 10.2

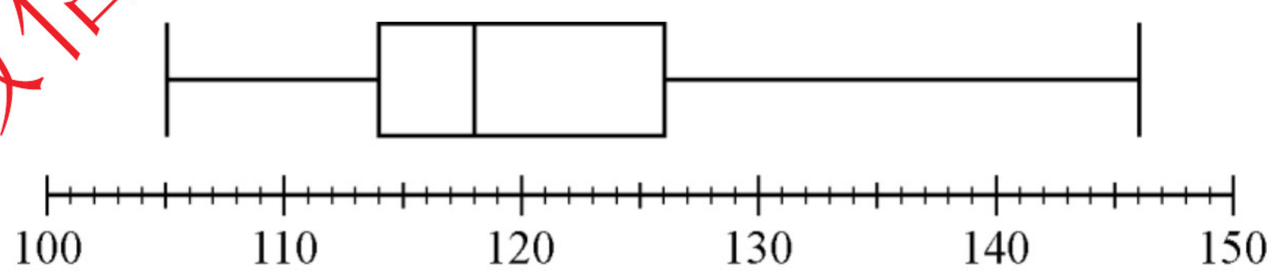
2. Find the mean and median of the values of the random variable X, whose relative frequency distribution is given in the table below.

X	Relative Frequency

0	0.18
1	0.33
2	0.10
3	0.06
4	0.33

Answer: mean = 2.03, median = 1

3. Eight hundred insects were weighed, and the resulting measurements, in milligrams, are summarized in the boxplot below.



(a) What are the range, the three quartiles, and the interquartile range of the measurements?
(b) If the 80th percentile of the measurements is 130 milligrams, about how many measurements are between 126 milligrams and 130 milligrams?

Answer:
(a) range=41 Q_1=114 Q_2=118 Q_3=126 IQR=12
(b) 40 measurements

4. x < y-2

Quantity A: The average (arithmetic mean) of x and y
Quantity B: y-1

A. Quantity Ais greater.
B. Quantity B is greater.
C. The two quantities are equal
D. The relationship cannot be determined from the information given.

Answer: B

5. In a data set of 10,000 numbers varying from 20 to 80, the number 62 is the 60th percentile and the number 74 is the nth percentile.

Quantity A: n
Quantity B: 70

A. Quantity A is greater.
B. Quantity B is greater.

C. The two quantities are equal
D. The relationship cannot be determined from the information given.

Answer： D

6.

Mike, Scott, Jim, Kate, and Pete each have a different number of assignments this month. Pete has fewer assignments than Kate, Kate has more assignments than Mike, Mike has more assignments than Jim, and Jim has more assignments than Scott. Which of the following could be the person who has the median number of assignments this month for the five people listed?

Indicate all such people.

- ☐ Mike
- ☐ Scott
- ☐ Jim
- ☐ Kate
- ☐ Pete

Answer： Mike, Jim, Pete

7.

A research report states that the average (arithmetic mean) of 120 measurements was 72.5, the greatest of the 120 measurements was 92.8, and the range of the 120 measurements was 51.6.

The information given above is sufficient to determine the value of which of the following statistics?

Indicate all such statistics.

- ☐ The least of the 120 measurements
- ☐ The median of the 120 measurements
- ☐ The standard deviation of the 120 measurements
- ☐ The sum of the 120 measurements

Answer： AD

8.

In a certain club, the average (arithmetic mean) age of the 35 males is 24.2 years and the average age of the 25 females is 27.6 years.

Quantity A	Quantity B
The average age of all of the people in the club	25.9

Answer: B

4.3 Counting Methods（计算方法）

4.3.1 Objectives

A. 了解集合问题和维恩图的应用的计算
B. 了解阶乘、排列组合等计算方法
C. 了解乘法原则和加法原则

4.3.2 Key terms' definition

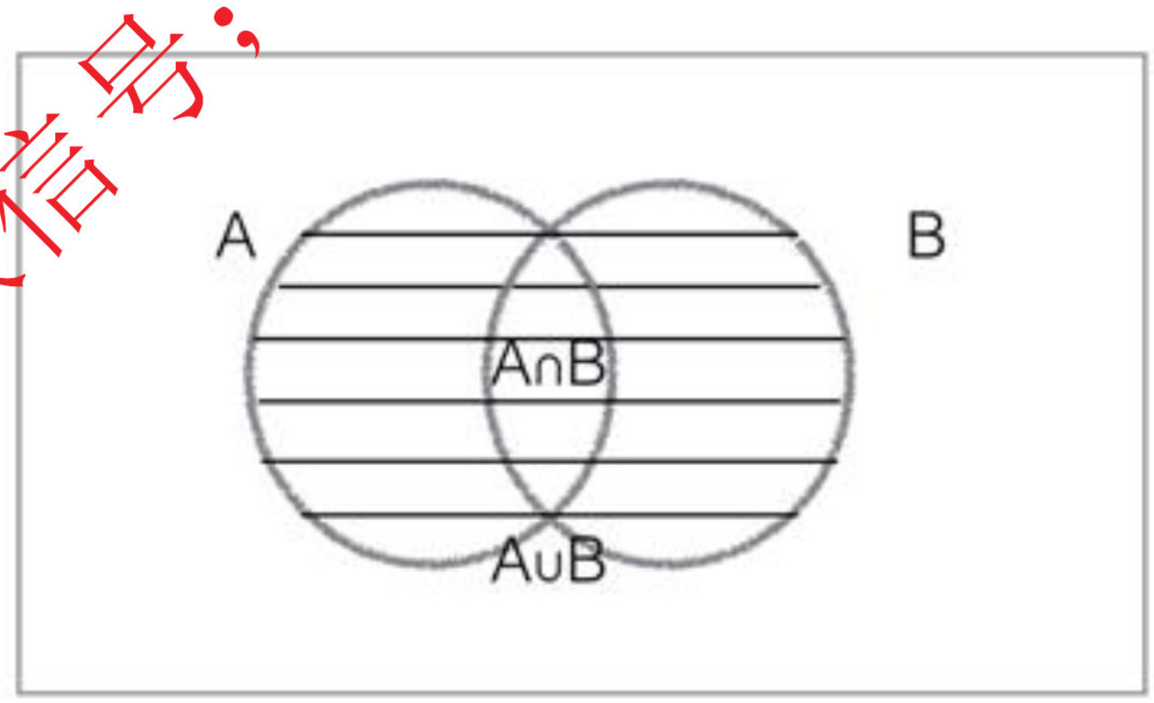
set 集合
finite set 有限集合
infinite set 无限集合
empty set 空集
nonempty 非空
element / member 元素
subset 子集
intersection 交集
union 并集
universal set 全集
Venn Diagram 维恩图
disjoint / mutually exclusive 互斥
list 数列
permutation 排列
combination 组合
factorial 阶乘

4.3.3 Strategies

list 和 set 的区别：list (数列) 里的数字是有顺序的

集合与维恩图：

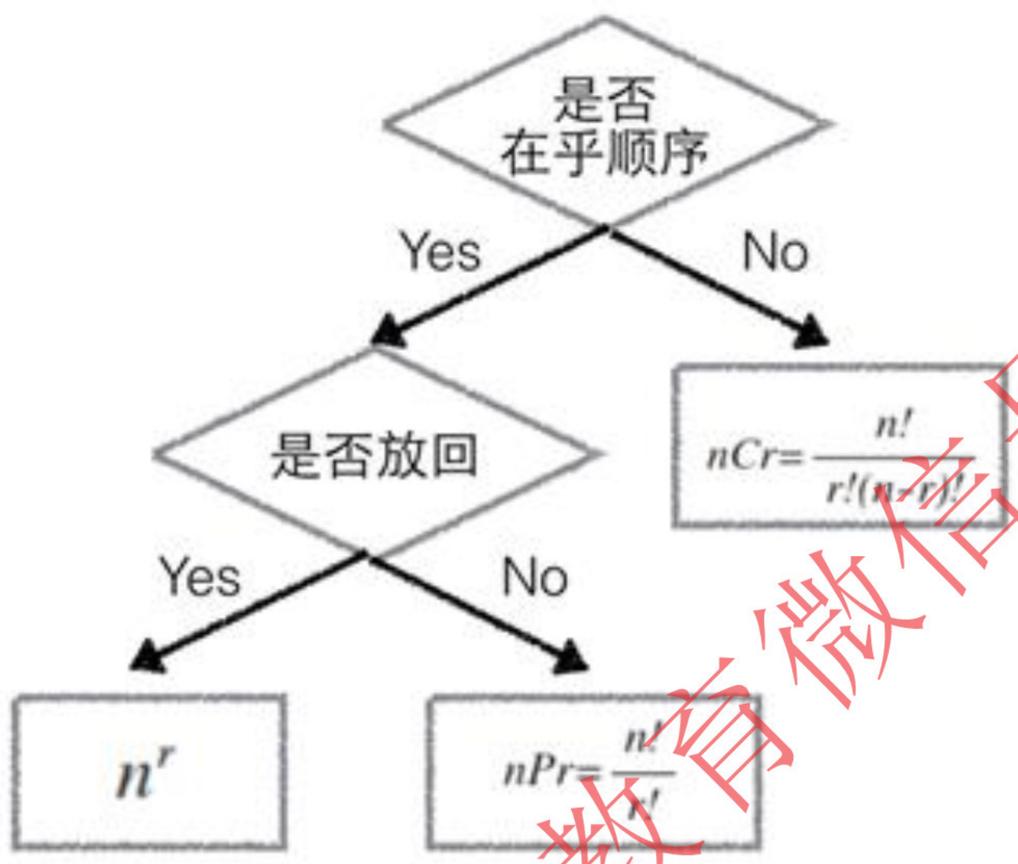
|A ∪ B|=|A|+|B|-|A ∩ B|



- **Factorial（阶乘）：**

  适用范围：natural numbers（自然数）
  表达符号：n!
  计算公式：n!=n(n-1)(n-2)……1
  特殊公式：0!=1

- **Combination and Permutation（排列组合）**



- **Multiplication Principle and Addition Principle （乘法原则和加法原则）**

当完成一件事是可以分步骤完成的，可以通过将每一步的possibilities相乘来计算完成该事件possibilities；
当完成一件事是可以分类完成的，可以通过将每一步的possibilities相加来计算完成该事件possibilities。

4.3.4 Exercises

1. Martha invited 4 friends to go with her to the movies. There are 120 different ways in which they can sit together in a row of 5 seats, one person per seat. In how many of those ways is Martha sitting in the middle seat?

Answer: 24

2. From a box of 10 light bulbs, you are to remove 4. How many different sets of 4 light bulbs could you remove?

Answer: 210

3. A talent contest has 8 contestants. Judges must award prizes for first, second, and third places, with no ties.
   (a) In how many different ways can the judges award the 3 prizes?
   (b) How many different groups of 3 people can get prizes?

Answer:
(a) 336 (b) 56

4.4 Probability 概率

4.4.1 Objectives

A. 了解乘法原则和加法原则
B. 了解概率的各项计算

4.4.2 Key terms' definition

possibility 可能性
probability 概率
dice / die 骰子

probability experiment 随机试验：

- A **probability experiment**, also called a **random experiment**, is an experiment for which the result, or outcome, is uncertain. We assume that all of the possible outcomes of an experiment are known before the experiment is performed, but which outcome will actually occur is unknown.

sample space 样本空间:

- The set of all possible outcomes of a random experiment is called the sample space.

event 事件:

- Any particular set of outcomes is called an event.

random 随机的

random selection 随机抽取:

- The assumption of random selection means that each of the names is equally likely to be selected.

mutually exclusive 互斥:

- Events that cannot occur at the same time are said to be mutually exclusive.

independent 独立:

- E and F are said to be independent if the occurrence of either event does not affect the occurrence of the other.

4.4.3 Strategies

- **Possibility and Probability（可能性和概率）**

在 GRE 考试中，一定要注意区分 Possibility 和 Probability 的区别。这两个单词虽然中文均指可能性，但是在数学考试中，前者能见到的问法大多是用 How many possibilities does EVENT have?的问句，而后者多见的是 What is the probability of EVENT?。通过疑问词的区别我们可以看出，possibility 在数学概念中指的是发生的情况的种类数目，而 probability 指的是概率，是一个介于 0 到 1（包括边界）的数字，概率越大，该事件越可能发生。比如，投掷一个六面骰，一共会有 6 possibilities，而投中“1”的 probability 是$\frac{1}{6}$。

- **Caculation of Probability（概率的计算）**

$$P(E) = \frac{\text{the number of outcomes in the event } E}{\text{the number of possible outcomes in the experiment}}$$

总体来说，一个event的probability的计算方法为，在一次random experiment中，该事件发生的possibilities和所有的possibilities之比。

概率遵循如下特征:

A. 概如果事件A一定发生，P(A) = 1.

B. 如果事件A一定不发生, P(A) = 0.

C. 如果时间A不一定发不发生, 0 < P(A) < 1.

D. 事件A不发生的概率为 1 - P (A).

E. 如果A为随机事件，那么A发生的概率为所有在A中所发生的结果的概率之和 .

F. 一个随机试验中，所有事件的概率之和为1.

G. P（A∪B）=P（A）+P（B）- P（A∩B）

H. 如果事件A和B mutually exclusive（互斥），P（A∩B）=0，也就意味着P（A∪B）=P（A）+P（B）

I. 如果A和B是independent（独立）的，那么P(A∩B)=P(A)P(B)，也就意味着P（A∪B）=P（A）+P（B）- P(A)P(B)

4.4.4 Exercises

1. If an integer is randomly selected from all positive 2-digit integers, what is the probability that the integer chosen has

a) a 4 in the tens place?
b) at least one 4 in the tens place or the units place?
c) no 4 in either place?

Answer:
(a) 1/9
(b) 1/5
(c) 4/5

2. In a box of 10 electrical parts, 2 are defective.

a) If you choose one part at random from the box, what is the probability that it is not defective?
b) If you choose two parts at random from the box, without replacement, what is the probability that both are defective?

Answer:
(a) 4/5
(b) 1/45

3. The table shows the distribution of a group of 40 college students by gender and class.

	Sophomores	Juniors	Seniors
Males	6	10	2
Females	10	9	3

If one student is randomly selected from this group, find the probability that the student chosen is

a) not a junior
b) a female or a sophomore

c) a male sophomore or a female senior

Answer:

(a) 21/40

(b) 7/10

(c) 9/40

4. Let A,B,C, and D be events for which P(A or B)=0.6, P(A)=0.2, P(C or D)=0.6, and P(C)=0.5. The events A and B are mutually exclusive, and the events C and D are independent.

a) Find P(B)

b) Find P(D)

Answer:

(a) 0.4

(b) 0.2

5. Lin and Mark each attempt independently to decode a message. If the probability that Lin will decode the message is 0.80 and the probability that Mark will decode the message is 0.70, find the probability that

a) both will decode the message

b) at least one of them will decode the message

c) neither of them will decode the message

Answer:

(a) 0.56

(b) 0.94

(c) 0.06

6.

If an integer greater than 100 and less than 1,000 is to be selected at random, what is the probability that the integer selected will be a multiple of 7 ?

- ○ $\frac{142}{999}$
- ○ $\frac{142}{900}$
- ○ $\frac{142}{899}$
- ○ $\frac{128}{900}$
- ○ $\frac{128}{899}$

Answer： E

4.5 Distributions of Data, Random Variables, and Probability Distributions 数据、随机变量和概率的分布

4.5.1 Objectives

A. 了解各类分布的特征
B. 了解正态分布和标准正态分布的各项指标的计算方法
C. 熟练掌握 Z-score 的标准化计算

4.5.2 Key terms' definition

random variable 随机变量:

- In data analysis, variables whose values depend on chance play an important role in linking distributions of data to probability distributions. Such variables are called random variables.

distribution curve 分布曲线:

- The distribution can be modeled by a smooth curve that is close to the tops of the bars in the bar charts or histograms, and if the vertical scale can be adjusted (stretched or shrunk) so that the sum of the areas of the bars is 1. With this vertical scale adjustment, the area under the curve that models the distribution is also 1. This model curve is called a distribution curve, but it has other names as well, including density curve and frequency curve.

discrete random variable 离散随机变量:

- The variables that their values consist of discrete points on a number line.

uniform distribution 均匀分布:

- The probability is distributed uniformly over all possible outcomes.

normal distribution 正态分布:

- Many natural processes yield data that have a relative frequency distribution shaped somewhat like a bell, as in the distribution below with mean *m* and standard deviation *d*.

continuous random variable 连续随机变量:

- The area of the region under the curve is 1, and the areas of vertical slices of the region—similar to the bars of a histogram—are equal to probabilities of a random variable associated with the distribution. Such a random variable is called a continuous random variable.

standard normal distribution 标准正态分布:

- The standard normal distribution is a normal distribution with a mean of 0 and standard deviation equal to 1.

expected value 期望

4.5.3 Strategies

- **Distribution of Data（数据分布）**

A. 分布曲线的作用在于，抛开数据的分类不看，通过观察曲线，该组数据大体的分布趋势可以被直观地观测到。随意确定一个横轴上的区间，这个区间内所对应的去线下的面积代表着，在该组数据内，数据位于该区间的概率。

B. 为了更精确地表达出数据的分布，在分布曲线的横轴上均值（m）和中位数（M）以及标准差（d）会被表达出来。并且有多少数据回落在 m±3d 这个区间内一般被认为是衡量一组数据是否集中地判断标准之一。

C. 中位数（M）根据其定义会将一组数据分为小的部分和大的部分两个等份。而均值（m）则是将所有数据都纳入计量后得出的中间水平，所以可能大于也可能小于中位数。如果 m 大于 M，我们认为这组数据有一些极大的数字；反之，我们认为这组数据有一些极小的数字。

- data 所对应的 relative frequency 和 variable 所对应的 probability 是一件事，因此 variable 也会有其对应的图像，并且有均值和中位数。

- **Expected value （期望）**： Variable X的均值可以被称为**mean of the random variable X**，也可以称为**expected value**，记做**E(X)**。期望为离散型随机变量的一切可能的取值xi与对应的概率pi乘积之和.

$$E(X)=\sum_{i=1}^{n} x_i \cdot p_i$$

- **Normal Distribution（正态分布）**

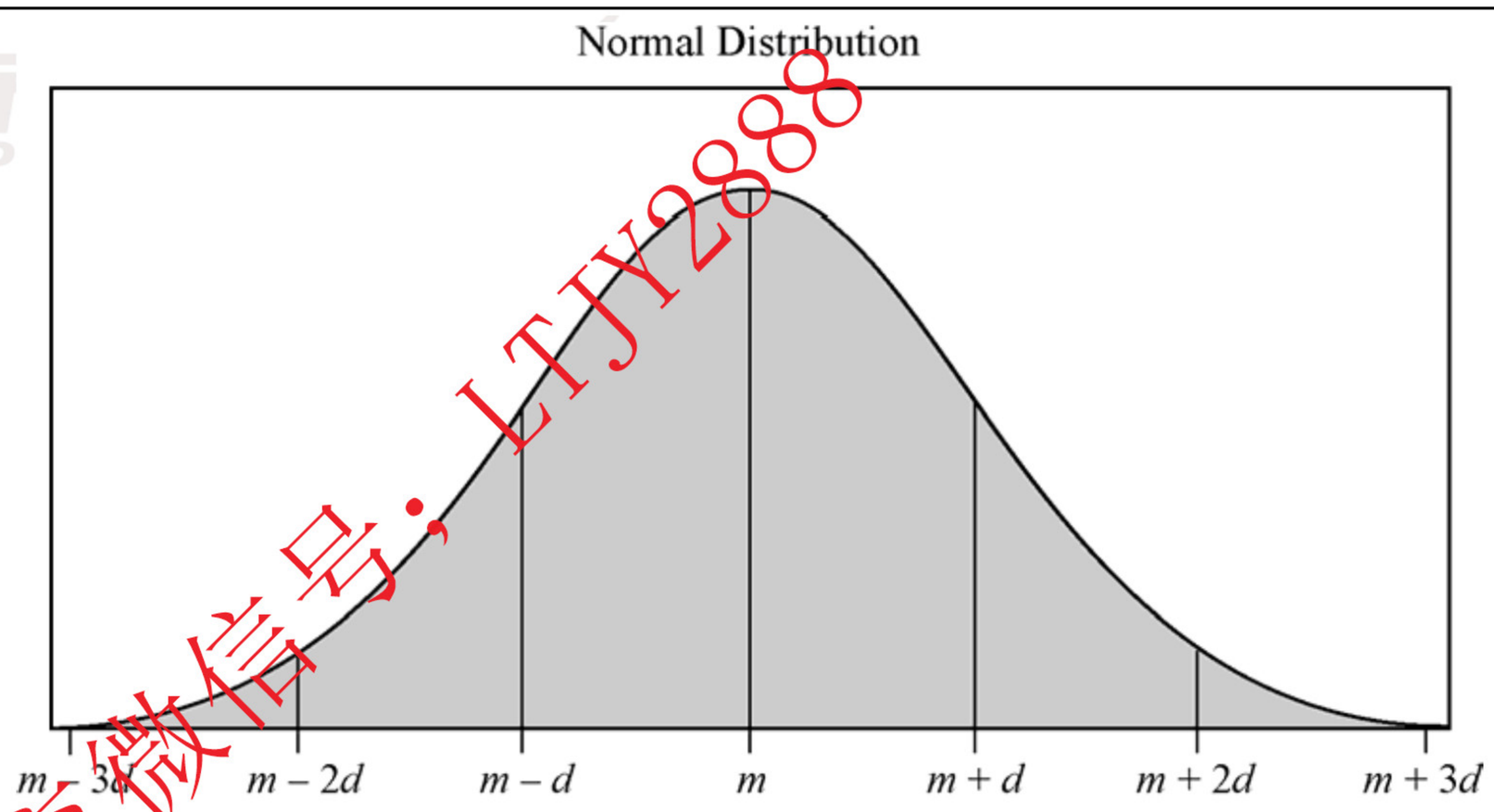
A. 正态分布的均值、中位数和众数是近乎相同的

B. 68-95-99.7法则：在正态分布中，有68%的数据落入m±d的区间内，有95%的数据落入m±2d的区间内，有99.7%的数据落入m±3d的区间内

C. 一组变量的均值决定了正态分布的中心位置，而标准差决定了他的陡峭程度

D. 正态分布适用于连续变量，而在连续变量中，取到具体一个值的概率为0

E. 正态分布线下的面积为1，任意横轴区间内所对应的线下面积，为变量的值取到该区间内的概率

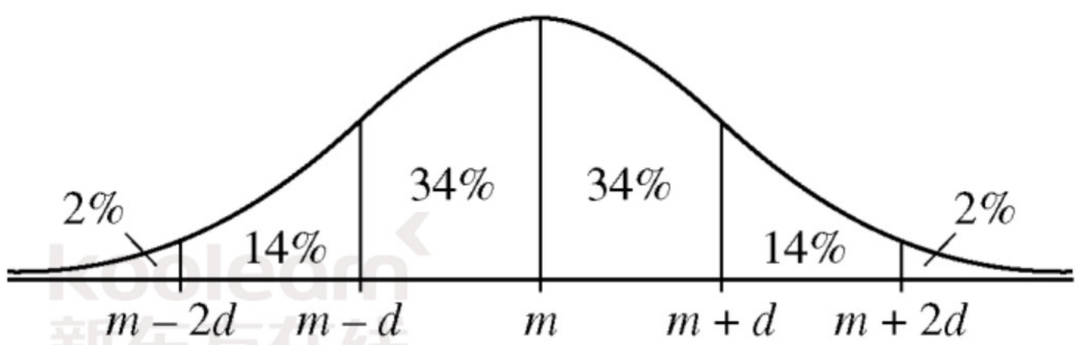


- **Standard Normal Distribution（标准正态分布）**

标准正态分布指均值为0，标准差为1的正态分布，任何正态分布都可以通过对其变量进行standardization（标准化）的变换而转换到标准正态分布。

4.5.4 Exercises

1.



The figure shows a normal distribution with mean m and standard deviation d, including approximate percents of the distribution corresponding to the six regions shown.

Suppose the heights of a population of 3,000 adult penguins are approximately normally distributed with a mean of 65 centimeters and a standard deviation of 5 centimeters.

(a) Approximately how many of the adult penguins are between 65 centimeters and 75 centimeters tall?

(b) If an adult penguin is chosen at random from the population, approximately what is the probability that the penguin's height will be less than 60 centimeters? Give your answer to the nearest 0.05.

Answer: (a) 1440 (b) 0.15

2.

In a distribution of the values of the variable x, the 50th percentile is 48.5 and the 60th percentile is 56.5.

Quantity A	Quantity B
The 40th percentile of the distribution of the values of x	40.5

Answer: D

Appendix 常见英文单位表达与换算

(1) 金钱单位

cent 一美分（硬币）
penny 一美分（硬币）
nickle 五美分（硬币）
dime 十美分/角（硬币）
quarter 二十五美分（硬币）
dollar 美元

(2) 数量单位

half 一半，二分之一
quarter 四分之一
dozen 打（12 个）
score 廿（20 个）

(3) 温度单位

Centigrade 摄氏
Fahrenheit 华氏
F = C * 1.8 + 32

(4) 体积单位

liter 升 l
milliliter 毫升 ml
gallon 加仑 gal
quart 夸脱 qt
pint 品脱 pt （干量或液量的单位）
gill 及耳 gi
ounce 盎司 oz （小额重量单位，液体，贵金属）

(5) 长度单位

meter 米 m
kilometer 千米 km
decimeter 分米 dm
centimeter 厘米 cm
millimeter 毫米 mm
inch（复数 inches）英寸 in
foot（复数 feet） 英尺 ft
yard 码 yd
mile 英里 mi
1 ft = 12 in
1 yd=3 ft
1 mi = 1760 yd

(6) 质量单位

gram 克 g
kilogram 千克 kg
ton 吨 t

pound 磅 lb

ounce 盎司 oz

1 pound = 0.454 kg

Vocabulary

abscissa 横坐标
absolute value 绝对值
acute angle锐角
add 加
adjacent angle 邻角
algebraic term 代数项
alternate interior angle 内错角
angles 角
arc 弧
area 面积
arithmetic mean 算术平均数
axis 轴（圆柱中心线）
bar chart 柱状图
bar graph 柱状图
base 底
base 底数
bisect 平分
bisector平分线
bivariate 双变量的
cent 一美分（硬币）
center 圆心
Centigrade摄氏
centimeter 厘米 cm
central angle 圆心角
central tendency 中心趋势
chord 弦
circle graph 饼状图
circle 圆
circular cylinder 圆柱体
circumference 圆的周长
circumscribe 外接 / 外切
combination 组合
common denominator 公分母
complementary angle 余角
complementary 互余
composite number 合数
compound interest 复利
concentric circles 同心圆
conduct 实施
cone 圆锥
congruent angles 等角

congruent line segments 全等线段
congruent triangle 全等三角形
consecutive number 连续数
constant 常数
continuous random variable 连续随机变量
coordinate system 坐标系
Corresponding angle 同位角
count 频数
cube root 立方根
cube 正方体
cylinder 圆柱
decagon 十边形
decimal 小数
decimeter 分米 dm
denominator 分母
diagonal对角线
diagram 图
diameter 直径
dice 骰子
die 骰子
difference 差
digit 数位
dime 十美分/角（硬币）
dimension 维度
directly proportional to 正比于
discount 折扣
discrete random variable 离散随机变量
disjoint 互斥
dispersion 扩散
distinct 不同的
distribution curve 分布曲线
distribution of data 数据分布
divide 除
divisible 可被整除的
divisor 因数
dollar 美元
domain 定义域
dozen 打（12个）
edge 棱
element 元素
elimination: 消元法
empty set 空集
endpoint 端点
equation 等式，方程

equilateral triangle 等边三角形
equivalent equations 等价方程式
even integer 偶数
event 事件
expected value 期望
exponent 指数
exterior angle 外角
externally tangent 外切
face 面
factor 因数
factorial 阶乘
factoring 因式分解
Fahrenheit 华氏
feet 英寸（复数）
Figure not drawn to scale 图形不按比例绘制
finite set 有限集合
foot 英尺 ft
formula 公式
fourth root 四次方根
fraction 分数
frequency distribution 频数分布
frequency 频数
function 函数
gallon 加仑 gal
gill 及耳 gi
global 球体
gram 克 g
greater than or equal to 大于等于
greater than 大于
greatest common divisor 最大公因数
greatest common factor 最大公因数
half 一半，二分之一
height 高
hexagon 六边形
histogram 直方图
hundreds digit 百位
hundredths digit 百分位
hypotenuse 斜边
identity 恒等式
improper fraction 假分数
inch 英寸 in
inches 英寸（复数）
inclusive 包含的
independent 独立

individual 个体
Inequalities 不等式
infinite set 无限集合
inscribe 内接 / 内切
integer 整数
intercept 截距
interest rate 利率
interest 利息
interior angle 内角
interior angles on the same side 同旁内角
internally tangent 内切
interquartile range 四分位差
intersect 相交
intersection 交集
interval 区间
inversely proportional to 反比于
irrational number 无理数
isosceles triangle 等腰三角形
kilogram 千克 kg
kilometer 千米 km
lateral surface 侧面（圆柱侧曲面）
least common multiple 最小公倍数
leg 直角边
length 长
length 长度
less than or equal to 小于等于
less than 小于
like terms 同类项
line of symmetry 对称轴
line segment 线段
linear equation 线性方程
lines 直线
list price 标价
list 数列
liter 升 l
literal coefficient 字母系数
margin 利润
mark down 降价
mark up 涨价
mean
median 中位数
member 元素
meter 米 m
midpoint 中点

mile 英里 mi
milliliter 毫升 ml
millimeter 毫米 mm
minus 减
mixed number 带分数
mode 众数
more than 大于
multiple 倍数
multiply 乘
mutually exclusive 互斥
negative number 负数
nickel 五美分（硬币）
nonempty 非空
normal distribution 正态分布
number line数轴
numerator 分子
numerical coefficient 数字系数
object 个体
obtuse angle钝角
octagon 八边形
odd integer 奇数
ones digit 个位
opposite angles 对顶角
ordinate 纵坐标
origin 原点
original price 原价
ounce 盎司 oz （小额重量单位，液体，贵金属）
outlier 异常值
parabola 抛物线
parallel 平行
parallelogram 平行四边形
penny 一美分（硬币）
pentagon 五边形
percent change 百分比变化
percent decrease 百分比减
percent increase 百分比增
percent 百分比
percentiles 百分位数
perimeter 周长
permutation 排列
perpendicular 垂直
pie chart 饼状图
piecewise-defined function 分段函数
pint 品脱 pt (干量或液量的单位)

plus 加
point of tangency 切点
polygon 多边形
population standard deviation 总体标准差
population 群体，总体
positions 位置
positive number 正数
possibility 可能性
pound 磅 lb
power 次方，幂
prime divisor 质因数
prime factor 质因数
prime number 质数
principal 资本
prism 棱柱
probability experiment 随机试验
probability 概率
product 积
profit 利润
proportion 比例
proportional to 正比于
purchasing price 买价
pyramid 棱锥
Pythagorean theorem勾股定理
quadrant 象限
quadratic equation 二次方程
quadrilateral 四边形
quadruple 四倍
quantity 数量
quart 夸脱 qt
quarter 二十五美分（硬币）
quarter 四分之一
quartiles 四分位数
questionnaire 问卷
quotient 商
radian弧度
radii 半径（复数）
radius 半径
random experiment 随机试验
random selection 随机抽取
random variable 随机变量
random 随机的
range 值域
range 极差

rate 率
ratio 比例
rational numbers 有理数
ray 射线
real number line 实数轴
real number 实数
reciprocal 倒数
rectangle 长方形
rectangular coordinate system 直角坐标系
rectangular solid 长方体
regular polygon 正多边形
relative frequency distribution 频率分布
relative frequency 频率，相对频数
remainder 余数
repeat decimal 循环小数
retail value 零售价
rhomb 菱形
right angle 直角
right circular cylinder 直圆柱
right triangles 直角三角形
round to the nearest 四舍五入
sale price 卖价
sales tax 消费税
sample space 样本空间
sample standard deviation 样本标准差
scatter plot 散点图
score 廿 (20个)
sector 扇区
sector 扇形
segmented bar graph 分段柱状图
semicircle半圆
set 集合
side 边，边长
similar terms 同类项
similar triangles 相似三角形
simultaneous equations 联立方程
single interest 单利
slope 斜率
solution set 解集
sphere 球体
square root 平方根
square 平方
square 正方形
standard deviation 标准差

standard normal distribution 标准正态分布
standardization 标准化
subset 子集
substitution: 连立法
subtract 减
sum 和
supplementary angle 补角
supplementary 互补
surface area 表面积
survey 调查
symmetric 对称
system of equations 方程组
tangent line 切线
tangent 相切
tax 税
tens digit 十位
tenths digit 十分位
terminate decimal 有限小数
thousands digit 千位
thousandths digit 千分位
time plot 时间图表
time series 时间图表
time 乘
ton 吨 t
trapezoid 梯形
trend 趋势
triangle 三角形
triple 三倍
twice 两倍
undefined 不可定义的
uniform distribution 均匀分布
union 并集
units digit 个位
univariate 单变量的
universal set 全集
variable 变量
Venn Diagram 维恩图
vertex 顶点
vertex 交点
vertical angles 对顶角
vertices 顶点（复数）
vertices 交点（复数）
volume 体积
weighted mean 加权平均数

weight权重
width 宽
x-axis x轴
x-coordinate 横坐标
xy-coordinate system 平面直角坐标系
xy-plane 平面直角坐标系
yard 码 yd
y-axis y轴
y-coordinate 纵坐标